Von der unsicheren Sicherheit zur sicheren Ungewissheit

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Reliability analysis

Structural Engineering is the art and science
of moulding Materials we do not fully understand;

into Shapes
we cannot precisely analyze;

to resist Forces
we cannot accurately predict;

all in such a way
that the society at large is given no reason to suspect
the extent of our ignorance.

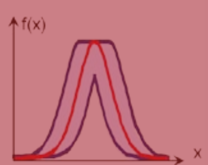
[E. H. Brown 1967]

Reliability analysis

physical model

uncertainty model

12.35
16.13
14.98
17.11
12.72
13.65
15.19



simulation

failure probabilities

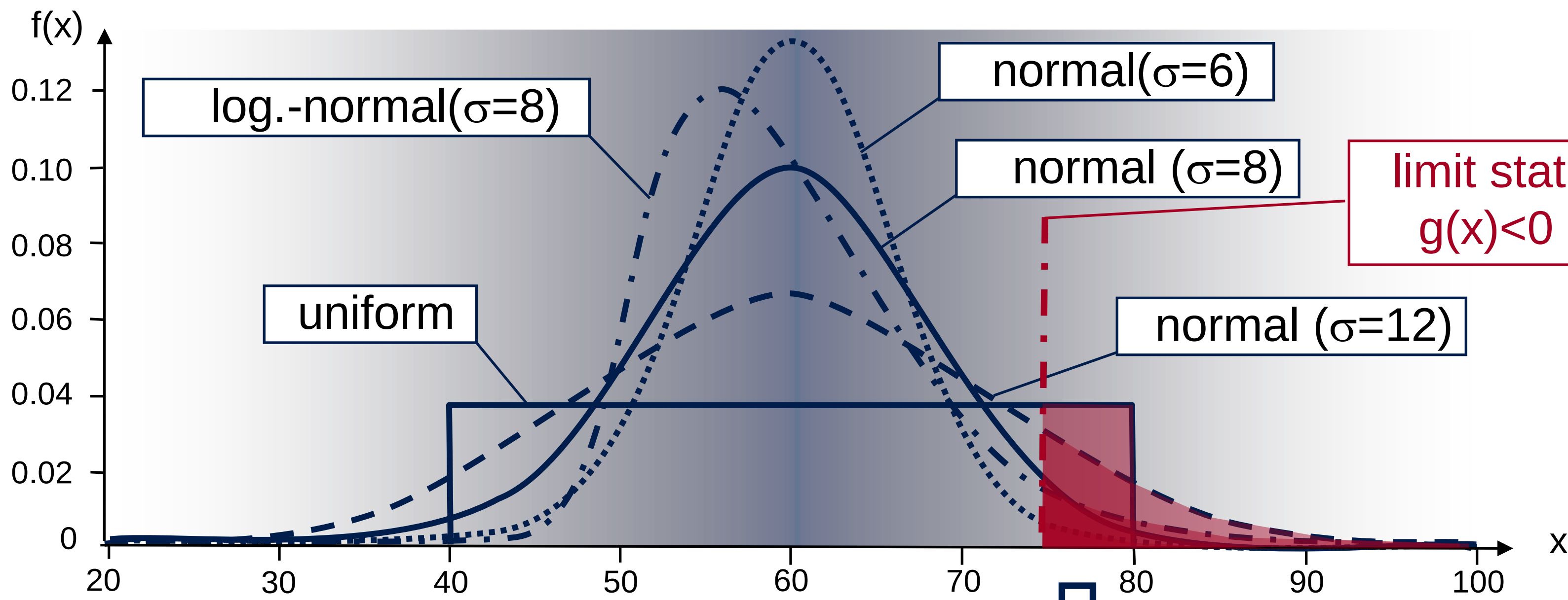
Difficulties with stochastic data models

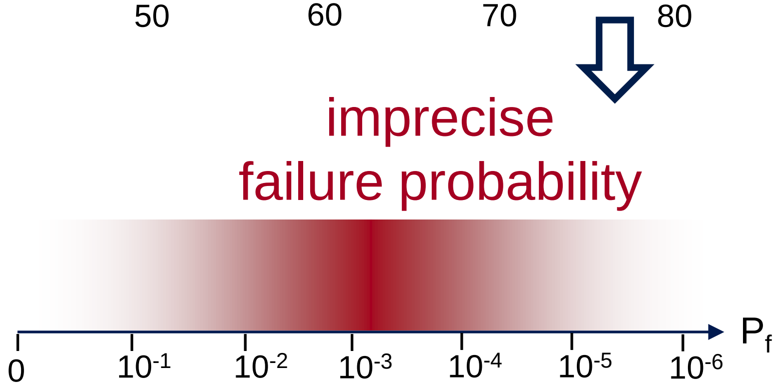
- failure probability of spoon

$$P_{f,\lim} = 10^{-5}$$

- weight of eggs

Difficulties with stochastic data models





Difficulties with stochastic data models

- failure probability of spoon

  $P_{f,\lim} = 10^{-5}$

- weight of eggs

o assumption:

1. assumption: normal distribution

$\mu = 60g$, $\sigma = 10\% \cdot \mu$

$\mu = 60g$, $\sigma = 12\% \cdot \mu$

$\mu = 64g$, $\sigma = 12\% \cdot \mu$

$w_{egg} = 85.7g$

→ $P_f = 1.0 \cdot 10^{-5}$ → $P_f = 1.78 \cdot 10^{-4}$ → $P_f = 1.29 \cdot 10^{-3}$

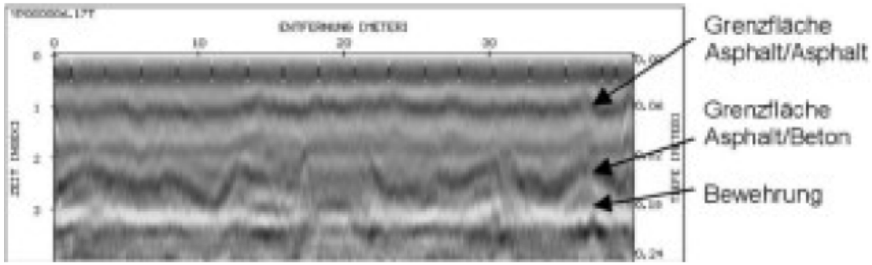
Imprecision

- measurements

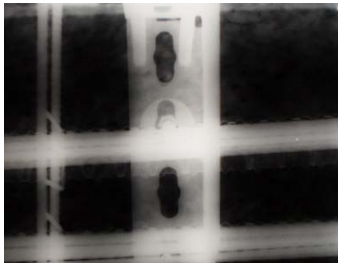
[www.digitalwaagen.de]

[www.wikipedia.de]

[Altmann]



[Radargramm – Maierhofer et al. 2000]



[Kaschmierzeck et al. 1999]

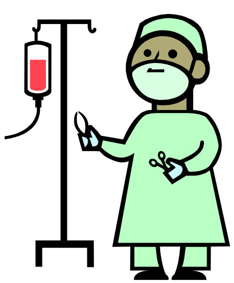
Linguistic assessment / expert knowledge

- value of objects

- loss due to shadow economy

- raised body temperature

- time of death



That ignorance of the cause of variation does not make variation random

A.M. Freudenthal

Overview

uncertainty

aleatoric	epistemic
randomness	perception/evaluation
objective data	subjective assessment
• abundant (objective) data • mass phenomena • discrete random events • gambling	• limited (subjective) data • imprecise measurements • linguistic quantification • experience/expert knowledge

Random variable

- definition

$$(\Omega, \Sigma, P) \qquad (\mathbb{R}, \mathcal{B}(\mathbb{R}))$$

$$X : \Omega \to \mathbb{R}$$

$$P : \Sigma \to [0;1]$$

$$P_X : I \to P(X^{-1}(I))$$

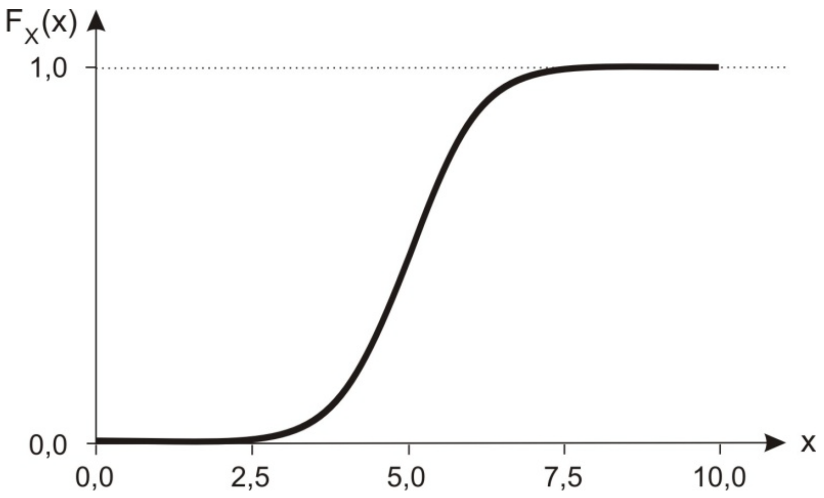
- example

many deterministic data

1	34,8
2	35,1
3	35,0
⋮	⋮
n	34,9

- representation

$$F_X(x) = P_X((-\infty; x])$$

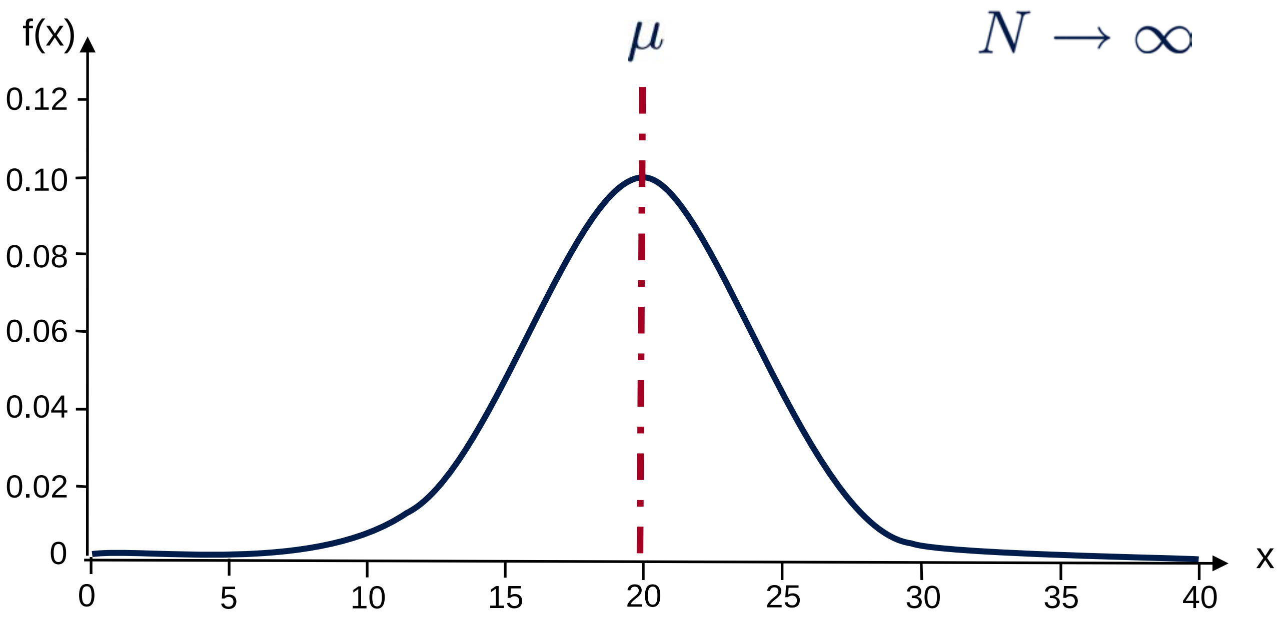


→ variability

Limits of stochastic models

sample size

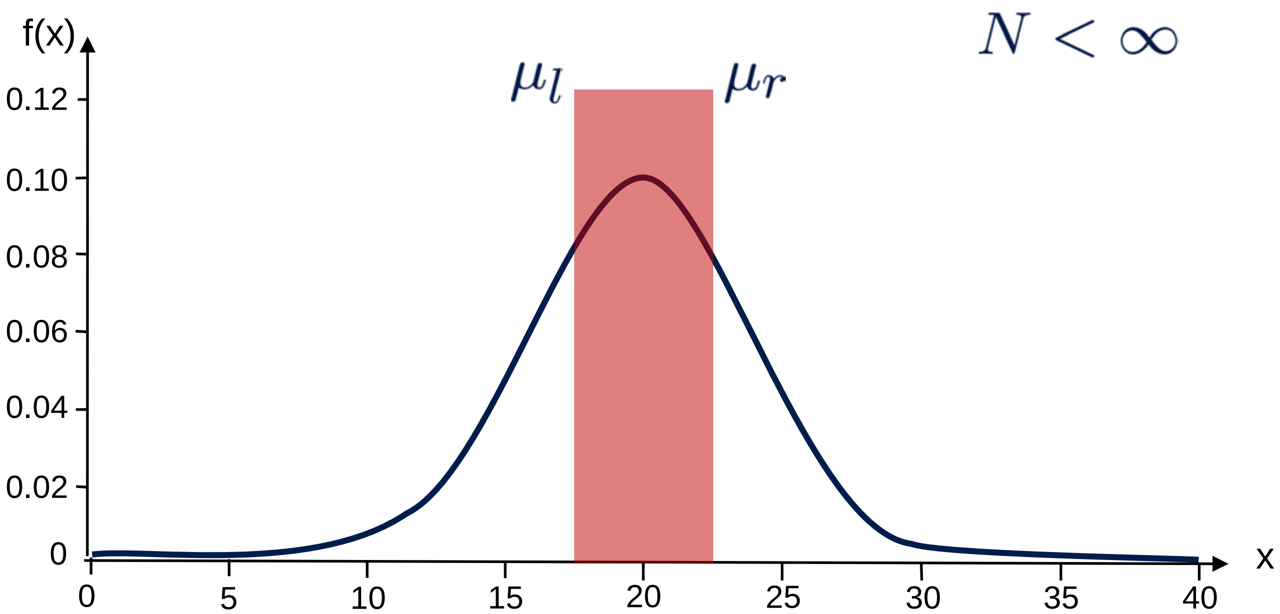
(confidence interval)



Limits of stochastic models

sample size

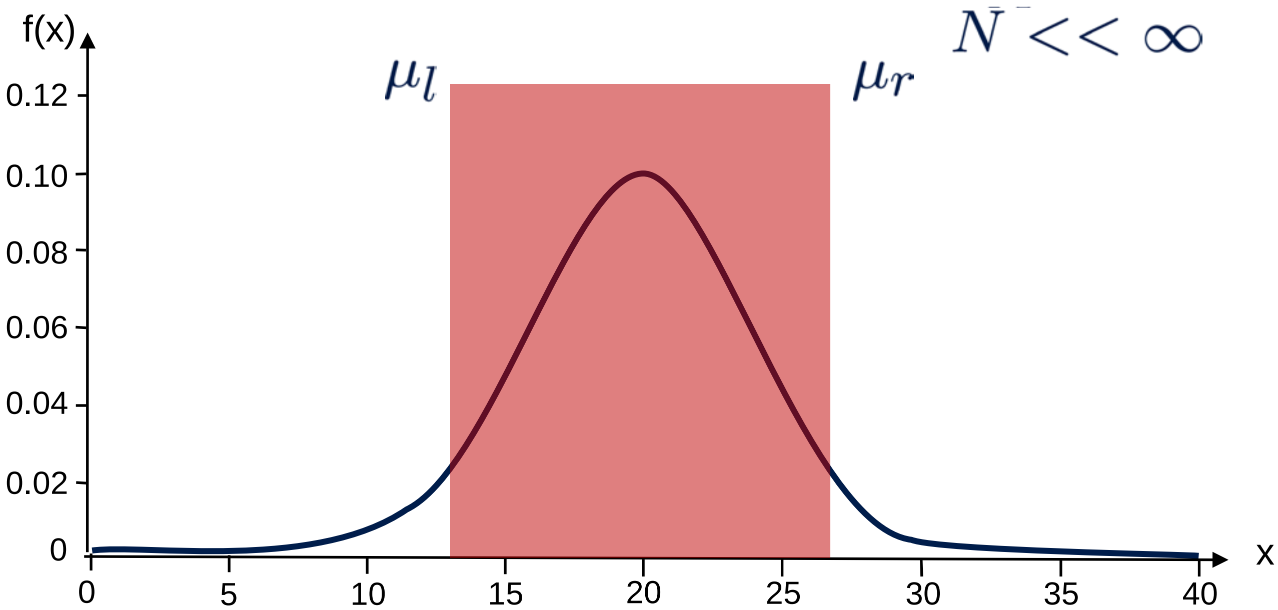
(confidence interval)



Limits of stochastic models

sample size

(confidence interval)



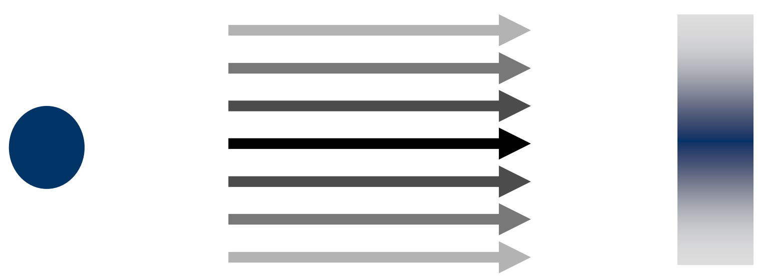
Limits of stochastic models

sample size

i.i.d. paradigm

Limits of stochastic models

- sample size
- i.i.d. paradigm
- imprecision



$X(\omega) \in \mathbb{R}$ $\quad$ $X(\omega) \in \mathcal{F}(\mathbb{R})$

Limits of stochastic models

- sample size
- i.i.d. paradigm
- imprecision

⇩

single distribution function ???

Overview

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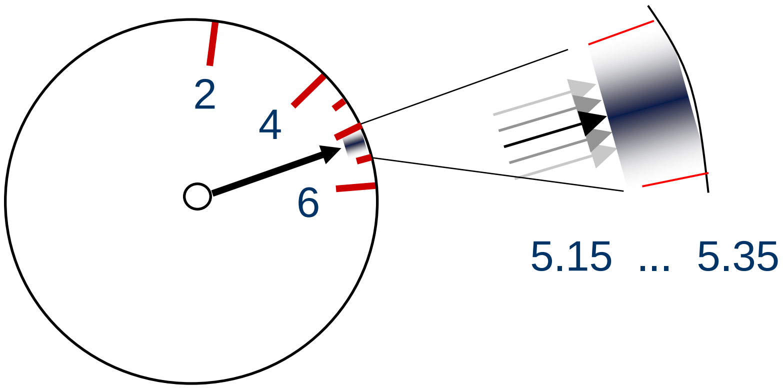
- discrete random events
- gambling

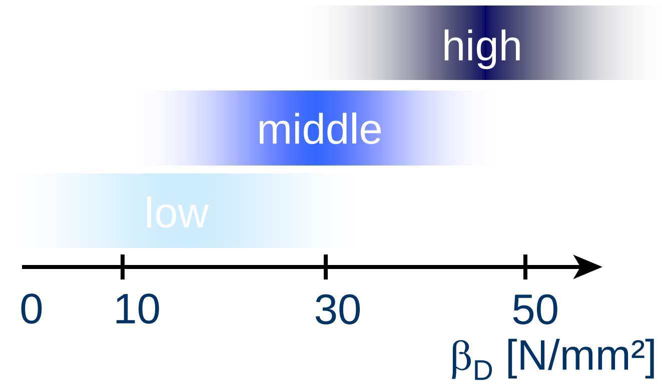
- limited (subjective) data
- imprecise measurements
- linguistic quantification
- experience/expert knowledge

Subjective information

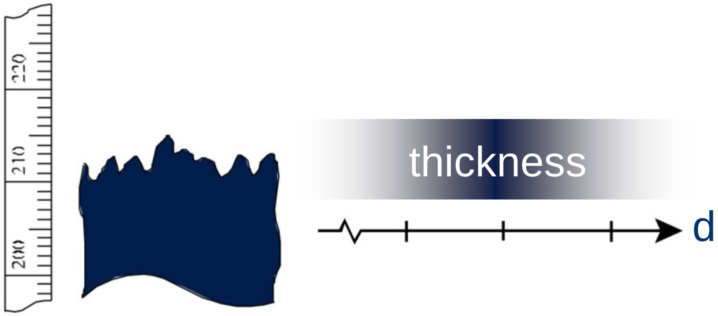
uncertainty of measurements



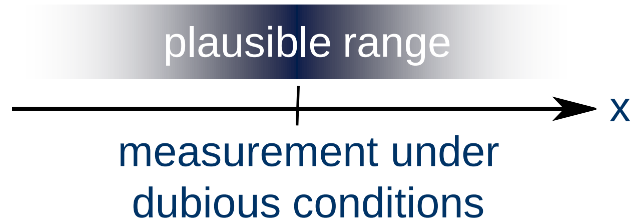
linguistic assessment



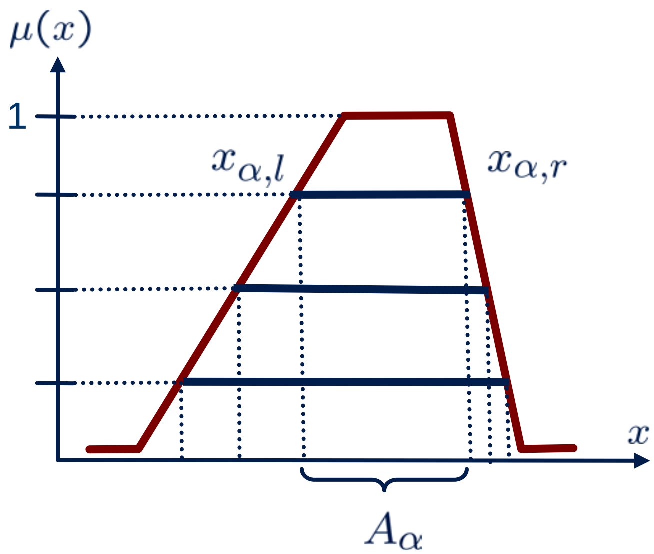
imprecise measure points



experience, expert knowledge



Fuzzy set representation for simulation



- general definition

$$\tilde{A} = \left\{ \left(x, \mu_{\tilde{A}}(x) \right) \mid x \in \mathbb{R}, \mu_{\tilde{A}}(x) \in [0,1] \right\}$$

$$\mu_{\tilde{A}} : \mathbb{R} \rightarrow [0,1]$$

- convexity

$$\forall \lambda \in [0,1],\ x_1, x_2 \in \mathbb{R} :$$

$$\mu_{\tilde{A}}\left(\lambda x_2 + (1-\lambda) x_1\right) \geq \min\left(\mu_{\tilde{A}}(x_1)\, \mu_{\tilde{A}}(x_2)\right)$$

- α-level discretization

$$\tilde{A} = (A_\alpha \mid \alpha \in [0,1])$$

$$A_\alpha = \left\{ x \in \mathbb{R} \mid \mu_{\tilde{A}} \geq \alpha \right\}$$

$$A_{\alpha_i} \subseteq A_{\alpha_k} \quad \forall\, \alpha_i \geq \alpha_k$$

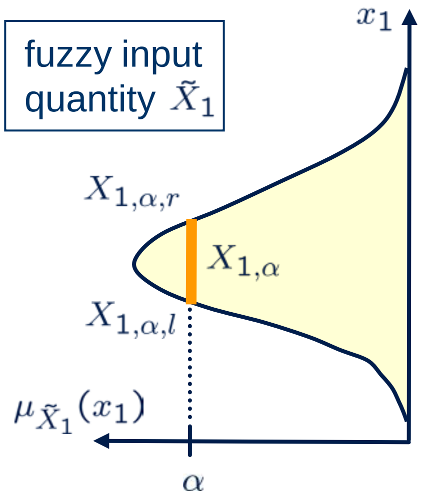
- α-level optimization

$$z_{\alpha,l} = \min_{x \in A_\alpha} f(x)$$

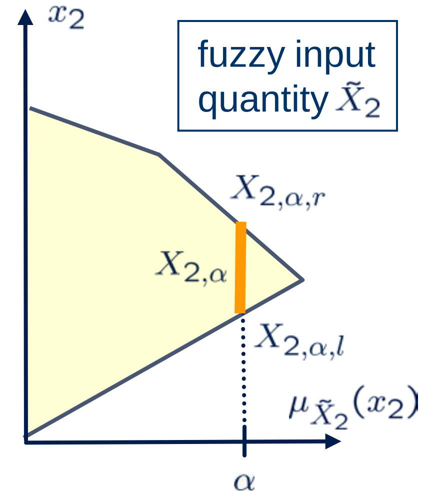
$$z_{\alpha,r} = \max_{x \in A_\alpha} f(x)$$

➡ imprecision, incompleteness, ...

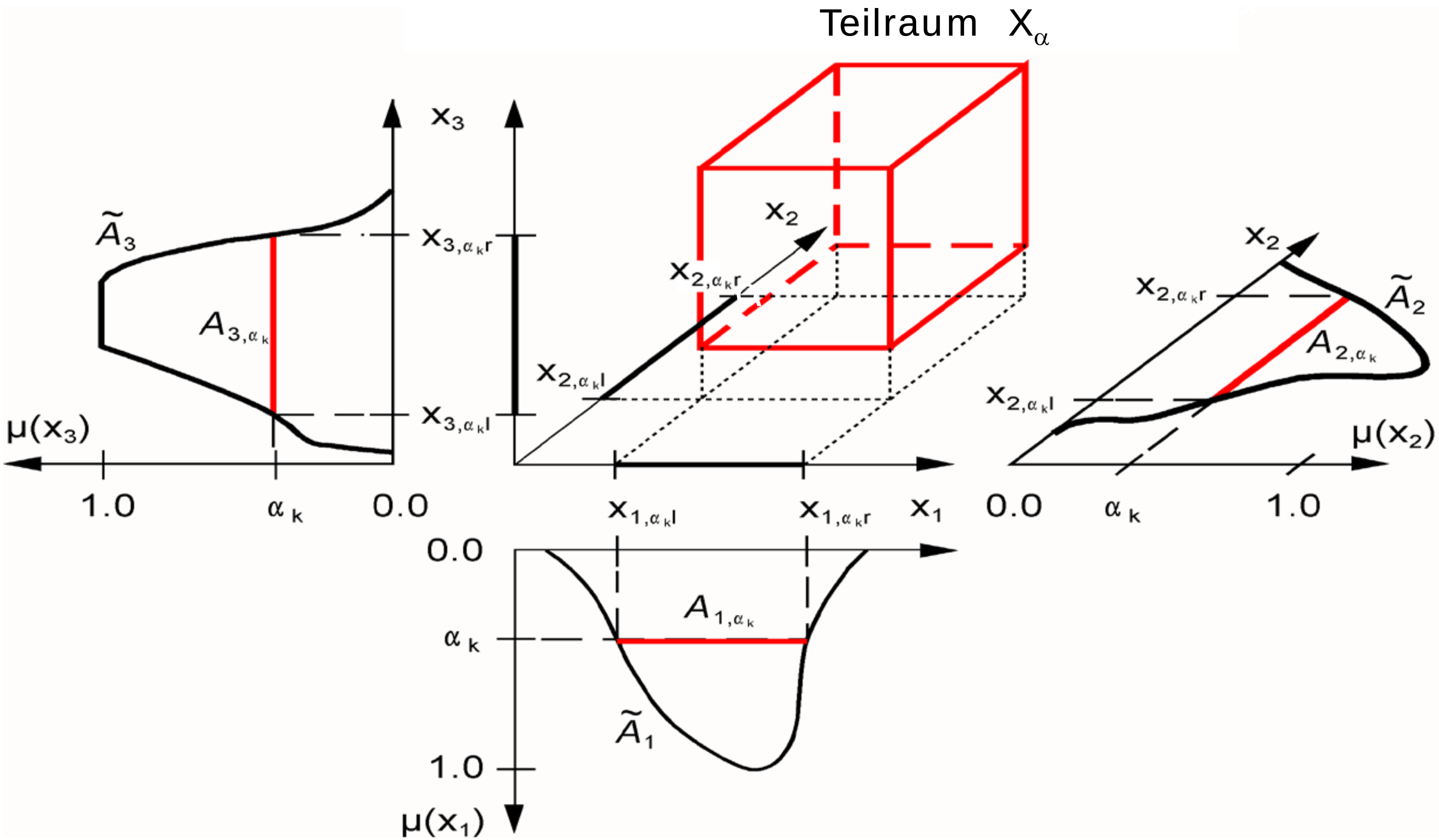
Fuzzy structural analysis



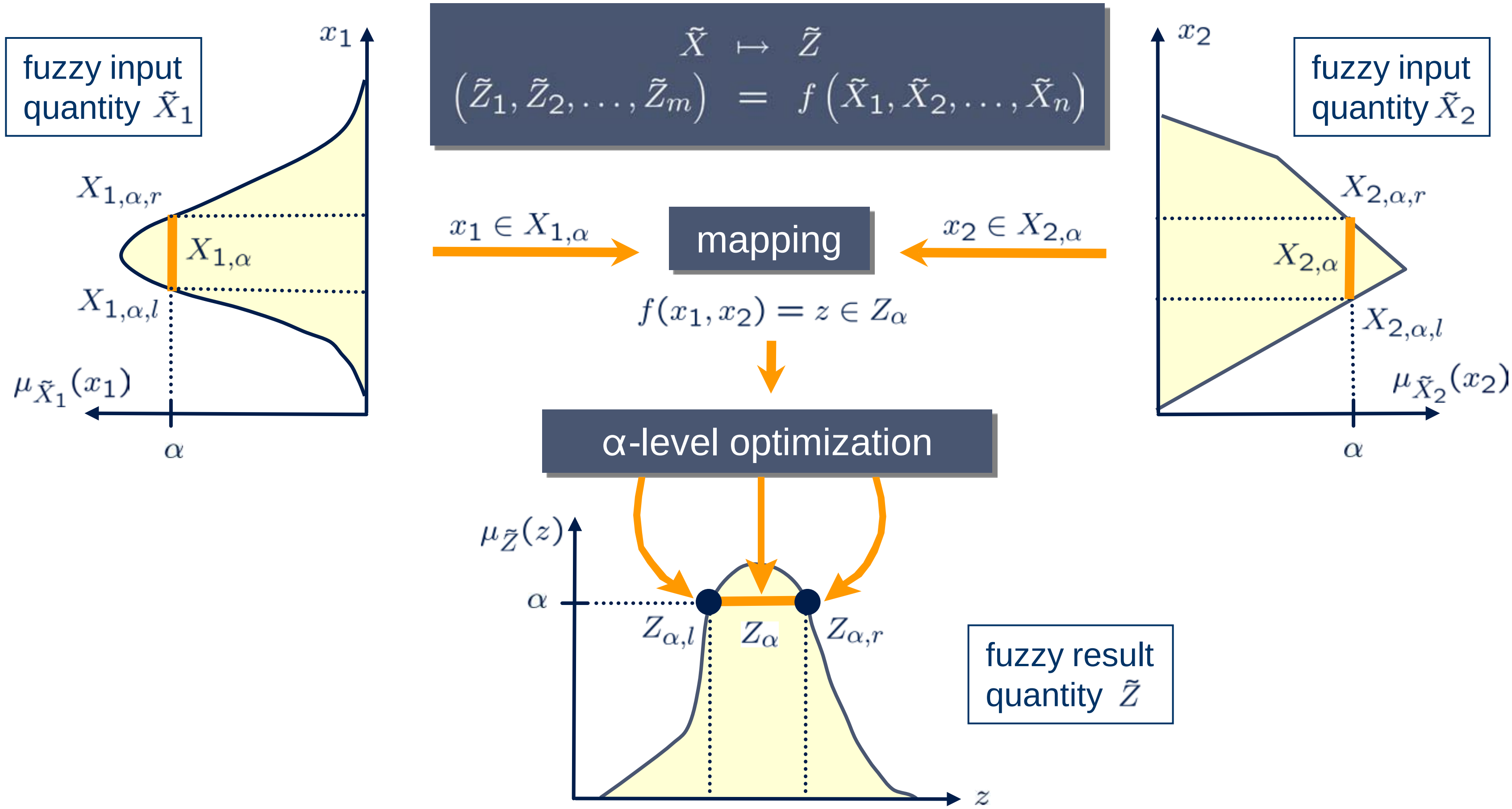
$$\tilde{X} \mapsto \tilde{Z}$$
$$\left(\tilde{Z}_1, \tilde{Z}_2, \ldots, \tilde{Z}_m\right) = f\left(\tilde{X}_1, \tilde{X}_2, \ldots, \tilde{X}_n\right)$$



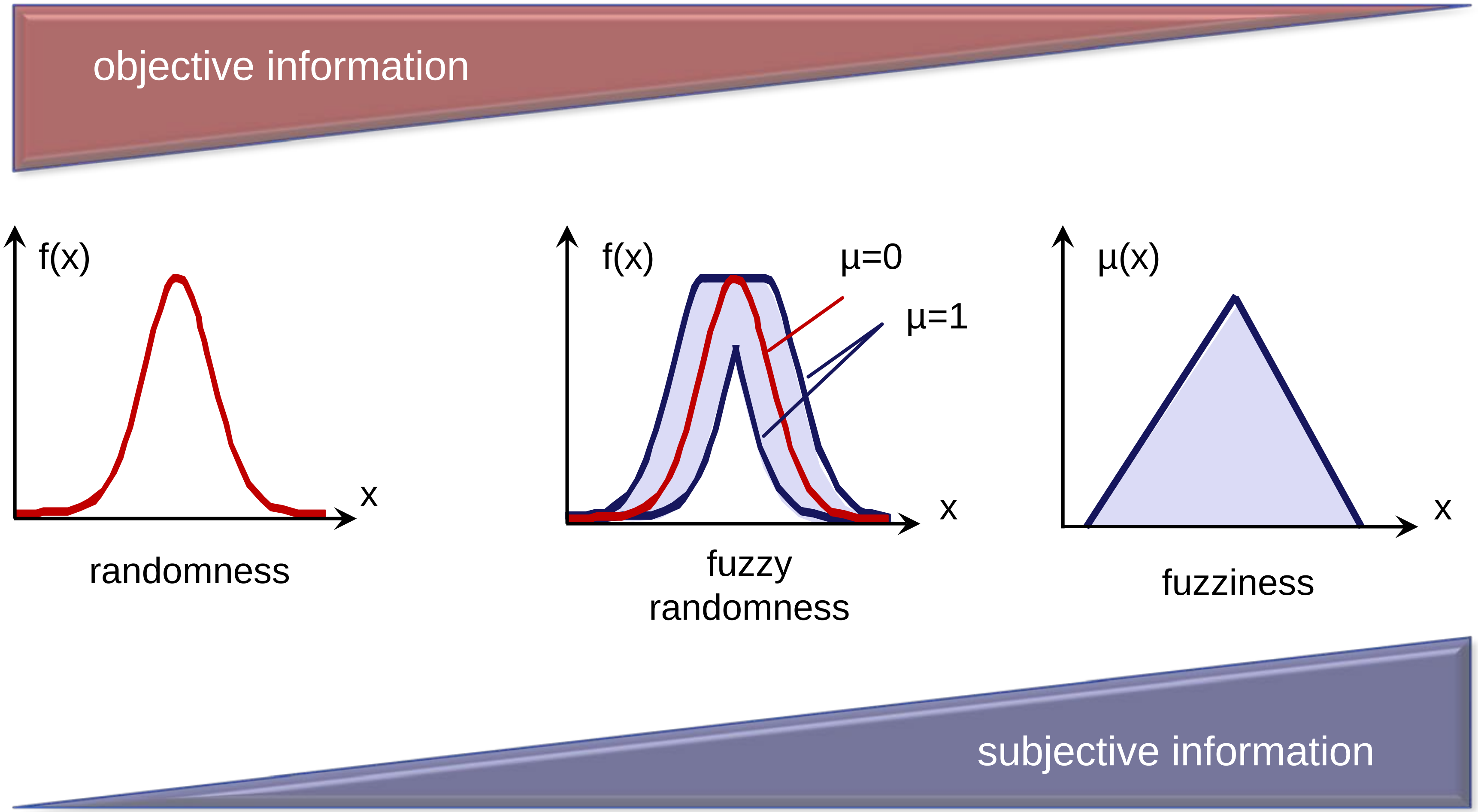
Fuzzy structural analysis



Fuzzy structural analysis



Fuzzy randomness



Overview

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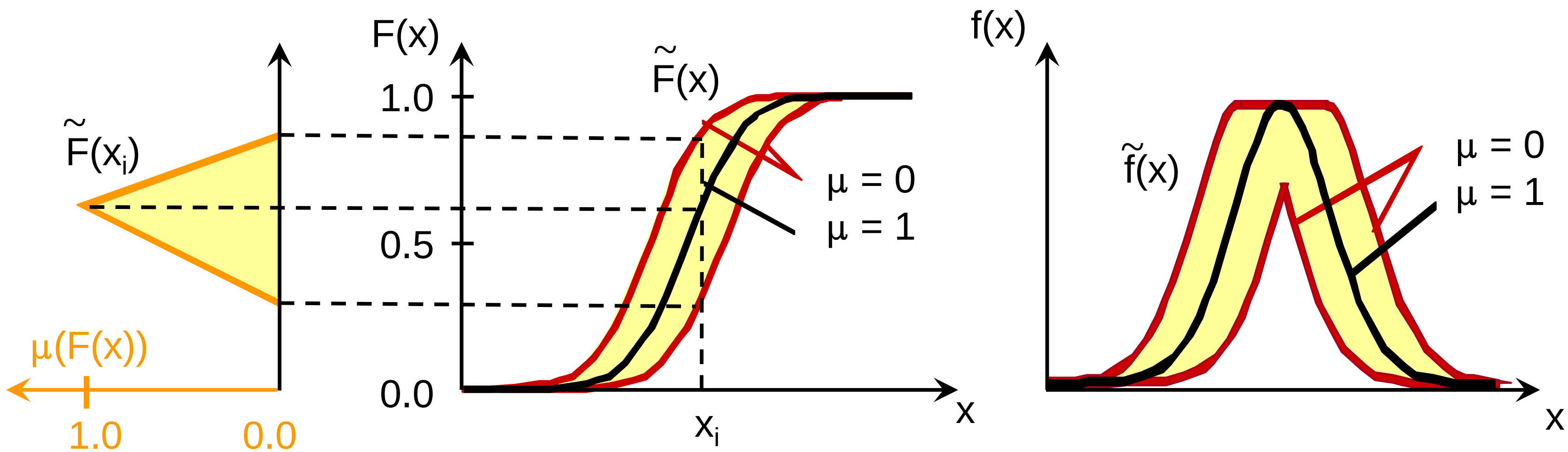
Fuzzy randomness

example: normal distribution with imprecise parameters

$$F(x) = \frac{1}{\tilde{\sigma}\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(-\frac{1}{2}\left(\frac{t-\tilde{\mu}}{\tilde{\sigma}}\right)^2\right) \mathrm{d}t\,.$$

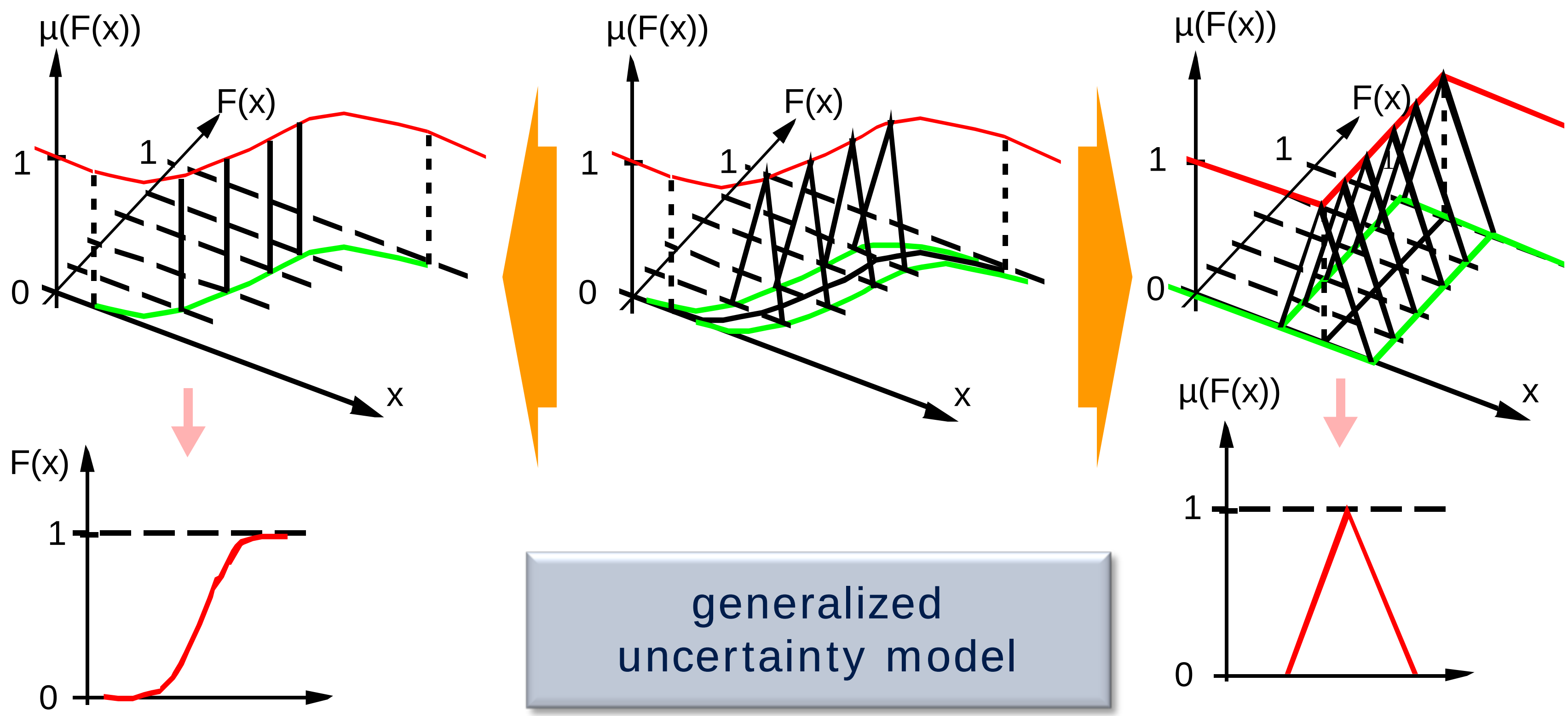
$\mu \in \tilde{\mu}$
$\sigma \in \tilde{\sigma}$



variability, imprecision, incompleteness, ...

Fuzzy randomness

random variable X | fuzzy random variable $\tilde{X}$ | fuzzy variable $\tilde{x}$



Ellsberg paradox

experiment I: 100 balls

= ? = ?

decision under ambiguity

experiment II: 100 balls

= 50 = 50

decision under risk

uncertainty model

reflection of real available information

Decision under ambiguity

reality

fuzzy input

fuzzy structural analysis

possible results

explanatory power of results corresponds to initial information

assumption

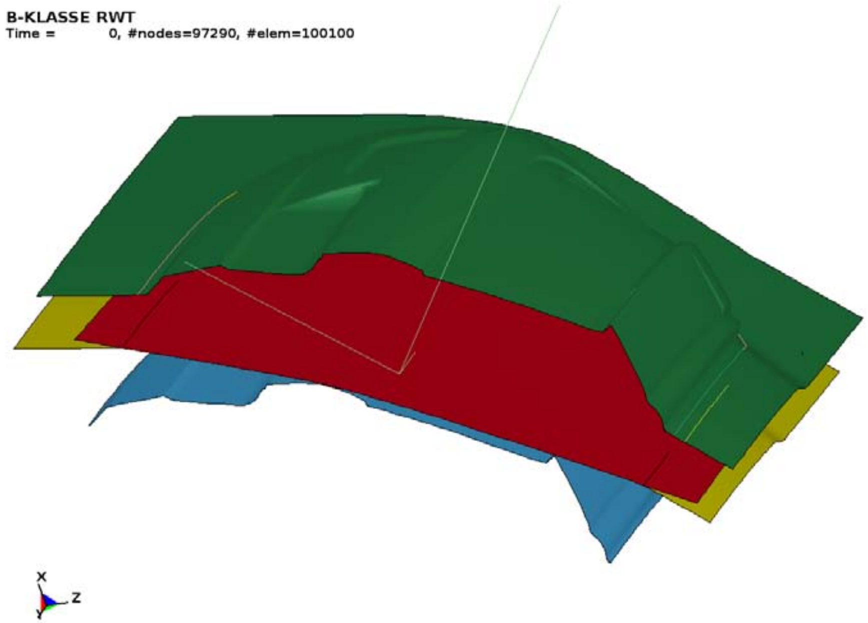
stochastic input

stochastic structural analysis

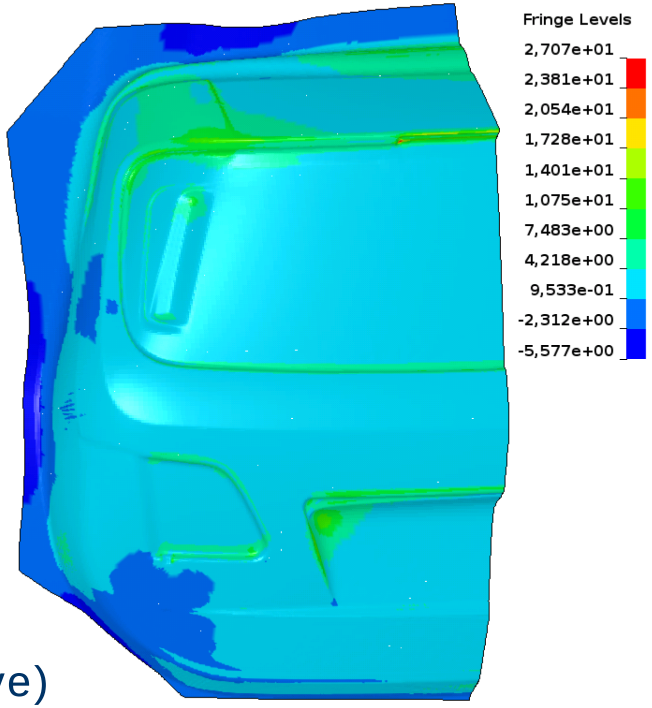
failure probability

results provide *if-then* study

Sheet metal forming



thickness reduction in %



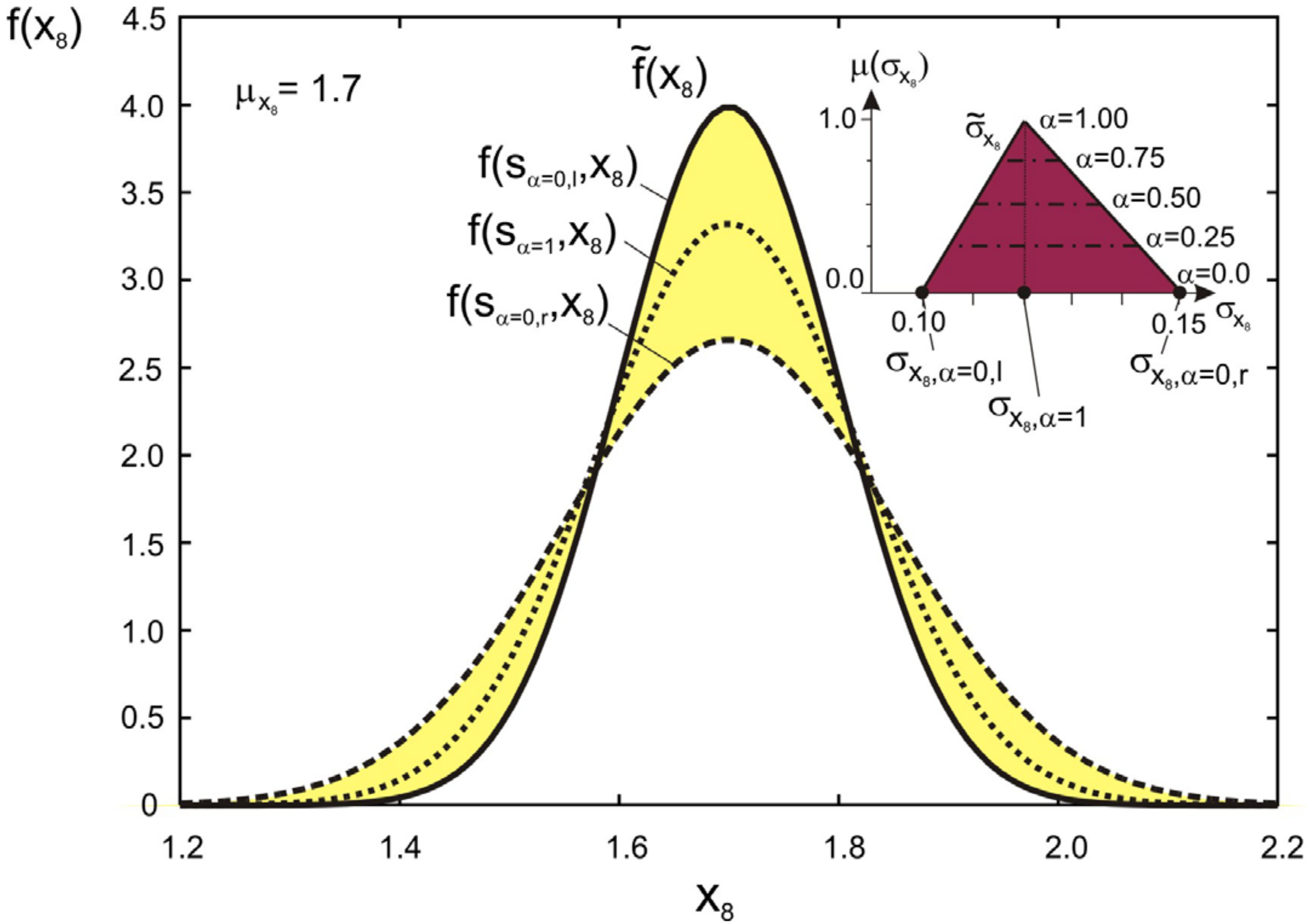
- steel grade DC06(1.0873)
- 1st step: gravity simulation (implicit)
  2nd step: stamping simulation (explicit, adaptive)
- Selective Mass Scaling (SMS)
- evaluated result quantities: shell thickness reduction

DAIMLER Mercedes-Benz by courtesy of Daimler AG

Sheet metal forming

fuzzy random quantity		normal distribution	
		mean value	standard deviation
yield strength f_y	$\tilde{X}_1$	0.14	<0.0067; 0.008; 0.01>
strength coefficient K	$\tilde{X}_2$	0.55	<0.0367; 0.044; 0.055>
hardening exponent n	$\tilde{X}_3$	<0.23; 0.275; 0.3>	0
friction coefficient μ	$\tilde{X}_4$	<0.05; 0.075; 0.1>	0
perturbation longitudinal p_1	$\tilde{X}_5$	<-0.005; 0.0; 0.005>	0
perturbation lateral p_2	$\tilde{X}_6$	<-0.005; 0.0; 0.005>	0
material parameter r_0	$\tilde{X}_7$	2.25	<0.0833; 0.1; 0.125>
material parameter r_{45}	$\tilde{X}_8$	1.70	<0.1; 0.12; 0.15>
material parameter r_{90}	$\tilde{X}_9$	2.85	<0.167; 0.14; 0.175>
draw bead force 1	$\tilde{X}_{10}$	d_2	<4; 5; 6>
:	:	:	:
draw bead force 6	$\tilde{X}_{15}$	d_7	<4; 5; 6>
binder force	$\tilde{X}_{16}$	d_1	<40; 50; 60>

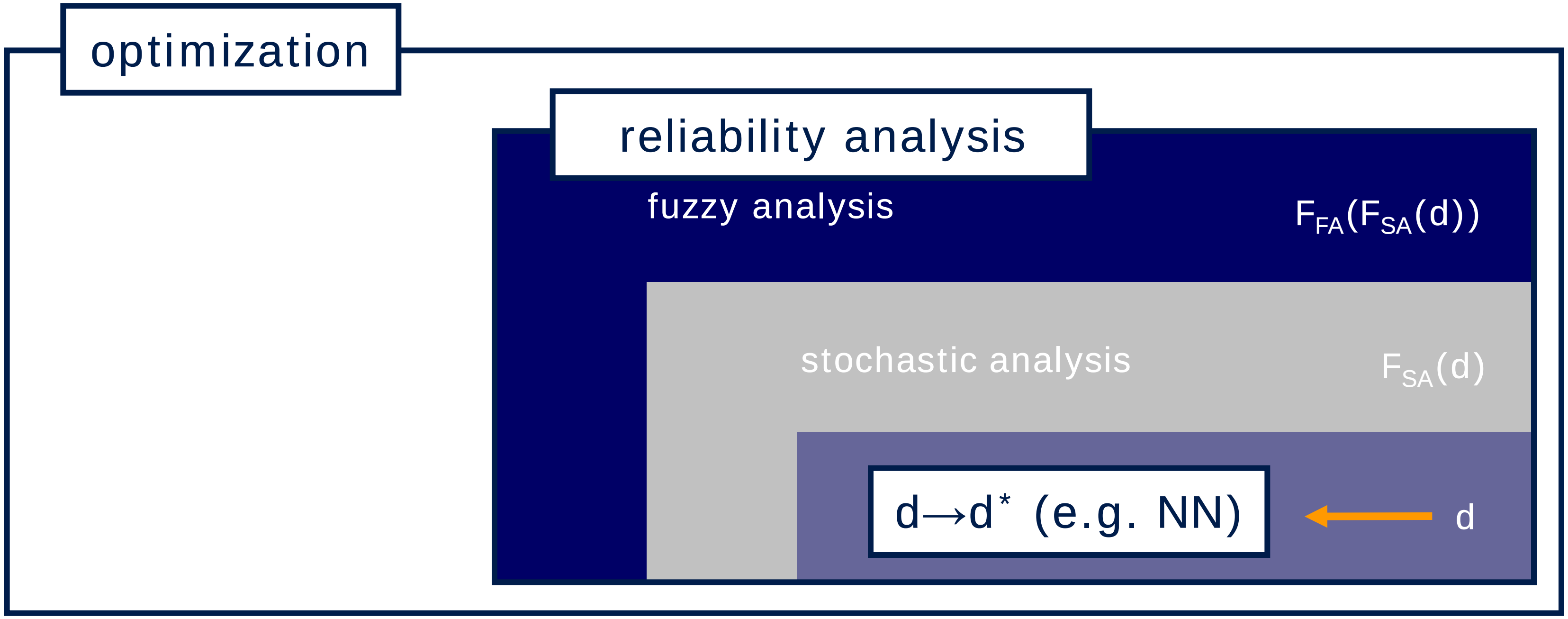
Sheet metal forming



Sheet metal forming

optimize the reliability of the production process

reliability based optimization (RBO)



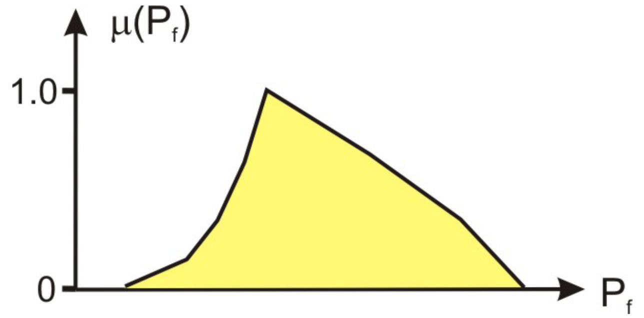
Sheet metal forming

optimize the reliability of the production process

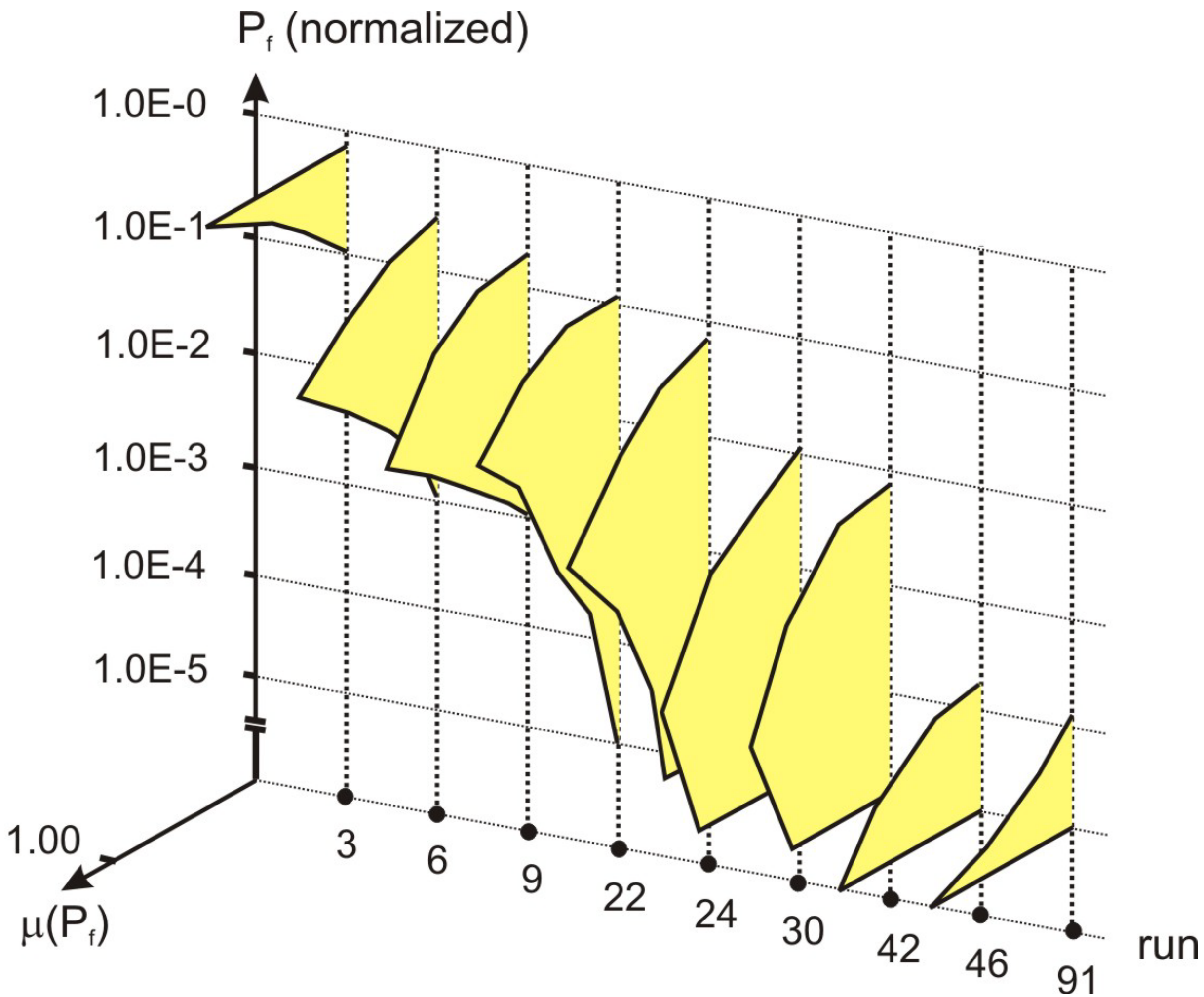
reliability based optimization (RBO)

optimization

reliability analysis

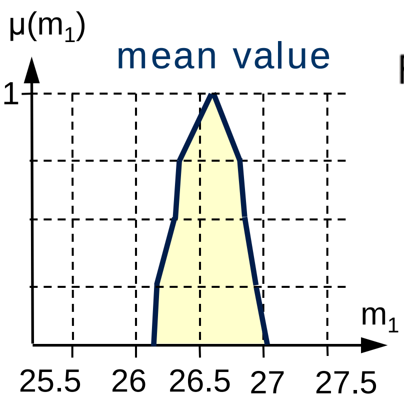


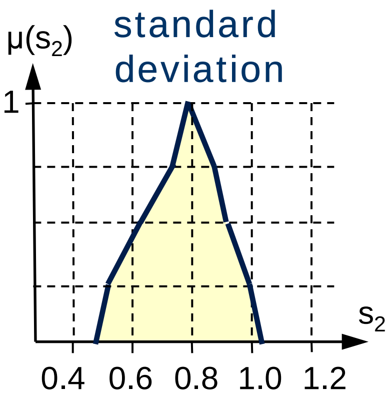
Sheet metal forming

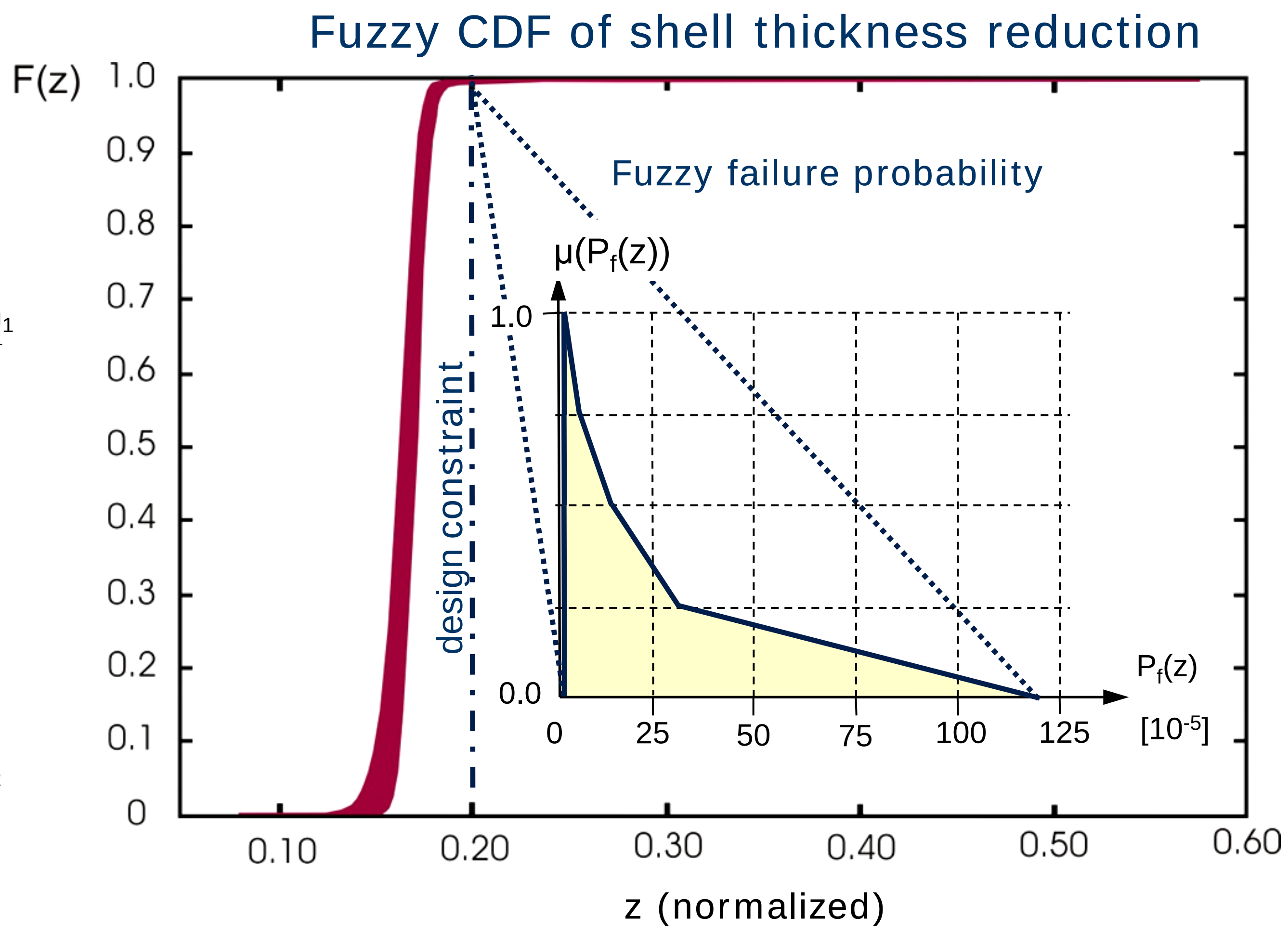


$$x_{d,opt} = \begin{pmatrix} 1414.81 \\ 25.23 \\ 200.0 \\ 50.0 \\ 60.0 \\ 77.5 \\ 20.0 \end{pmatrix}$$

Sheet metal forming





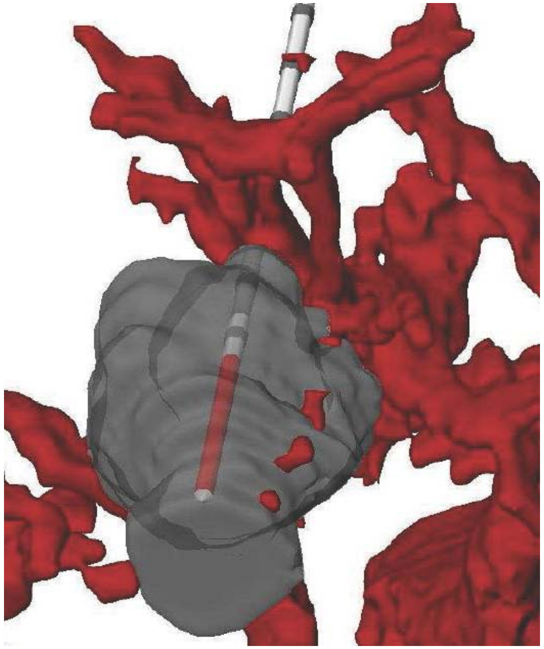


Radio frequency ablation

- thermal decomposition of tumor cells
- intervention planning
  (best applicator placement)

$$x \in [0;120]\,mm$$
$$y \in [0;80]\,mm$$
$$z \in [0;80]\,mm$$
$$\varphi \in [0;2\pi]$$
$$\psi \in [0;\pi]$$

- material parameters
  - electric conductivity
  - thermal conductivity
  - relative blood perfusion rate
  - parenchym
  - vessels
  - tumour

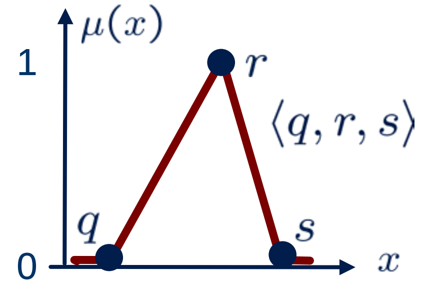


by courtesy of (Fraunhofer) MEVIS

Design task

- uncertain material parameters
  - fuzzy sets
    - electric conductivityLeitfähigkeit

$$\tilde{\sigma}^{\text{par}} = \langle 0,167; 0,384; 0,600 \rangle$$
$$\tilde{\sigma}^{\text{ves}} = \langle 0,667; 0,764; 0,860 \rangle$$
$$\tilde{\sigma}^{\text{tum}} = \langle 0,640; 0,800; 0,960 \rangle$$

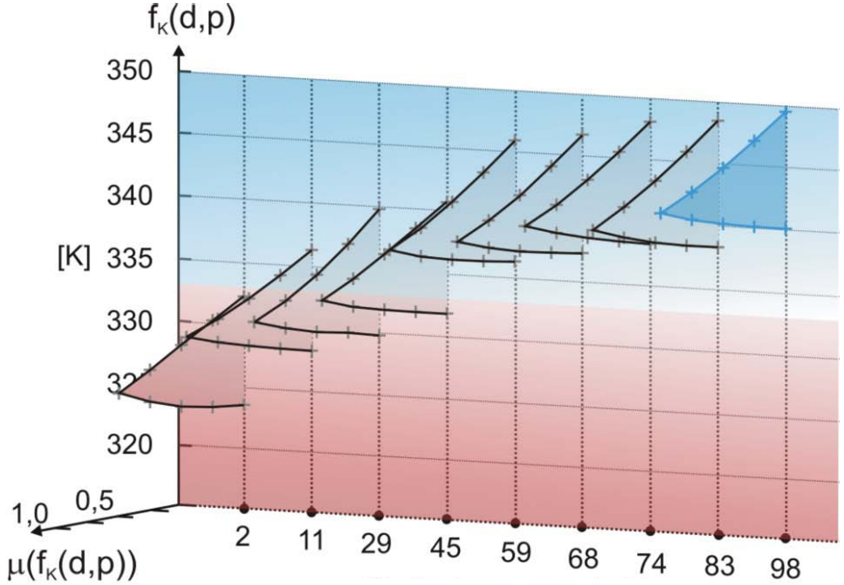


- objective
  - reliable intervention
  - maximizing the minimal temperature in tumors

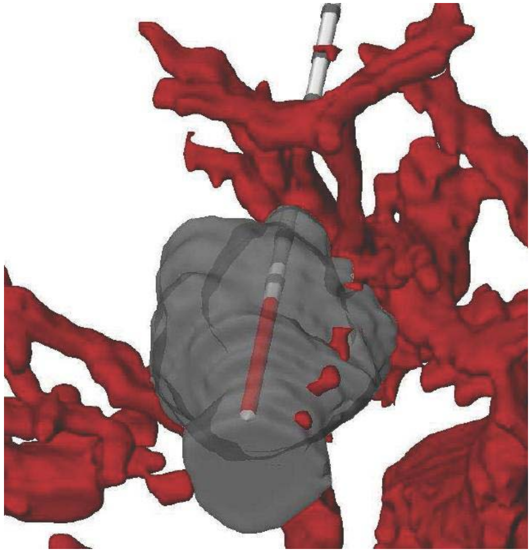
➡ optimization with uncertain quantities

Result

- progress of optimization



- best applicator placement



$$x = 0.0105\,mm \qquad \varphi = 3.166$$
$$y = 0.0080\,mm \qquad \psi = 0.149$$
$$z = 0.0083\,mm$$

Conclusion

- stochastic data models only part of a puzzle
- different uncertainty characteristics – aleatoric and epistemic
- uncertainty models – real available information
- profound knowledge about assumptions and limitations
- alternative uncertainty models applicable
  for industry-relevant problems

Far better an approximate answer to the right question,
which is often vague,

than the exact answer to the wrong question,
which can always be made precise.

John W. Tukey, 1962