TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Mechanical Engineering · Institute of Fluid Mechanics · Chair of Turbomachinery and Jet Propulsion

4. Dresdner Probabilistik-Workshop

Probabilistic Tutorial

“From the measured part to the probabilistic analysis“

Part I

Matthias Voigt (TU Dresden)

Outline

Outline

Part I

1. Introduction
2. Basic Statistics
3. Probabilistic methods (MCS)

Part II

4. Optical measurement (3D scanning)
5. Identification of geometrical Parameter and statistical analysis of scanned blades
6. Probabilistic Model - Meshmorphing
7. Generation of probabilistic samples
8. Analysis of probabilistic FE simulation

Part III

9. Analysis of cold-hot transformation
10. Deterministic CFD Modell
11. Analysis of probabilistic CFD simulation
12. Conclusion

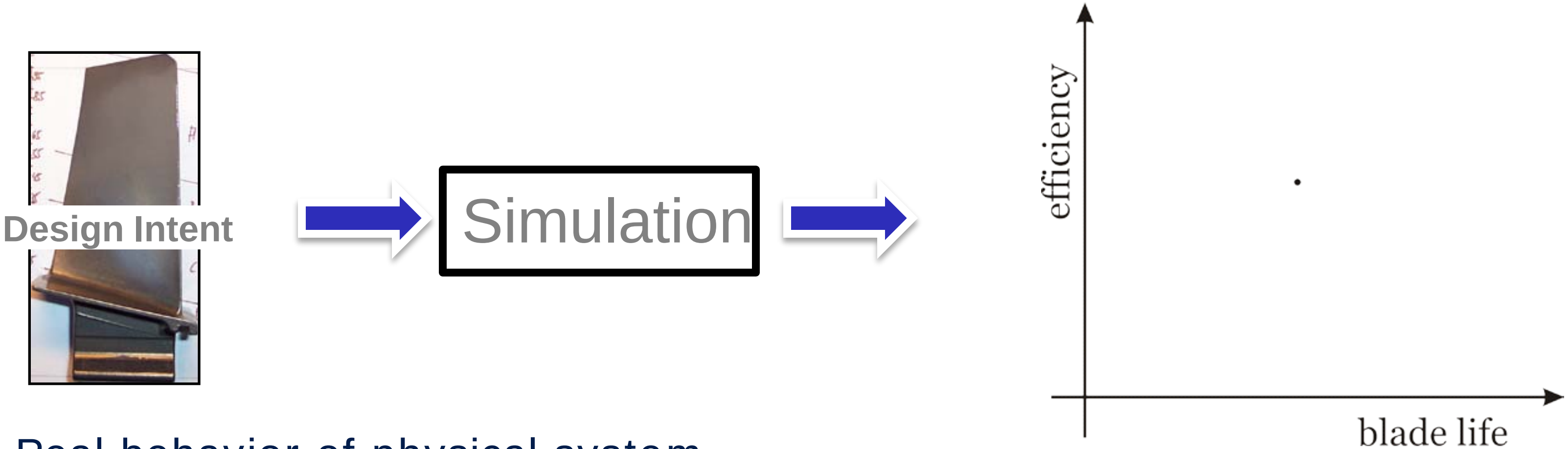
Two turbine blades from same engine…

… but with clearly different life.

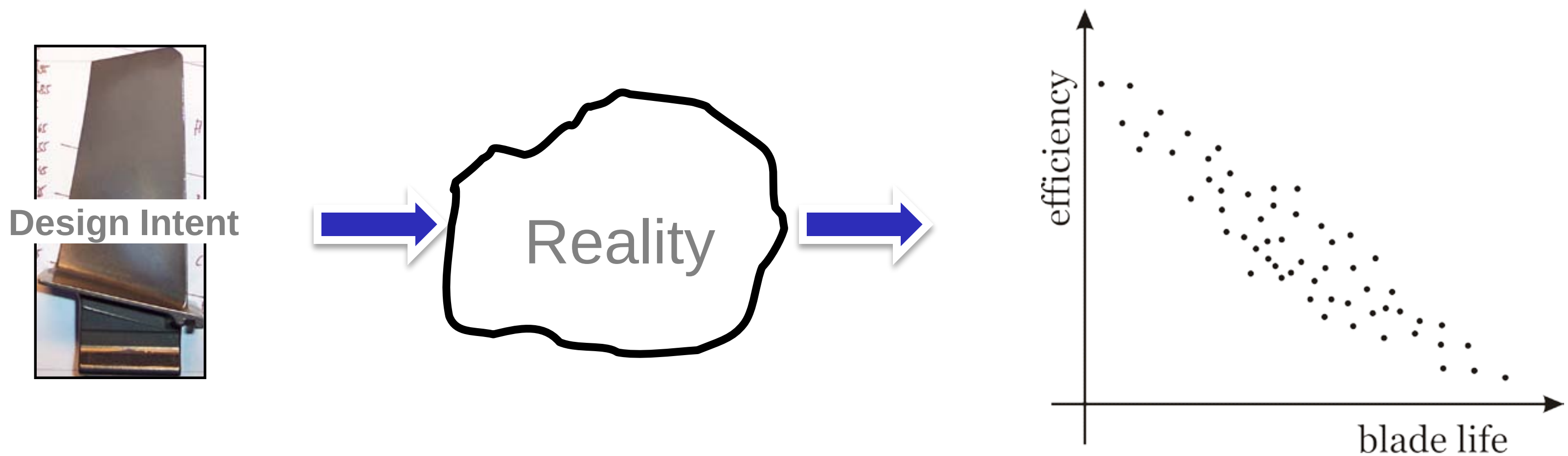
Massachusetts Institute of Technology, Prof. David L. Darmofal

Introduction

- Classic simulation-based approach used in design



- Real behavior of physical system



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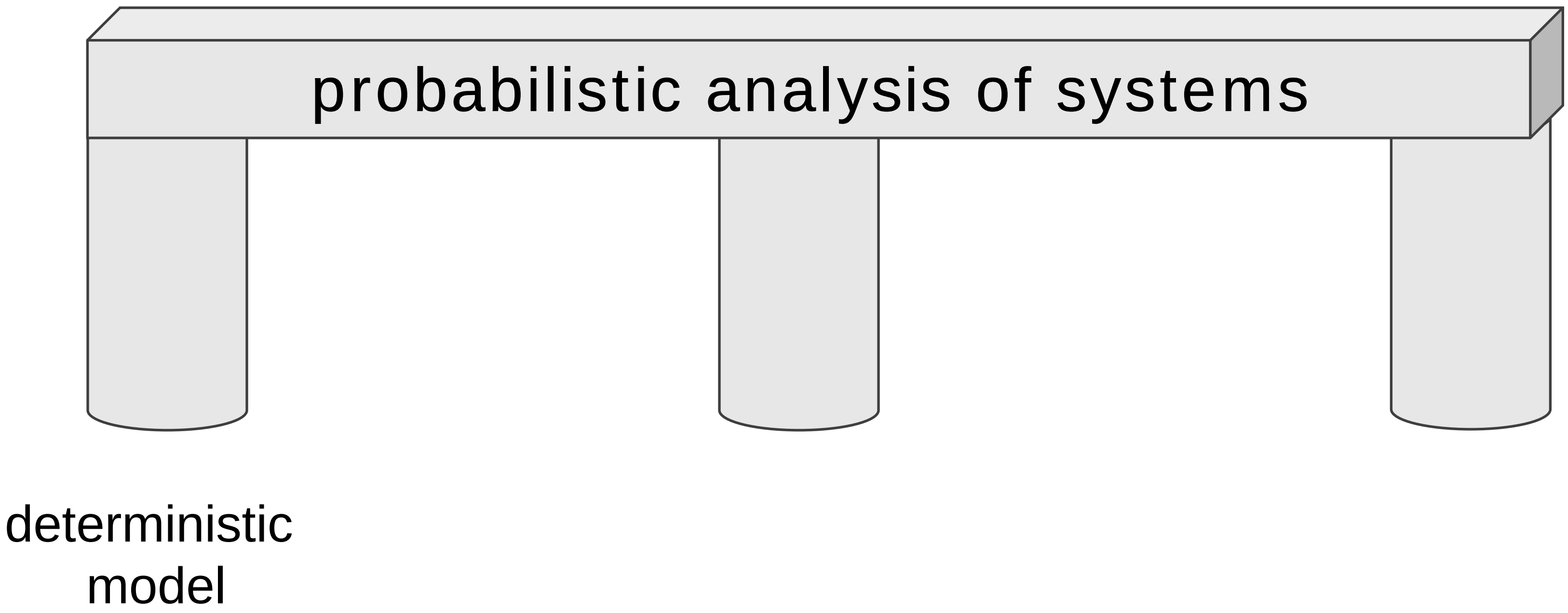
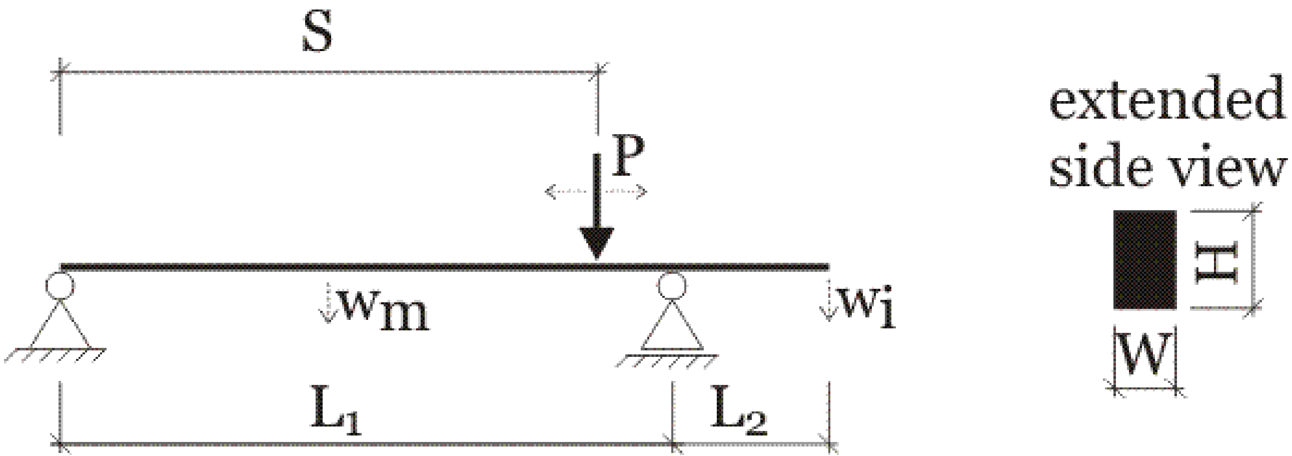
Boundary condition for probabilistic simulations



beam with a semibeam



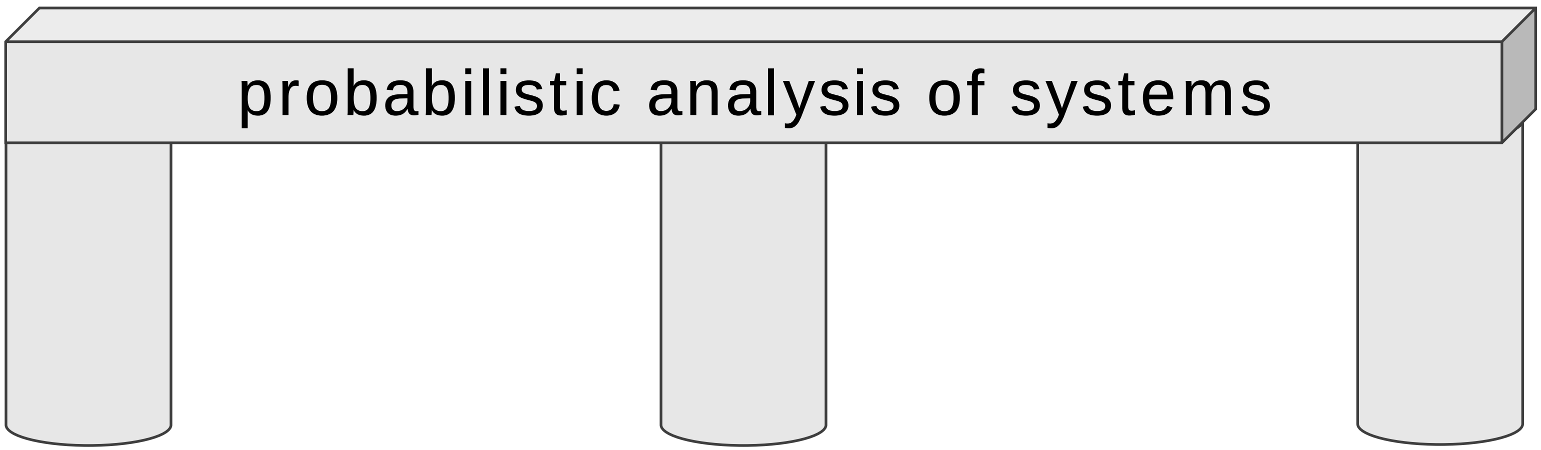
input values:

- beam height H
- beam width W
- E-Modulus
- point load P
- point load P position S

result values:

- deflections w_m, w_i

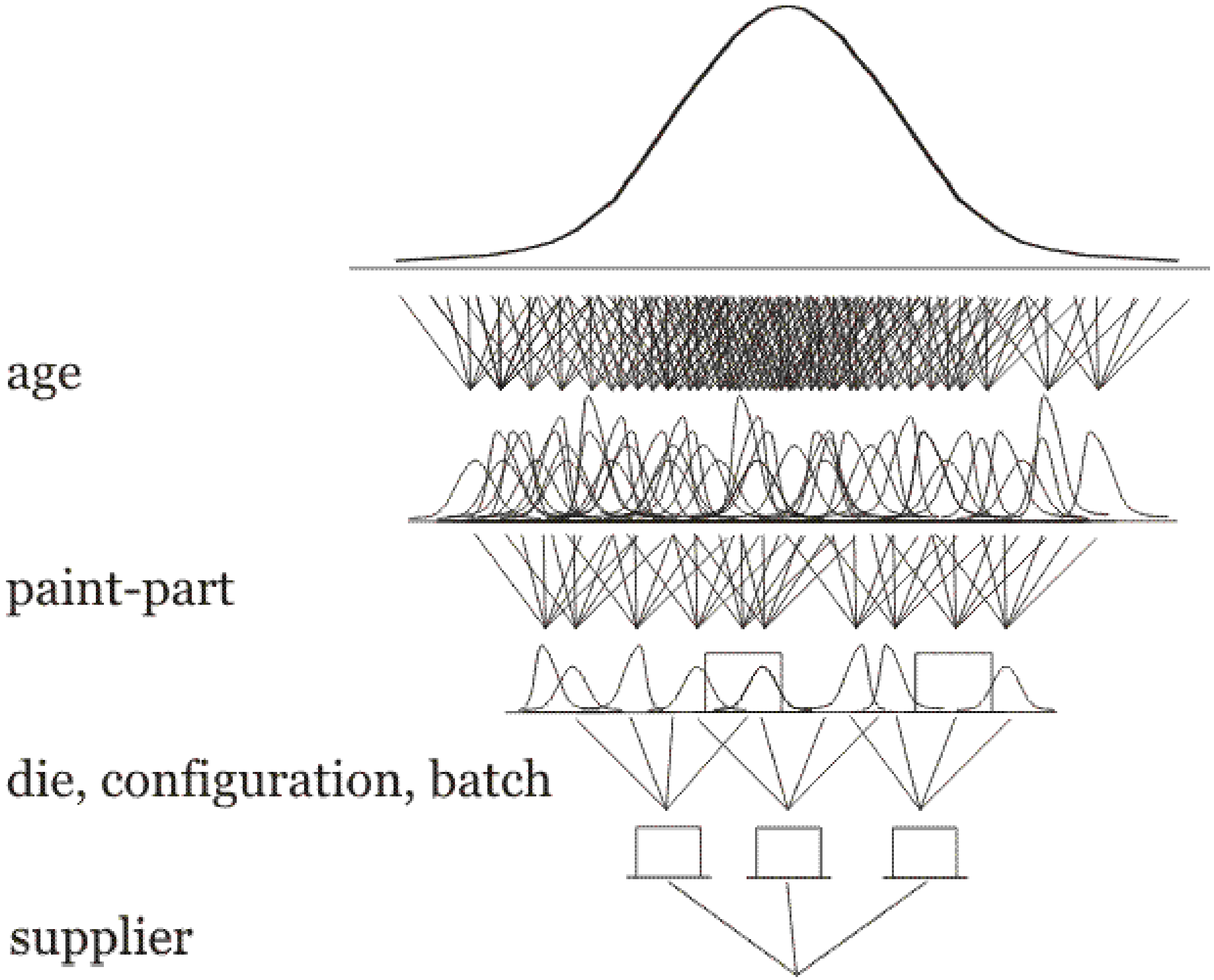
Boundary condition for probabilistic simulations



deterministic model

distribution type, distribution parameters, correlationen between input values

Reason for scatter

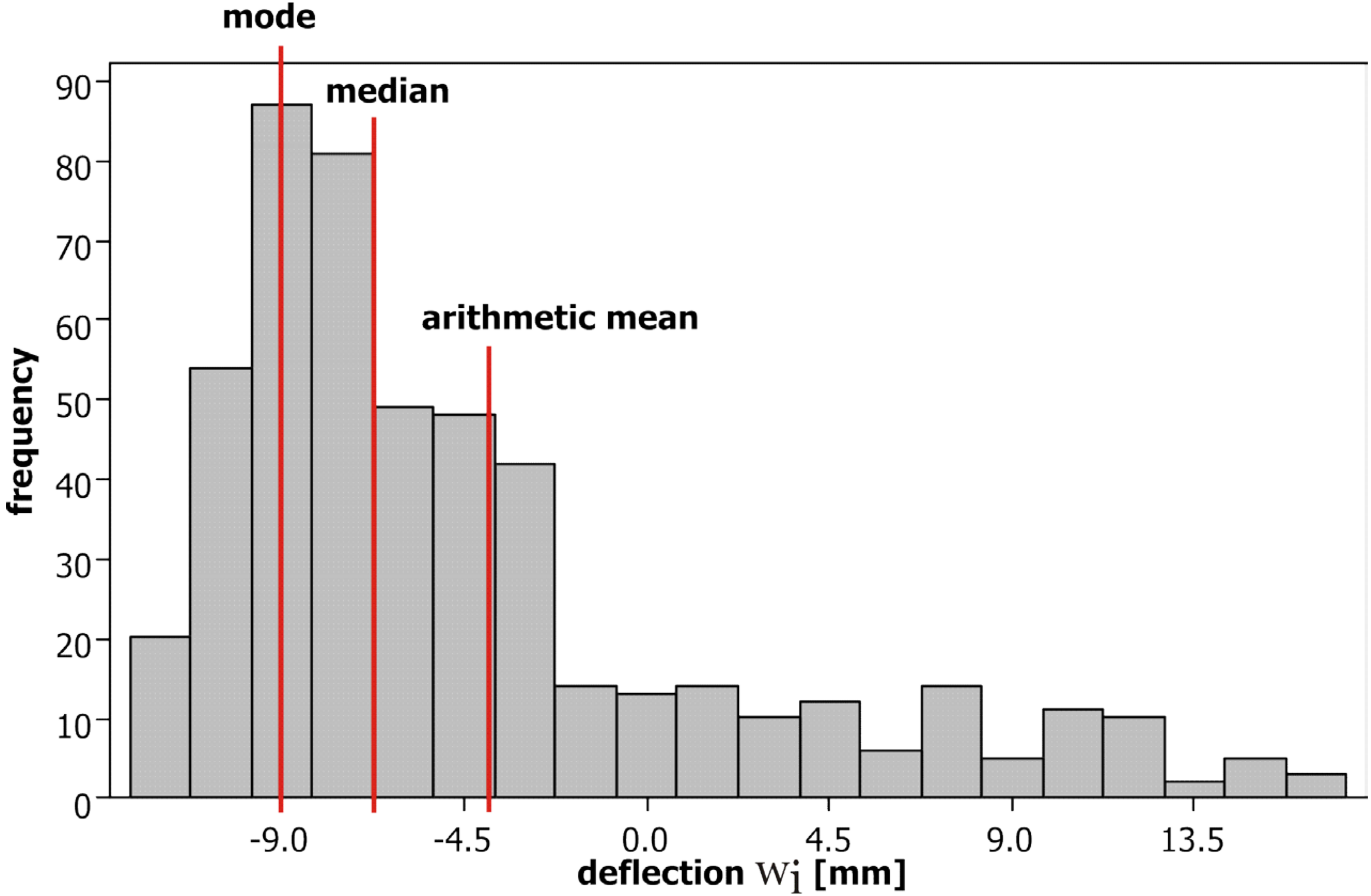


Statistical measures 1

[1]

- arithmetic mean: $$\bar{b}_{ri} = \frac{1}{n_{sim}} \sum_{k=1}^{n_{sim}} b_{ri,k}$$
  - non-robust estimator, outlier sensitive
- median: *is that value of the point that divides the area below the probability density function into two pieces of equal size*
  - a robust measure of central tendency
- modal value or mode: *is that value of the data set that occurs with the greatest frequency, it is not necessarily unique.*

Statistical measures 2



Statistical measures 3

[1]

- standard deviation:

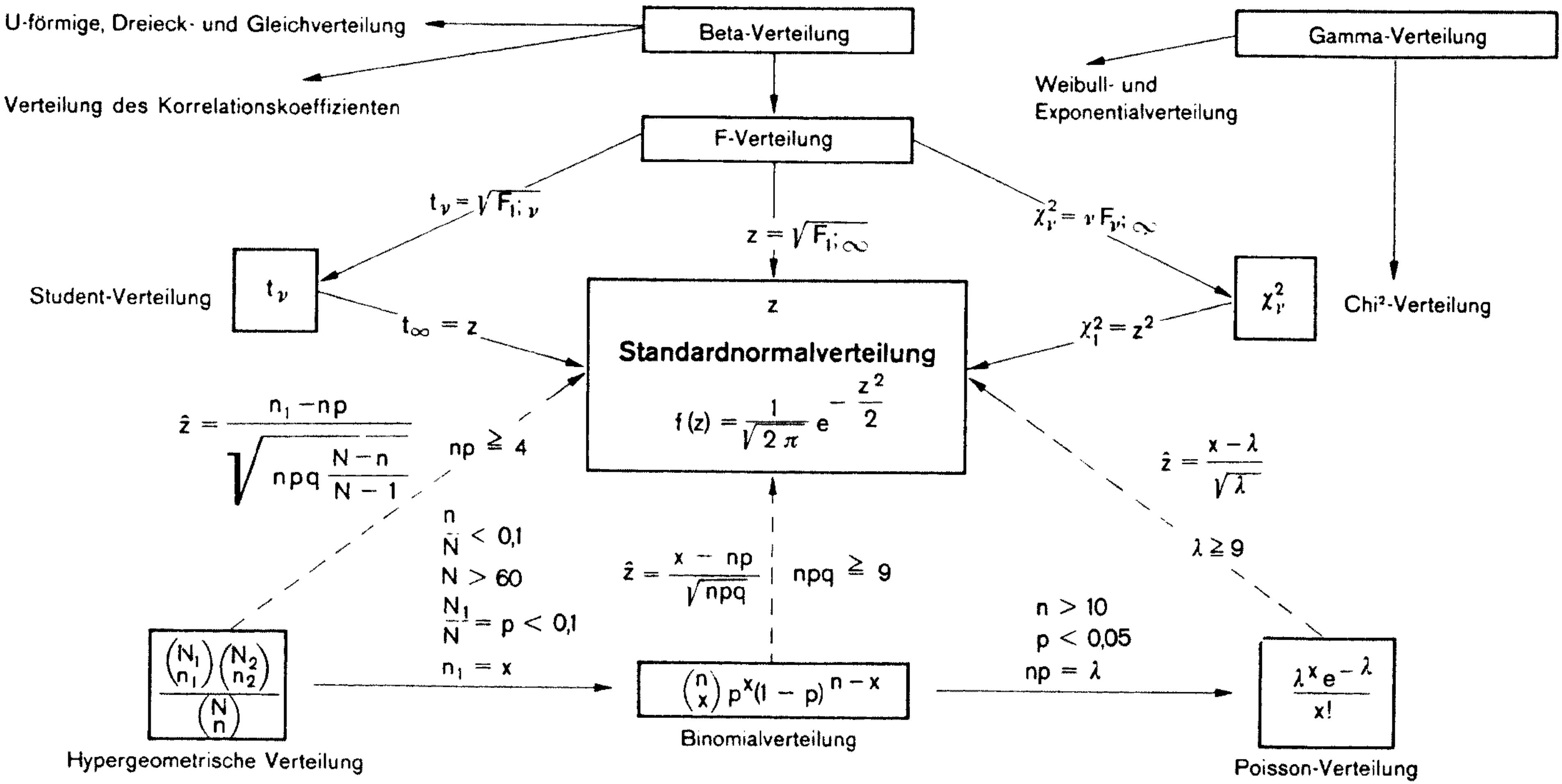
$$\sigma(\mathbf{b}_{ri}) = \sqrt{Var(\mathbf{b}_{ri})} = \sqrt{\frac{1}{n_{sim}-1}\sum_{k=1}^{n_{sim}}(b_{ri,k}-\bar{b}_{ri})^2}$$

- coefficient of variation:

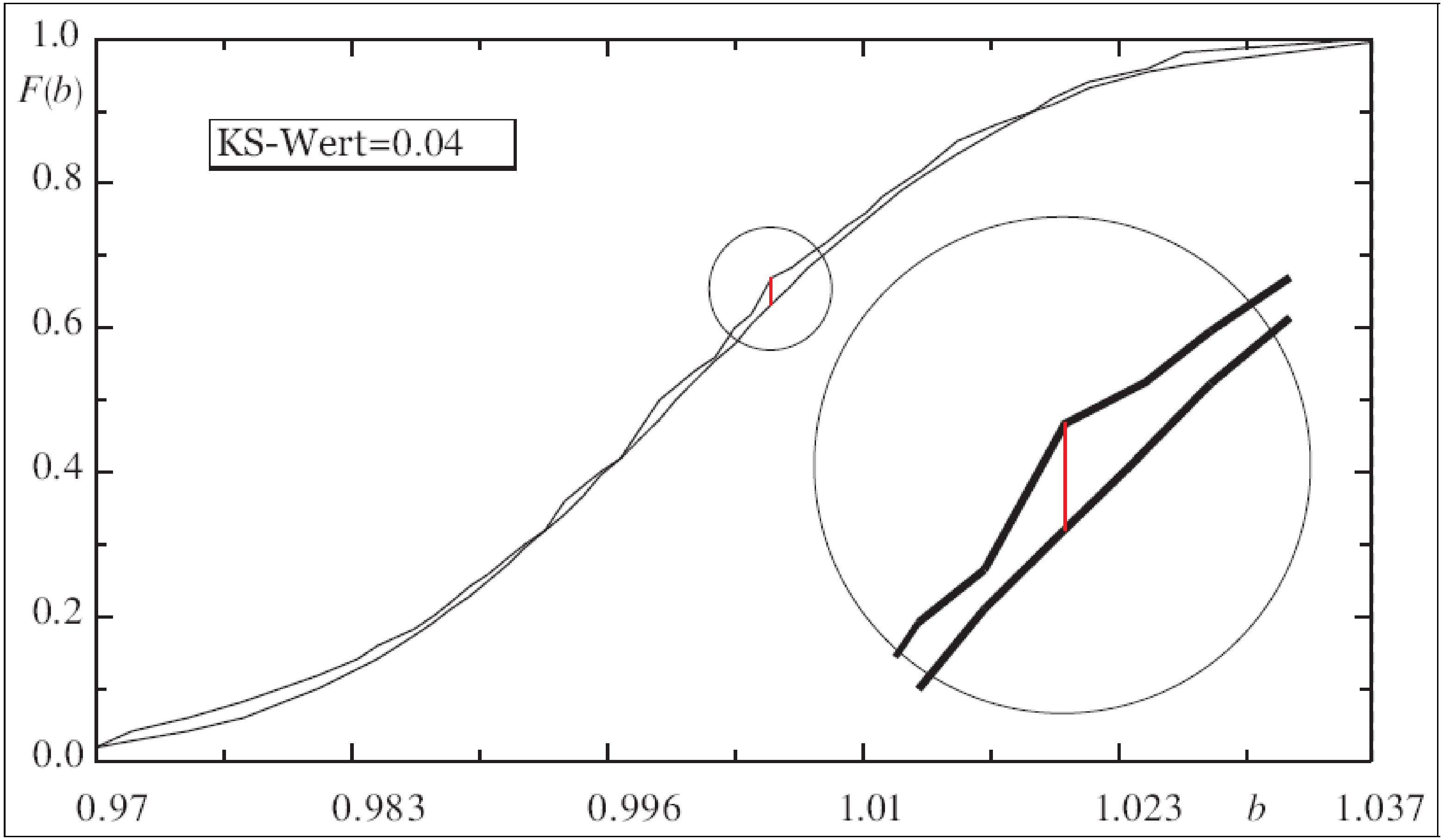
$$\delta(\mathbf{b}_{ri}) = \frac{\sigma(\mathbf{b}_{ri})}{\bar{b}_{ri}}$$

Statistical distribution

Übersicht 54. Zusammenhänge zwischen einigen Verteilungen [1]



Kolmogorow-Smirnow-Test



$$KS = \max_{-\infty < b_k < \infty} |F_d(b_k) - F_s(b_k)|$$

Anderson-Darling-Test

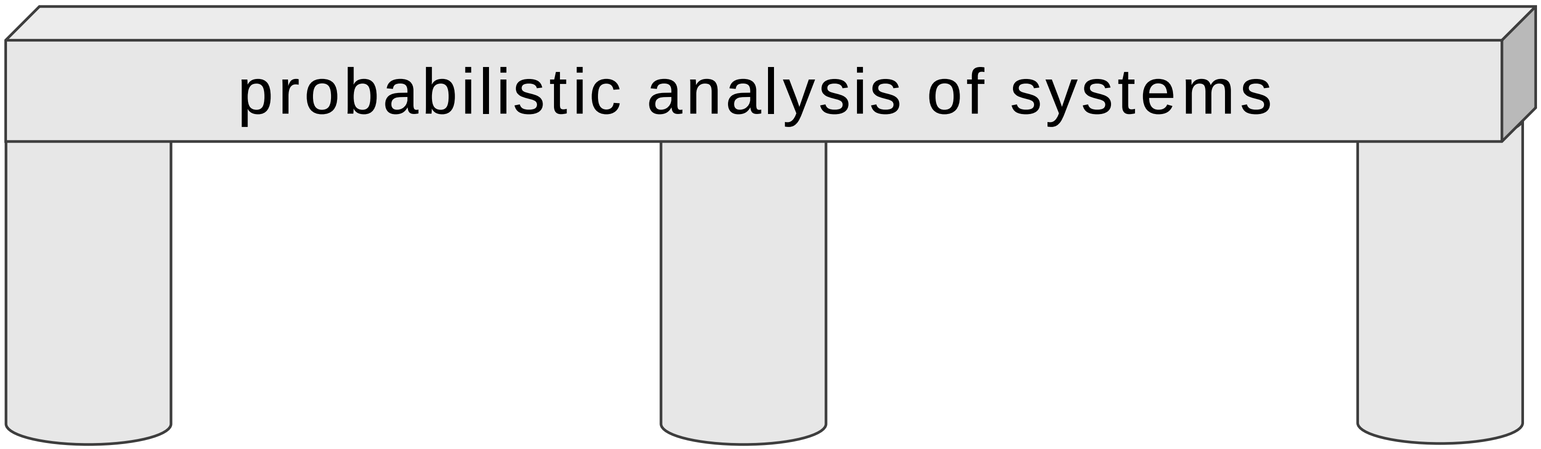
- the Anderson-Darling-Test is a modification of the Kolmogorow-Smirnow-Test
- this gives more weight to the tails than the K-S test does

[2]

$$A^2 = -n_{sim} - \frac{1}{n_{sim}} \sum_{k}^{n_{sim}} (2k-1)(\ln F_s(b_k) + \ln(1 - F_s(b_{n_{sim}+1-k}))$$

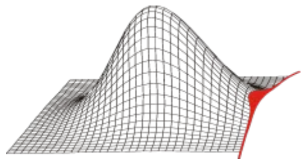
- F_s is the cumulative distribution function of the test data
- tables of critical values for A e.g. are available in [4] for different distributions

Boundary condition for probabilistic simulations



deterministic model

distribution type,
distribution parameters,
correlationen between input values



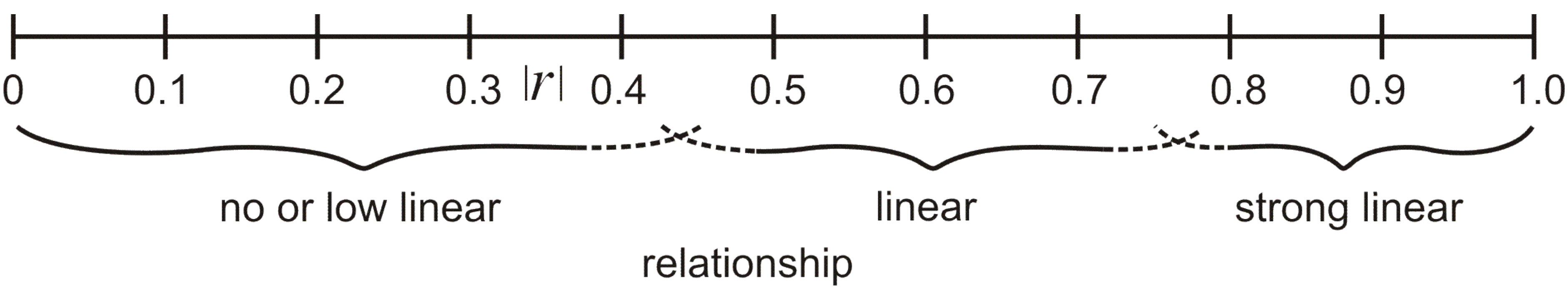
Pearson's correlation coefficient

[1]

$$r_{\mathrm{b}_{ri}\mathrm{b}_{rj}} = \frac{Cov(\mathrm{b}_{ri}, \mathrm{b}_{rj})}{\sqrt{Var(\mathrm{b}_{ri})}\sqrt{Var(\mathrm{b}_{rj})}}$$

$$Cov(\mathrm{b}_{ri}, \mathrm{b}_{rj}) = \frac{1}{n_{sim}-1}\sum_{k=1}^{n_{sim}}(b_{ri,k}-\overline{b}_{ri})(b_{rj,k}-\overline{b}_{rj})$$

range: [-1,1]



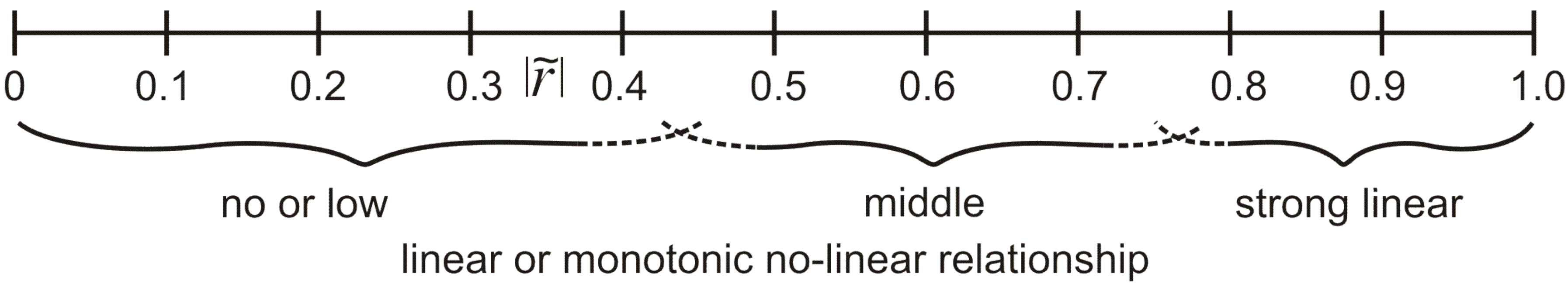
Spearman's Rank correlation coefficient

[1]

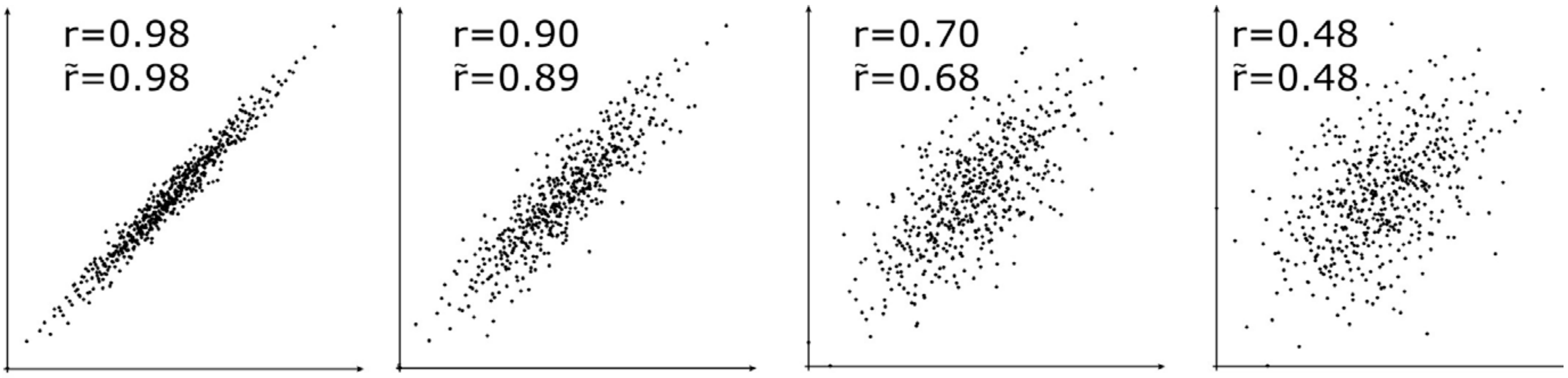
$$\mathrm{b}_{ri} = \begin{bmatrix} b_{ri,1} \\ \vdots \\ b_{ri,n_{sim}} \end{bmatrix} \Rightarrow \mathrm{Rang}\,(\mathrm{b}_{ri}) = \begin{bmatrix} R_{b_{ri,1}} = \mathrm{Rang\ von}\ \ b_{ri,1}\ \ \mathrm{in}\ \ \mathrm{b}_{ri} \\ \vdots \\ R_{b_{ri,n_{sim}}} = \mathrm{Rang\ von}\ \ b_{ri,n_{sim}}\ \ \mathrm{in}\ \ \mathrm{b}_{ri} \end{bmatrix}$$

$$\widetilde{r}_{\mathrm{b}_{ri}\mathrm{b}_{rj}} = \frac{\sum\limits_{k=1}^{n_{sim}} (R_{b_{ri,k}} - \overline{R}_{b_{ri}})(R_{b_{rj,k}} - \overline{R}_{b_{rj}})}{\sqrt{\sum\limits_{k=1}^{n_{sim}} (R_{b_{ri,k}} - \overline{R}_{b_{ri}})^2}\sqrt{\sum\limits_{k=1}^{n_{sim}} (R_{b_{rj,k}} - \overline{R}_{b_{rj}})^2}}$$

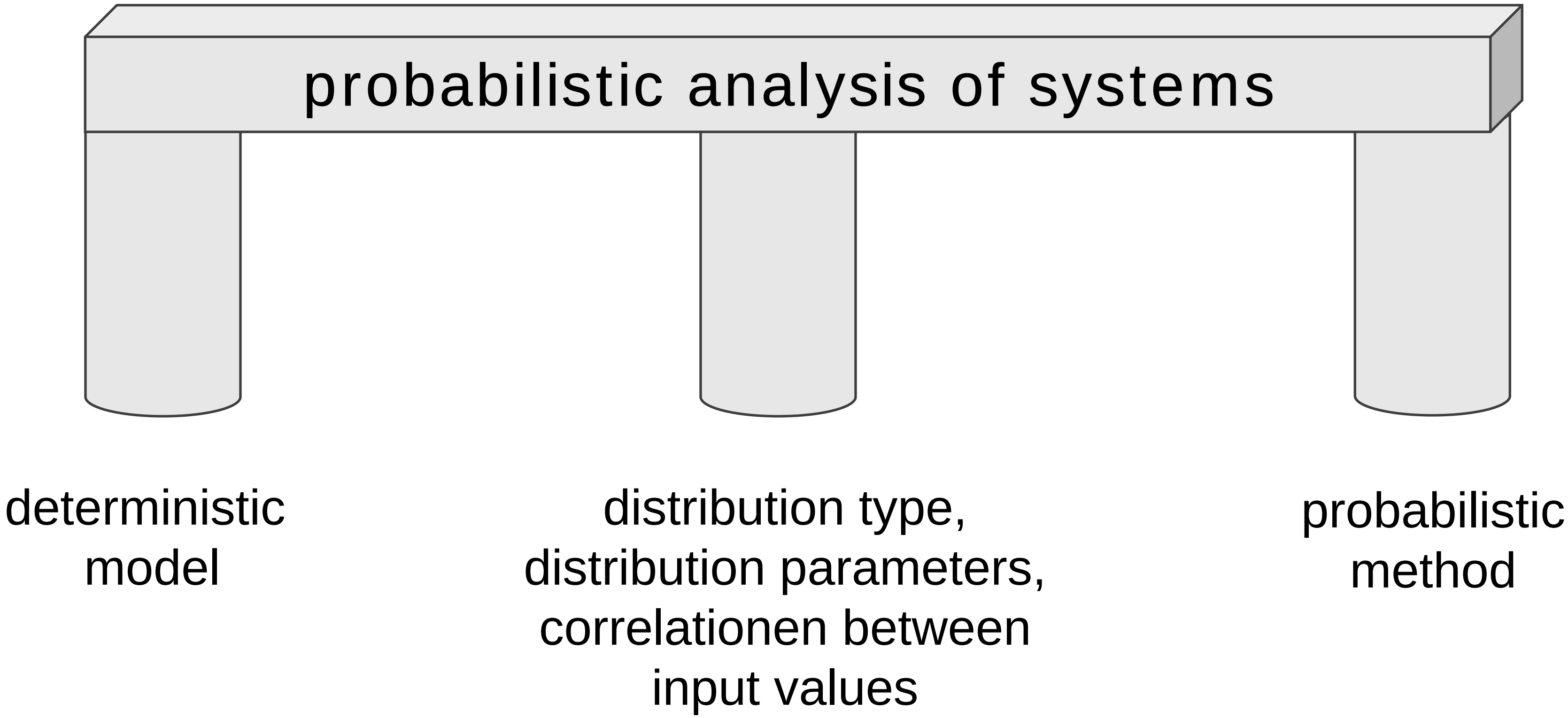
range: [-1,1]



Correlationen



Boundary condition for probabilistic simulations



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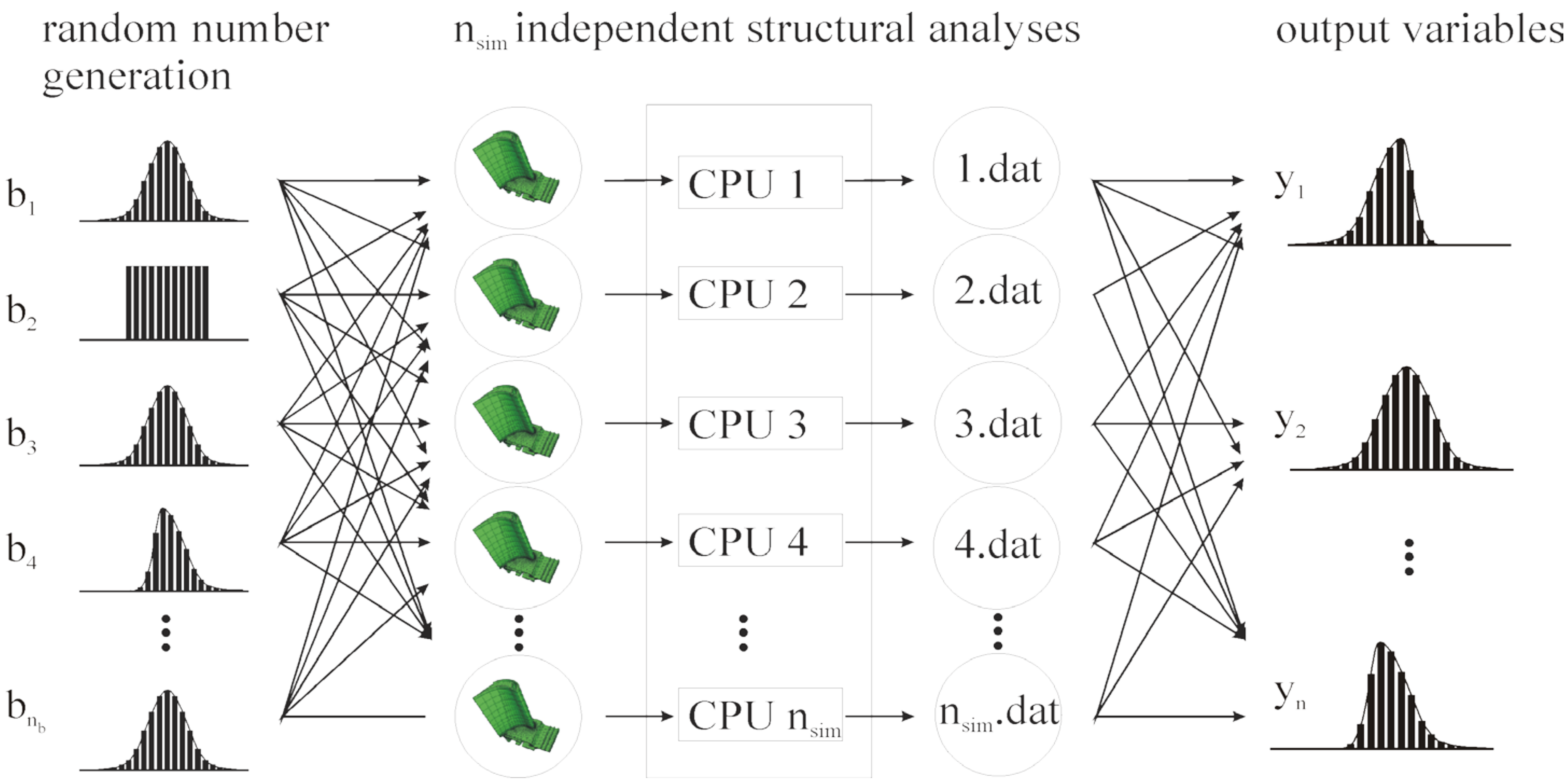
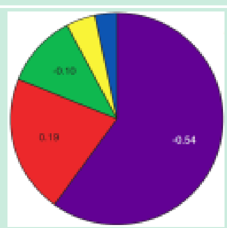
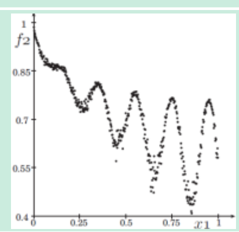
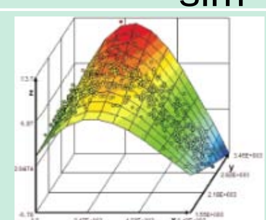
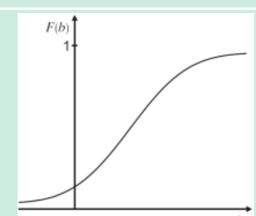
Monte-Carlo-Simulation (MCS)

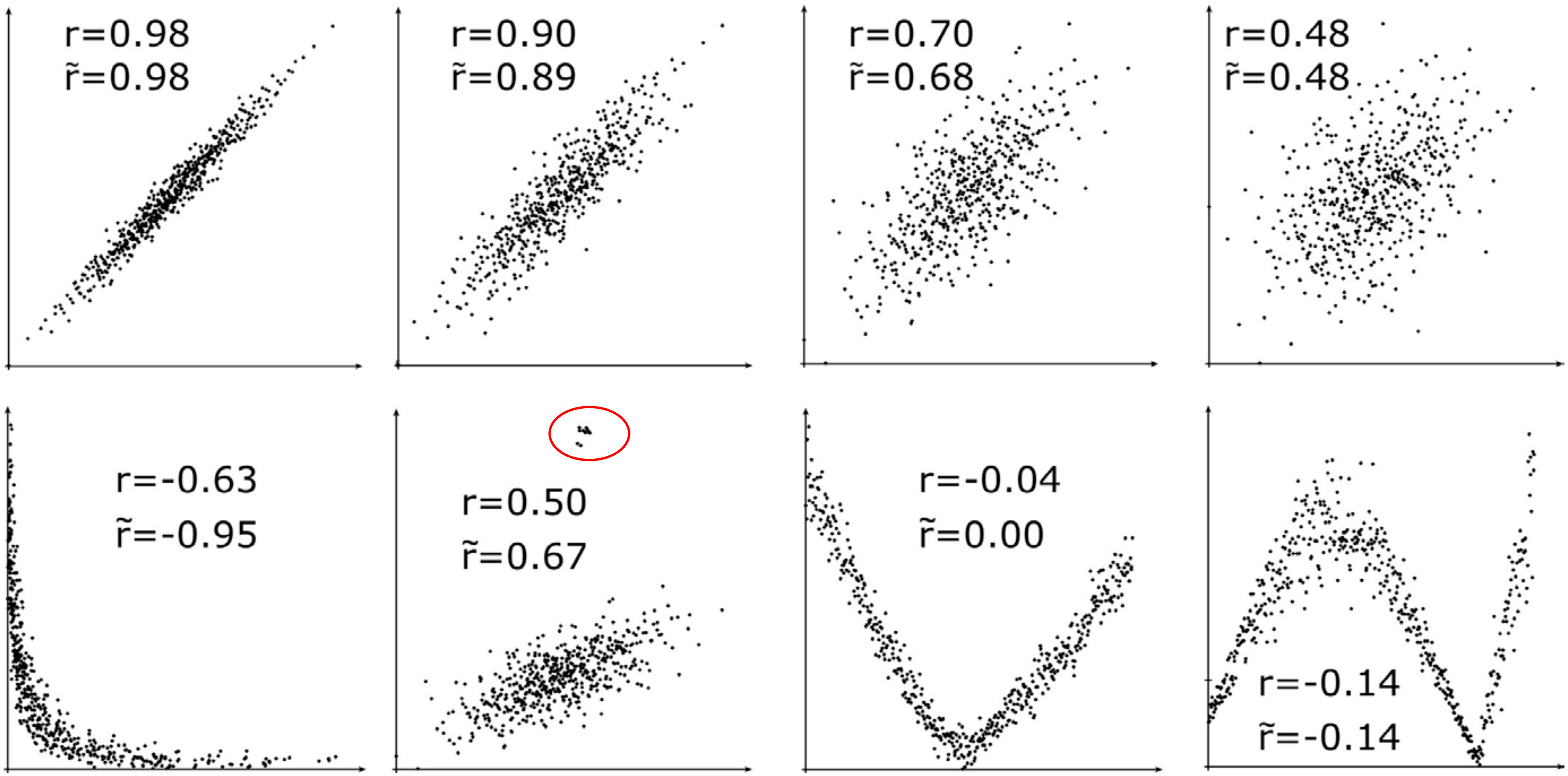


Results of MCS

Application of **Monte-Carlo** methods for probabilistic investigations using **optimized LHS** under consideration of **input parameter correlation**

Result of probabilistic Simulation	Sensitivities	Robustness	Optimization	Probability of failure
pdf of input variables - roughly known (as in industry)	✓	✓	✓	⛔
- precisely known (rarely)	✓	✓	✓	✓
required number of deterministic runs	**- almost independent to no. of parameters** - common: n_{sim} = 50…100 - minimum: n_{sim} = no. of par + 10…20 - better confidence intervals for higher n_{sim}			$P_f \approx \frac{10}{n_{sim}}$
output - one single MC Simulation provides all result quantities				

Correlationen



Correlationen

	R^2	E-Modul	Punktlast	Höhe	Breite	Position der Punktlast
w_m	0.97 / 0.42	0.01 / 0.02	0.02 / 0.03	0.06 / 0.06	0.02 / 0.04	0.95 / 0.39
w_i	0.97 / 0.39	0.01 / 0.02	0.01 / 0.02	0.01 / 0.03	0.01 / 0.03	0.97 / 0.38
max. w	0.80 / 0.14	0.01 / 0.00	0.06 / 0.06	0.19 / 0.10	0.07 / 0.03	0.68 / 0.02
max. |w|	0.51 / 0.06	0.00 / 0.00	0.01 / 0.00	0.05 / 0.02	0.01 / 0.00	0.48 / 0.04

Coefficient
- 0.8 ... 1.0
- 0.6 ... 0.8
- 0.4 ... 0.6
- 0.2 ... 0.4
- 0.0 ... 0.2

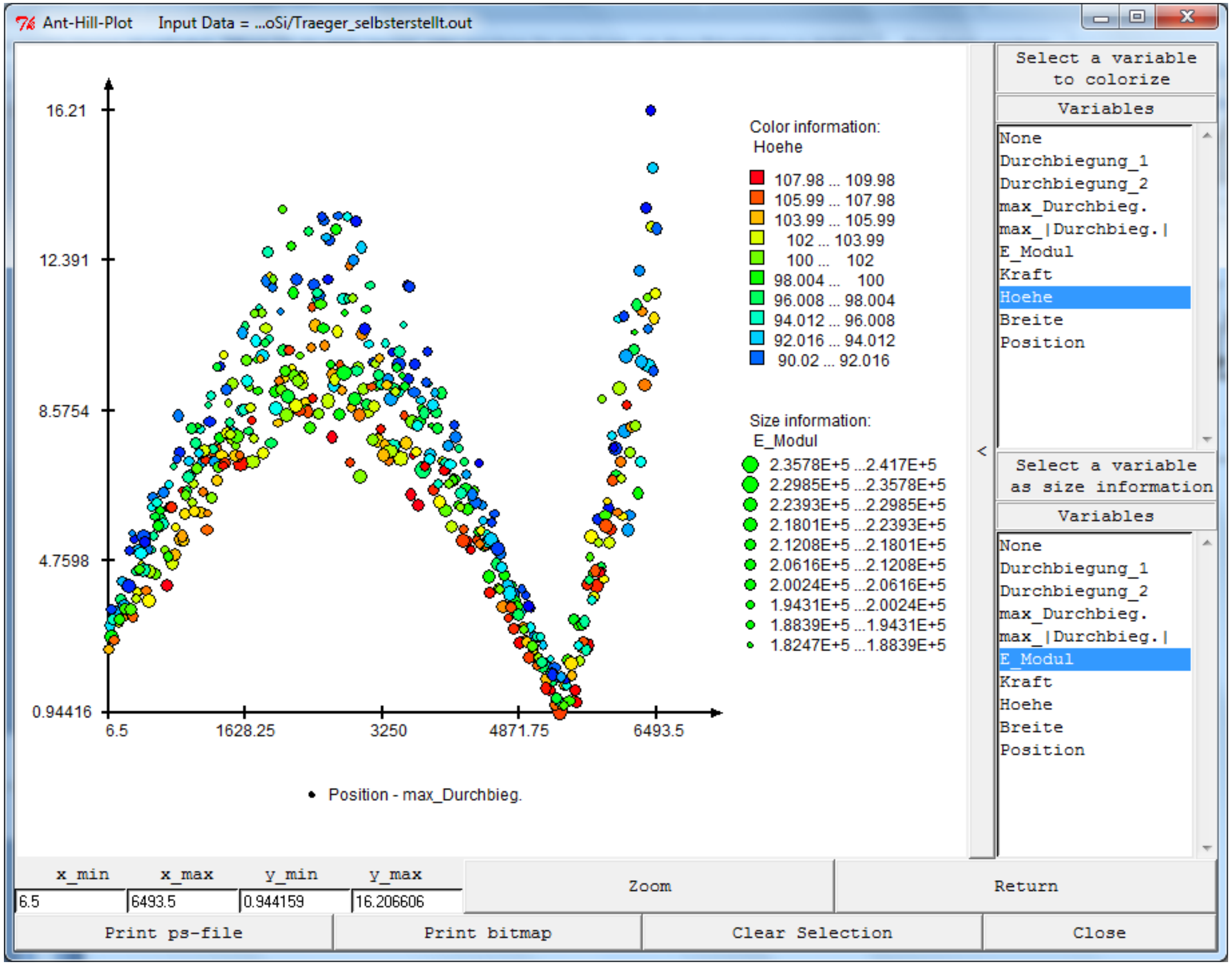
[3]

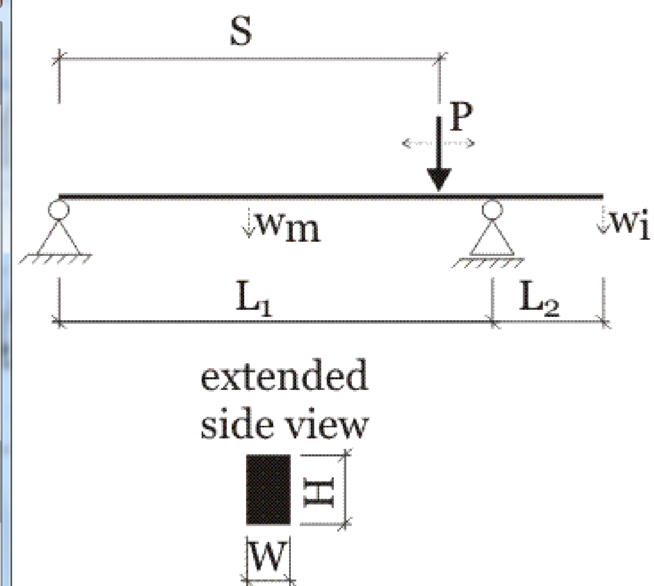
$$COI_e = R^2 - R_e^2$$

third-order response surface

first-order response surface

Robustness

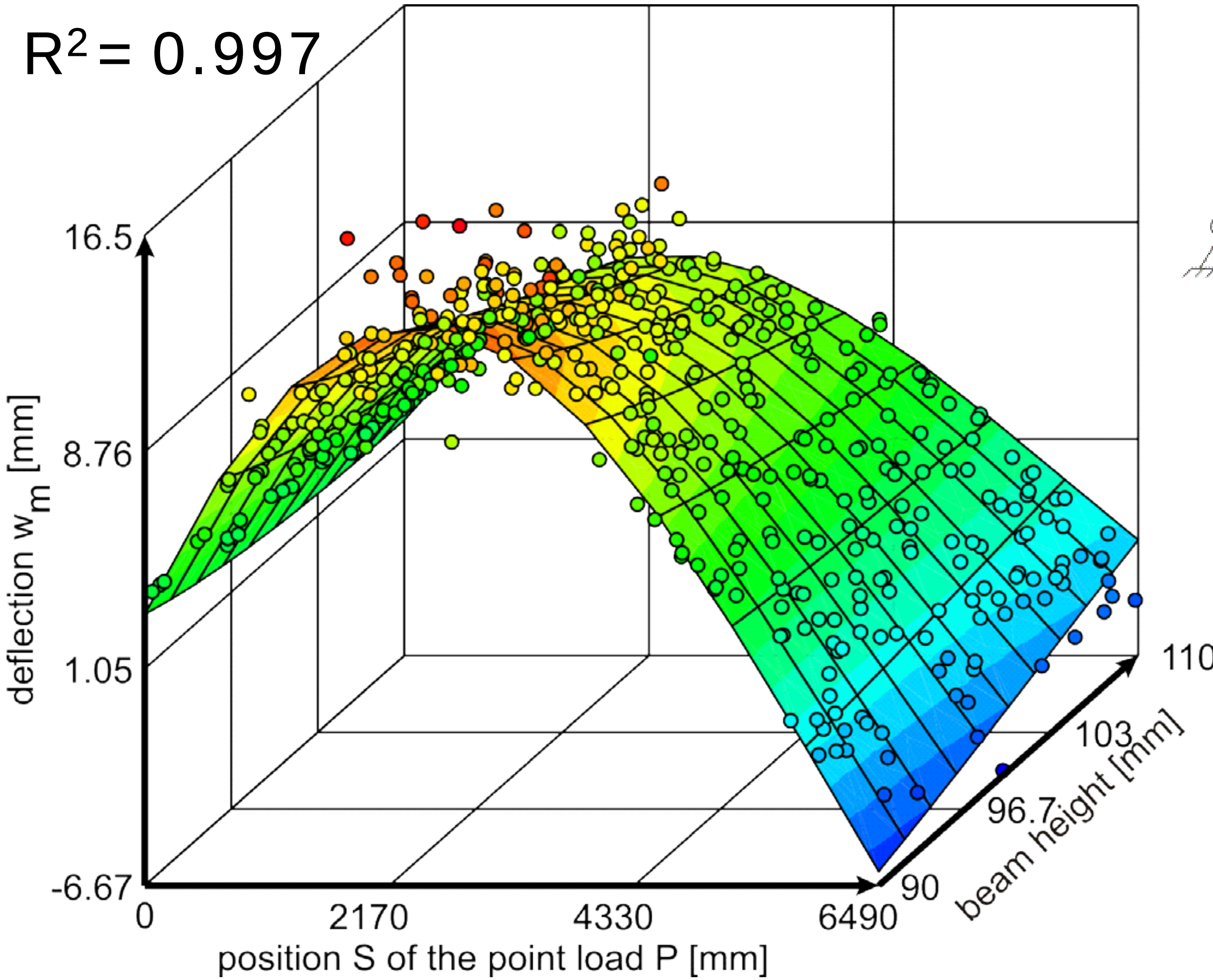


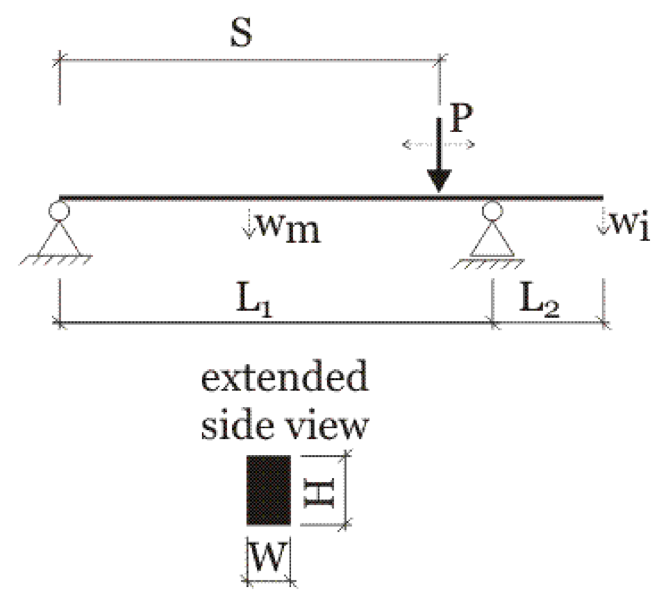


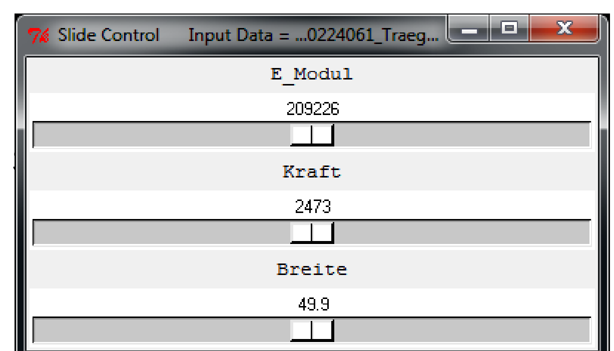
2D Ant Hill Plot

Meta model

$R^2 = 0.997$







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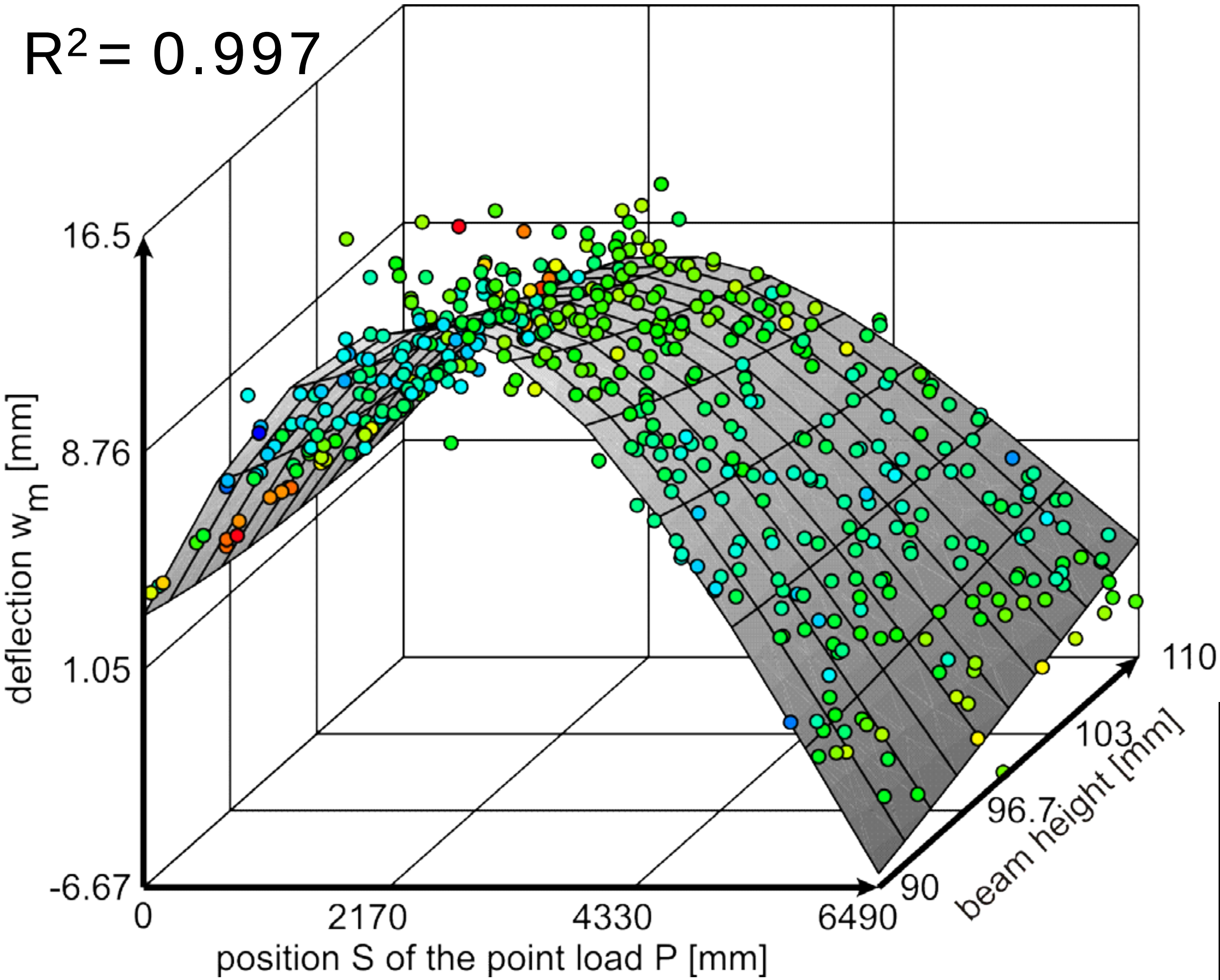
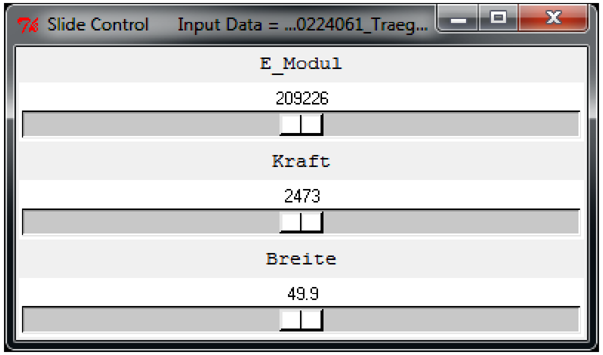
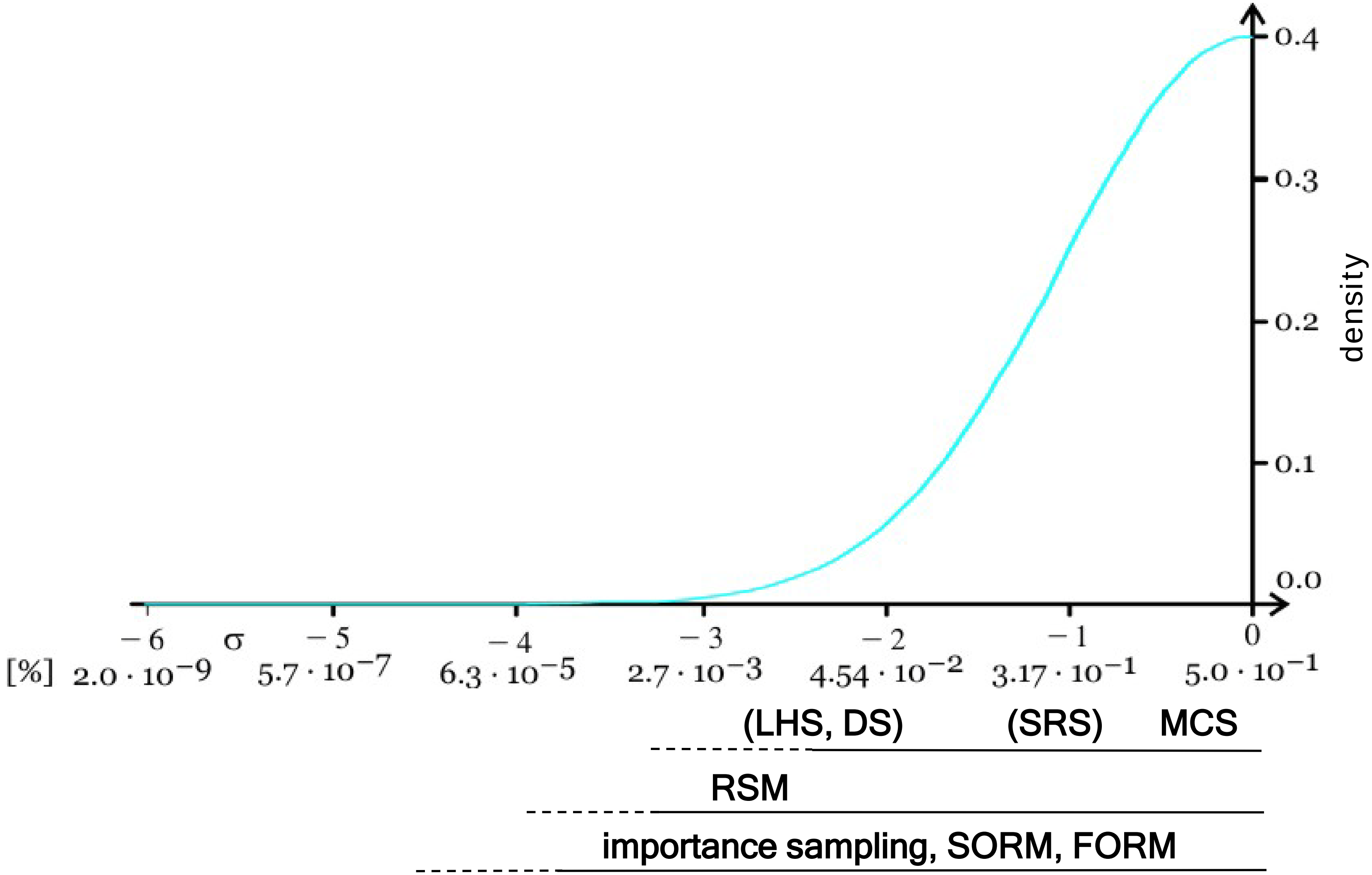




Probability of failure



Conclusions

problems / advantage

in the application of probabilistic methods

problems

- sufficient database for stochastic variables required
- increased engineering time
  - parametric model
  - validation of results
  - interpretation of results
- high computational cost due to multiple analysis

advantage

- probability to failure (no successive conservatism in the assessment)
- robustness of design
- sensitivity of result values w.r.t. stochastic variables
- cheaper design:
  - increased tolerances if possible
  - decreased tolerances if necessary

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references

[1] Sachs, L.: *Angewandte Statistik, Anwendung statistischer Methoden.* Springer-Verlag, Berlin/Heidelberg/New York, 2004.

[2] Anderson, T. W.; Darling, D. A.: *Asymptotic Theory of Certain "Goodness of Fit" Criteria Based on Stochastic Processes.* Annals of Mathematical Statistics 23, Seiten 193–212, 1952.

[3] Will, J.; Bucher, C.: *Statistische Maße für rechnerische Robustheitsbewertungen CAE gestützter Berechnungsmodelle.* Weimarer Optimierungs- und Stochastiktage 3.0, 2006.

[4] Stephens, M. A.: *EDF Statistics for Goodness of Fit and Some Comparisons.* Journal of the American Statistical Association, Vol. 69, Seiten 730–737, 1974.