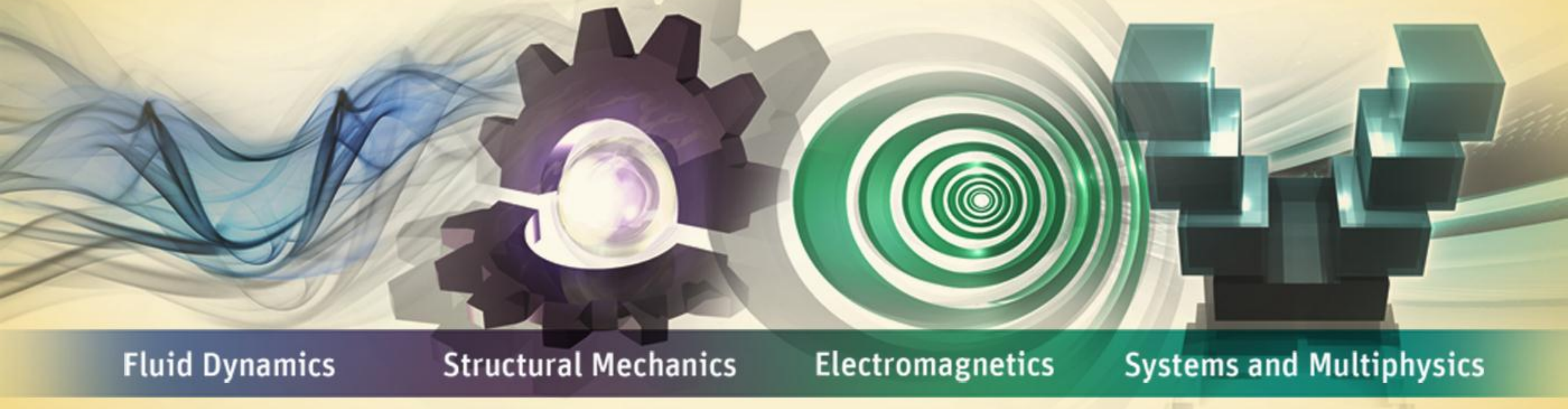
Realize Your Product Promise™

Multi-Physics Design Optimization of an Axial Compressor

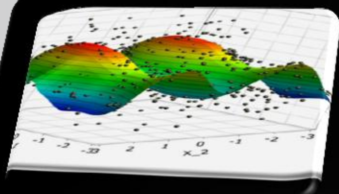
Application, Theory and Best-Practice Guide-Lines



Johannes Einzinger, ANSYS

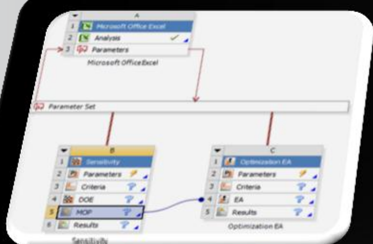
Thomas Most, Dynardo

Overview



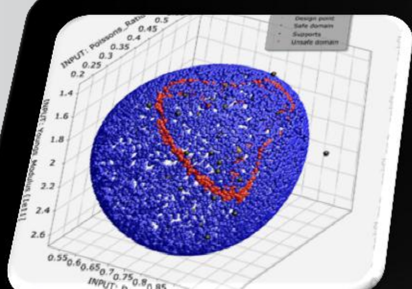
Meta-Model of Optimal Prognosis (MoP)

- *optiSLang inside Workbench*
- *MoP Theory*



Axial Compressor

- *Simulation Model*
- *Design Optimization*



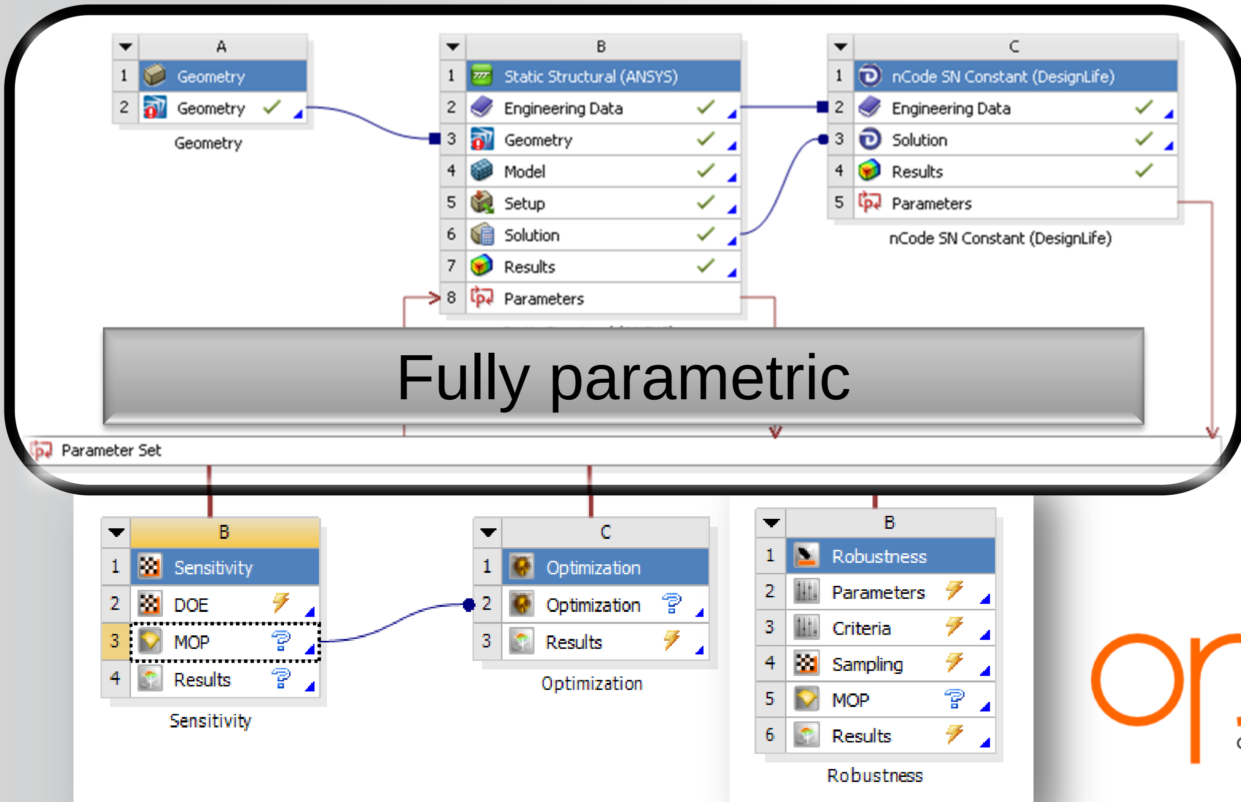
Robust Design Optimization

ANSYS

Workbench Framework

optiSLang inside Workbench

The Workbench Effect – easier to use



Easy parametric set up of complex simulations

optiSLang
optimizing structural language

easy use of best praxis automated flows inside Workbench

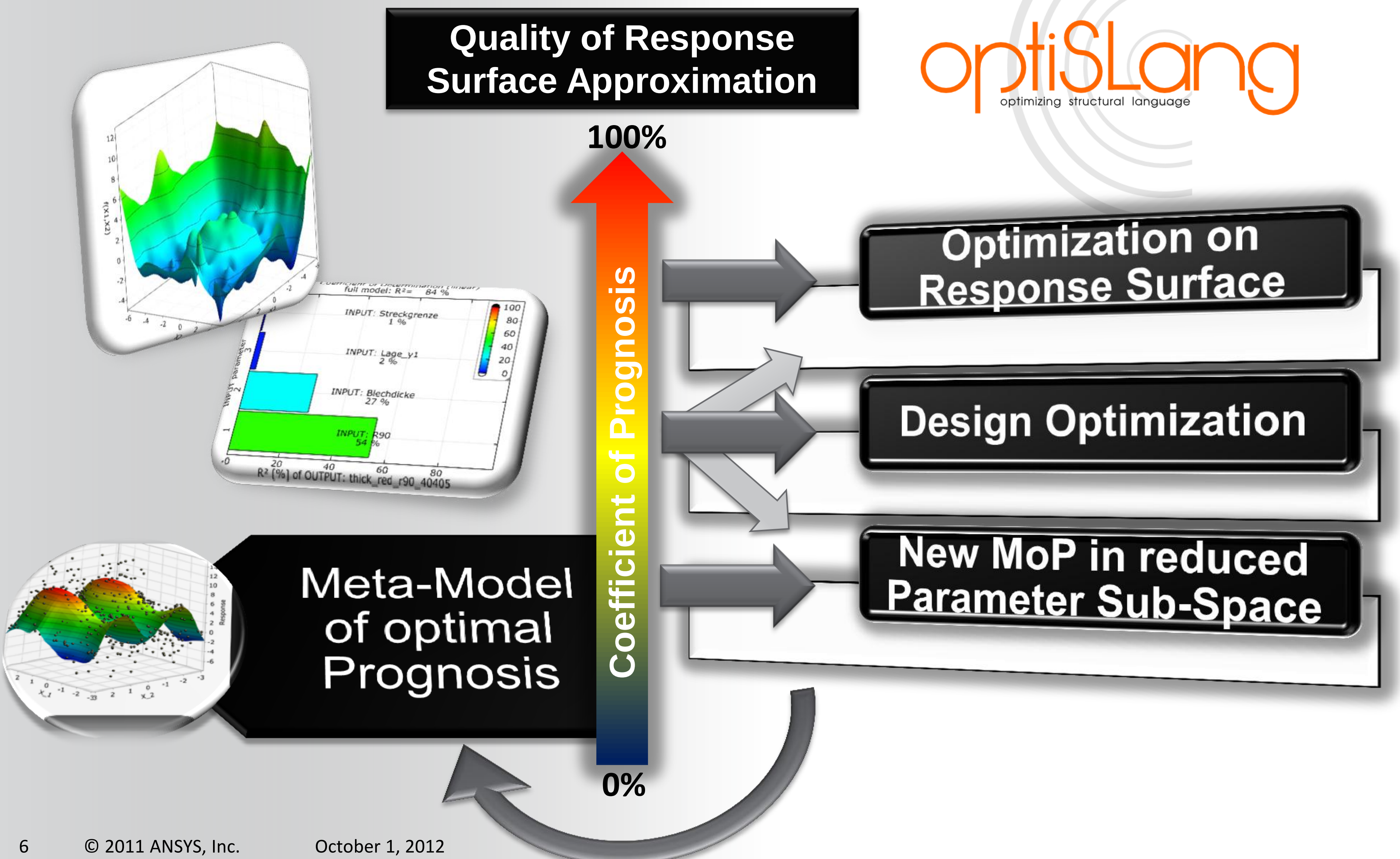
Optimization Strategy

General Procedure:

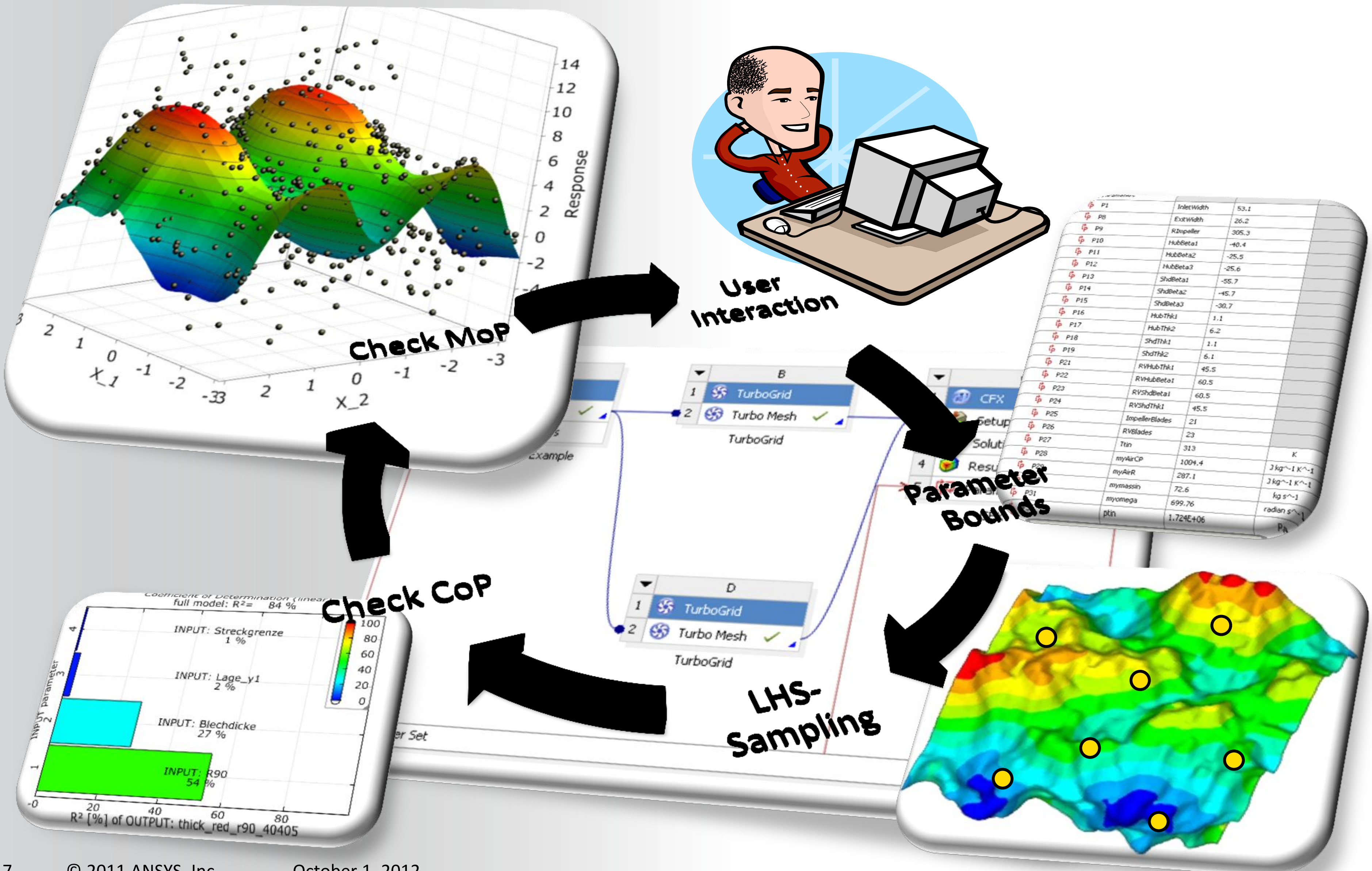
- Design Optimization
  - Gradient Based
  - Generic
  - Evolutionary
  - …

- Design of Experiments
  - Data Sampling
  - Detecting Correlations
  - Detecting Important Parameters
  - Parameter Space Reduction
  - Response Surface
- Design Optimization
  - …

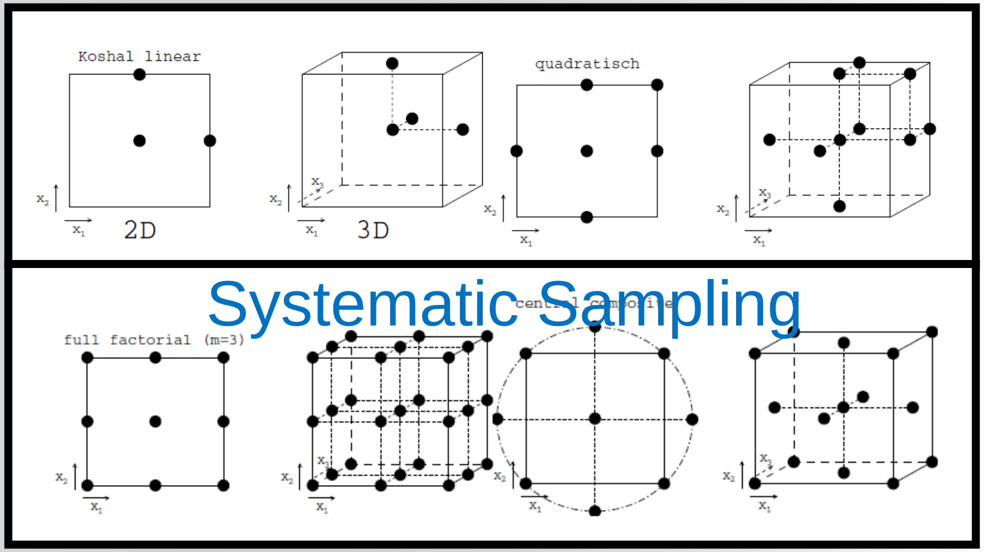
optiSLang Strategy

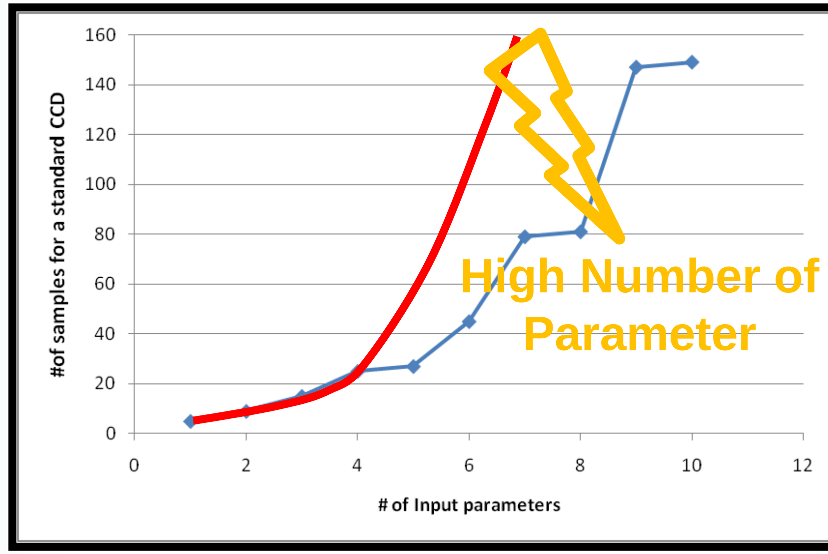


Design of Experiments



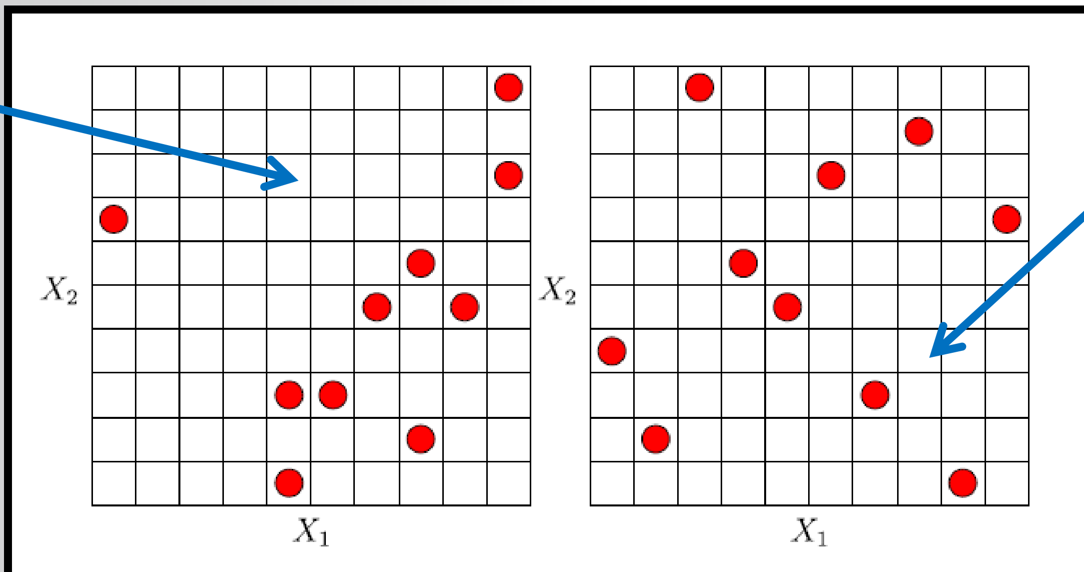
Design of Experiments, Sampling





Monte Carlo Sampling

"Random Correlation"



Latin Hypercube Sampling

Low effort for high number of parameter

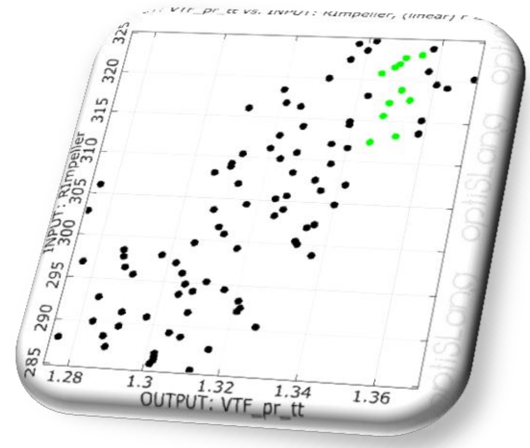
Linear Correlation

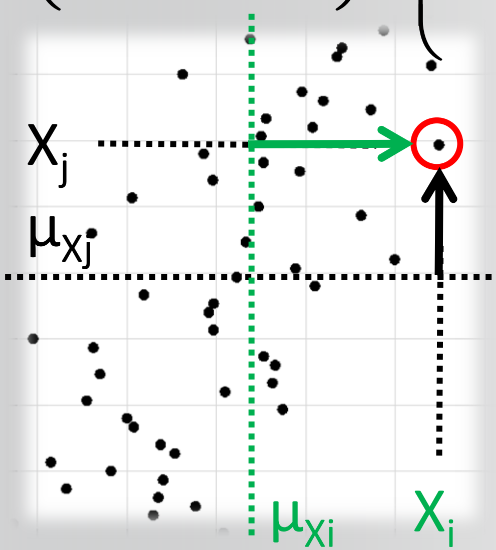
Mean value μ, variance σ² and standard Deviation σ:

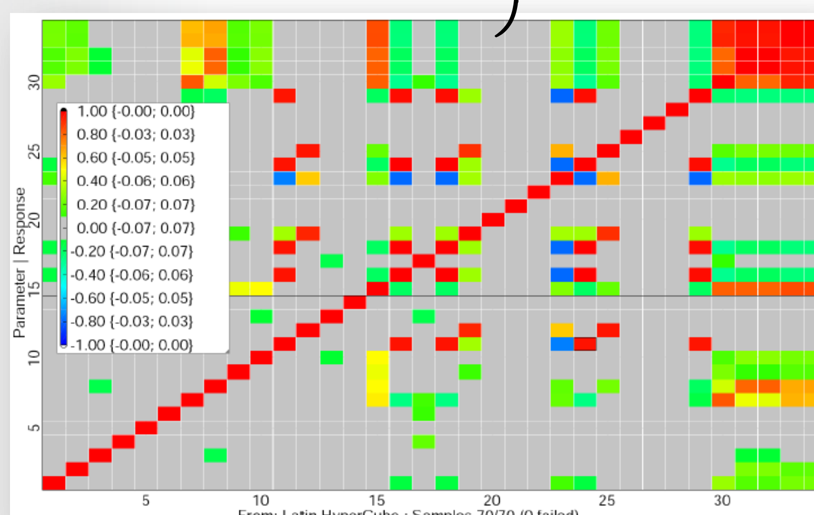
$$\mu_X = \frac{1}{N}\sum_{k=1}^{N} X_k; \quad \sigma_X^2 = \frac{1}{N-1}\sum_{k=1}^{N}\left(X_k - \mu_X\right)^2$$

Linear Coefficient of Correlation:

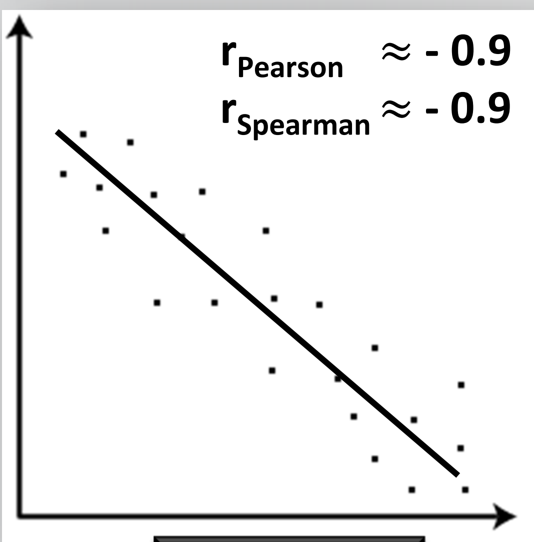
$$\rho_{ij} = \left(\frac{E\left(X_i \cdot X_j\right)}{\sigma_{X_i} \cdot \sigma_{X_j}}\right) = \left(\frac{\sum_{k=1}^{N}\left(X_i^{(k)} - \mu_{X_i}\right)\cdot\left(X_j^{(k)} - \mu_{X_j}\right)}{(N-1)\cdot\sigma_{X_i}\cdot\sigma_{X_j}}\right)$$



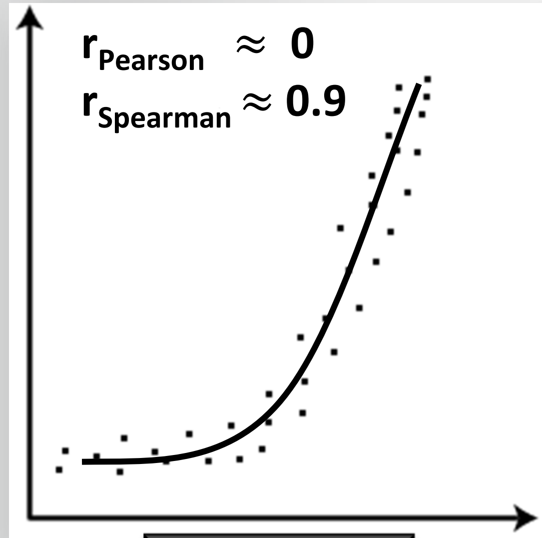




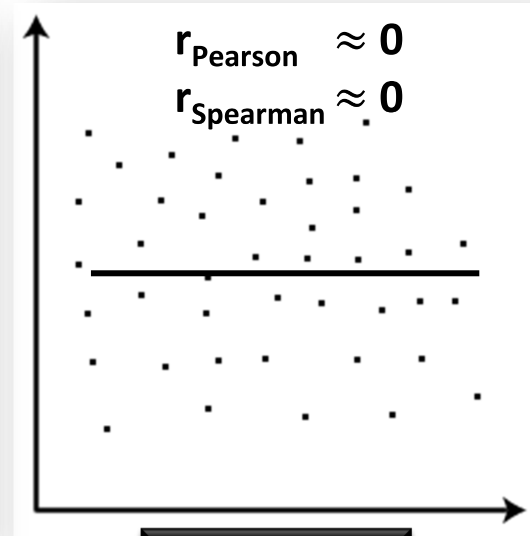
Significance Filter



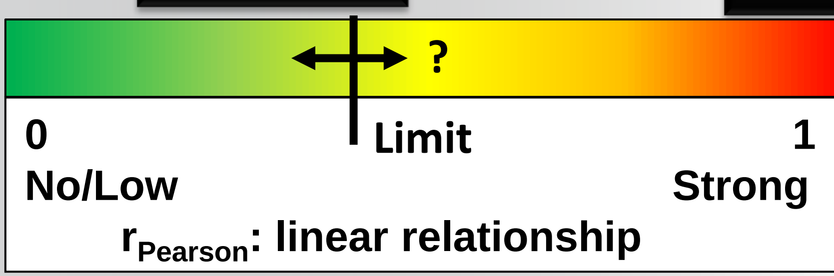
Linear monotonic correlation

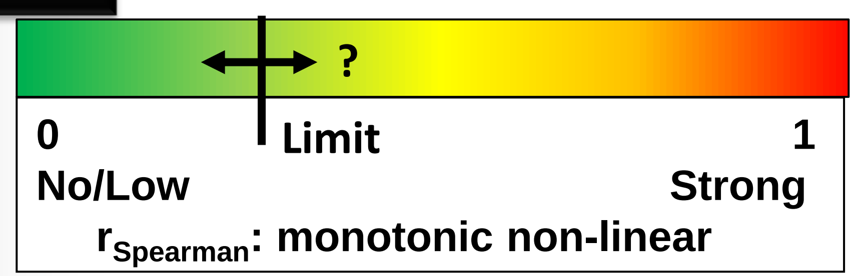


Monotonic non linear correlation



No Correlation





Correlation respects two parameters! Not more!

Polynomial Least Square

PLS: p polynomials h(x) and coefficients c

$$y(x) \approx \hat{y}(x) = h^T(x) \cdot c \qquad h^T(x) = \left(1, x_1, x_2, .., x_1^2, x_2^2, .., x_1 \cdot x_2, .\right)$$

Equations for all data points k=1…N with error ε_k (N>p)

$$y_k = H_k^T \cdot c + \varepsilon_k \qquad H_k^T = [N \times p]$$

Square error S → min

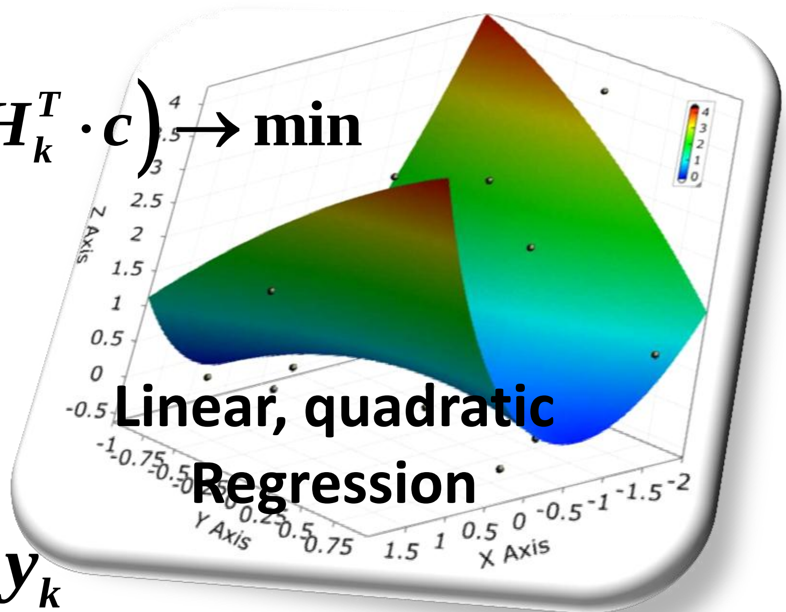
$$S(c) = \varepsilon_k^T \cdot \varepsilon_k = \left(y_k - H_k^T \cdot c\right)^T \cdot \left(y_k - H_k^T \cdot c\right) \to \min$$

Leads to equation for coefficients c:

$$\frac{\partial S}{\partial c} = H_k \cdot H_k^T \cdot c - H_k \cdot y_k = 0$$

and Polynomial Regression:

$$\hat{y}(x) = h^T(x) \cdot \left(H_k \cdot H_k^T\right)^{-1} \cdot H_k \cdot y_k$$



Polynomial Least Square

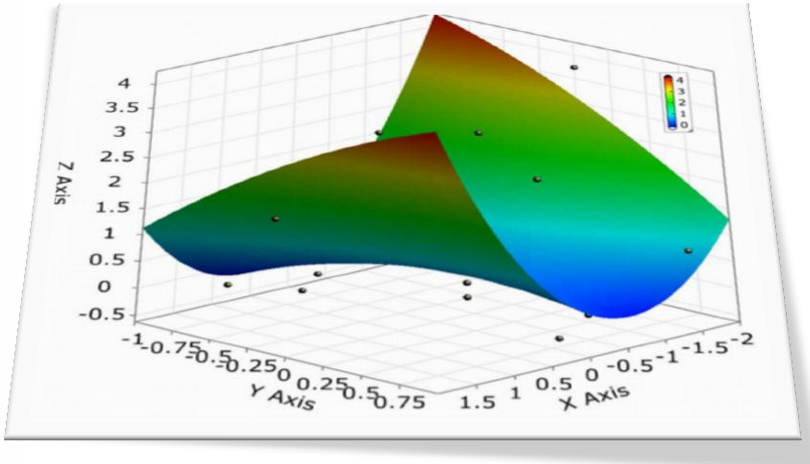
- **Number of Data Points n_p for N Input Parameter for Response Surface Y_k**
- **Polynomial respects multiple parameters!**
- **Mixed terms are not used: $n_p \sim N^2$**
- **Parameter Reduction with Significance Filter**

$$n_p < 1 + 2 \cdot N$$

$$Y_k = f(x_1, \cancel{x_2}, x_3, \cancel{x_4}, \cancel{x_5}, x_6, x_7, x_6, \dots, xN)$$

Polynomial	#Data Points
Const.	1
Linear x_i	N
Pure quadratic x_i^2	N
~~Mixed quadratic~~ $x_i x_j$	$0.5 \cdot N \cdot (N-1)$

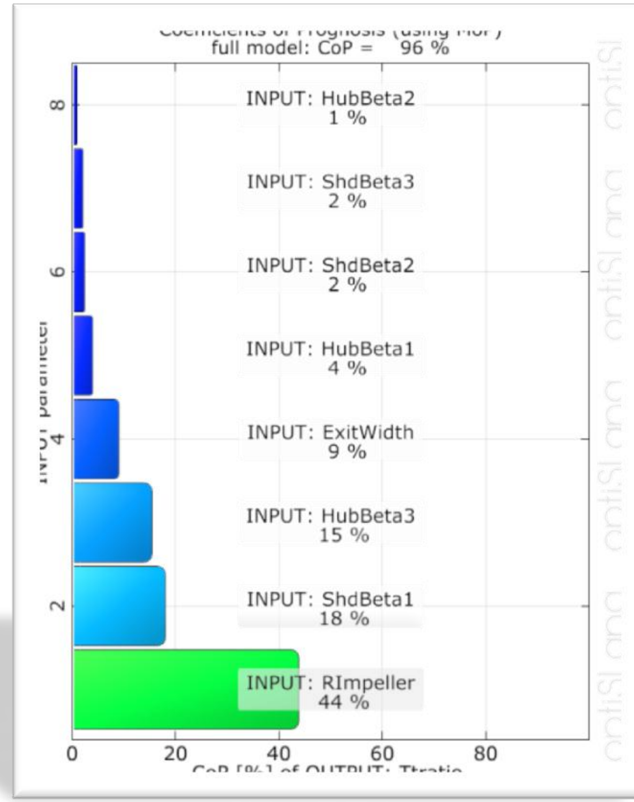
$$n_p = 1 + 2 \cdot N + \cancel{0.5 \cdot N \cdot (N-1)}$$



Coefficient of Importance

Estimation Operator:

$$\rho_{ij} = \left(\frac{E\left(X_i \cdot X_j\right)}{\sigma_{X_i} \cdot \sigma_{X_j}} \right) = \left(\frac{\sum_{k=1}^{N} \left(X_i^{(k)} - \mu_{X_i}\right) \cdot \left(X_j^{(k)} - \mu_{X_j}\right)}{\left(N-1\right) \cdot \sigma_{X_i} \cdot \sigma_{X_j}} \right)$$



Coefficient of Determination:

$$CoD = \left(\frac{E\left(Y \cdot \hat{Y}\left(X_k\right)\right)}{\sigma_Y \cdot \sigma_{\hat{Y}}} \right)^2$$

Coefficient of Importance:

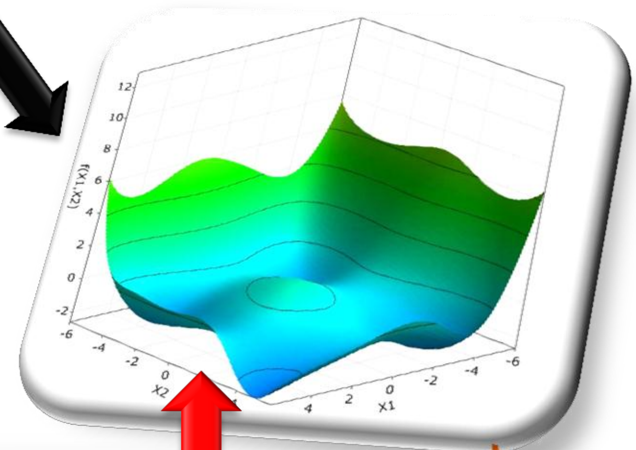
$$CoI_j = CoD\left(X_1 \ldots X_N\right) - CoD\left(X_1 \ldots X_{j-1}, X_{j+1} \ldots X_N\right)$$

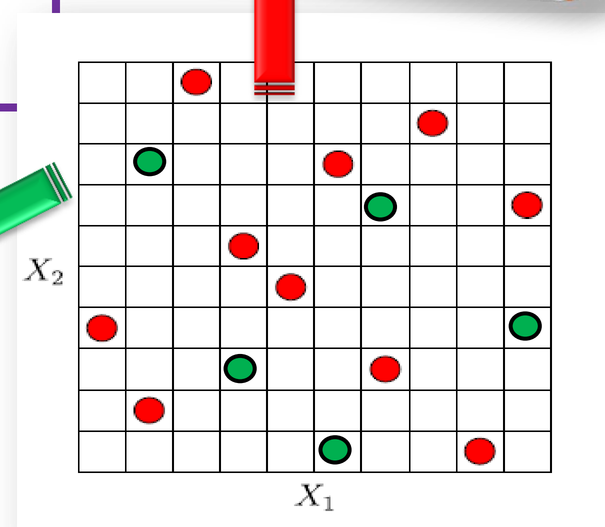
Importance Filter

- Significance Filter
- Importance Filter
- Remaining parameters are used for non-linear approximation
- Basic Points for Approximation
- Test Points for Quality Assurance

Data-Split

$$Y_k = f(x_1, \cancel{x_2}, \cancel{x_3}, \cancel{x_4}, \cancel{x_5}, x_6, \cancel{x_7}, x_6, \ldots, xN)$$





$$CoP = \left(\frac{E\left(Y \cdot \hat{Y}\right)}{\sigma_Y \cdot \sigma_{\hat{Y}}} \right)^2 = \left(\frac{\sum_{k=1}^{N} \left(y^{(k)} - \mu_y\right) \cdot \left(\hat{y}^{(k)} - \mu_{\hat{y}}\right)}{(N-1) \cdot \sigma_Y \cdot \sigma_{\hat{Y}}} \right)^2$$

Moving Least Square

MLS: p polynomials h(x) and coefficients a(x)

$$y(x) \approx \hat{y}(x) = h^T(x) \cdot a(x)$$

Weighted square error S → min

$$S(a) = \varepsilon_k^T \cdot W_k \cdot \varepsilon_k = \left(y_k - H_k^T \cdot a\right)^T \cdot W_k \cdot \left(y_k - H_k^T \cdot a\right) \rightarrow \min$$

Leads to equation for coefficients a(x):

$$\frac{\partial S}{\partial a} = H_k \cdot W_k \cdot H_k^T \cdot a - H_k \cdot W_k \cdot y_k = 0$$

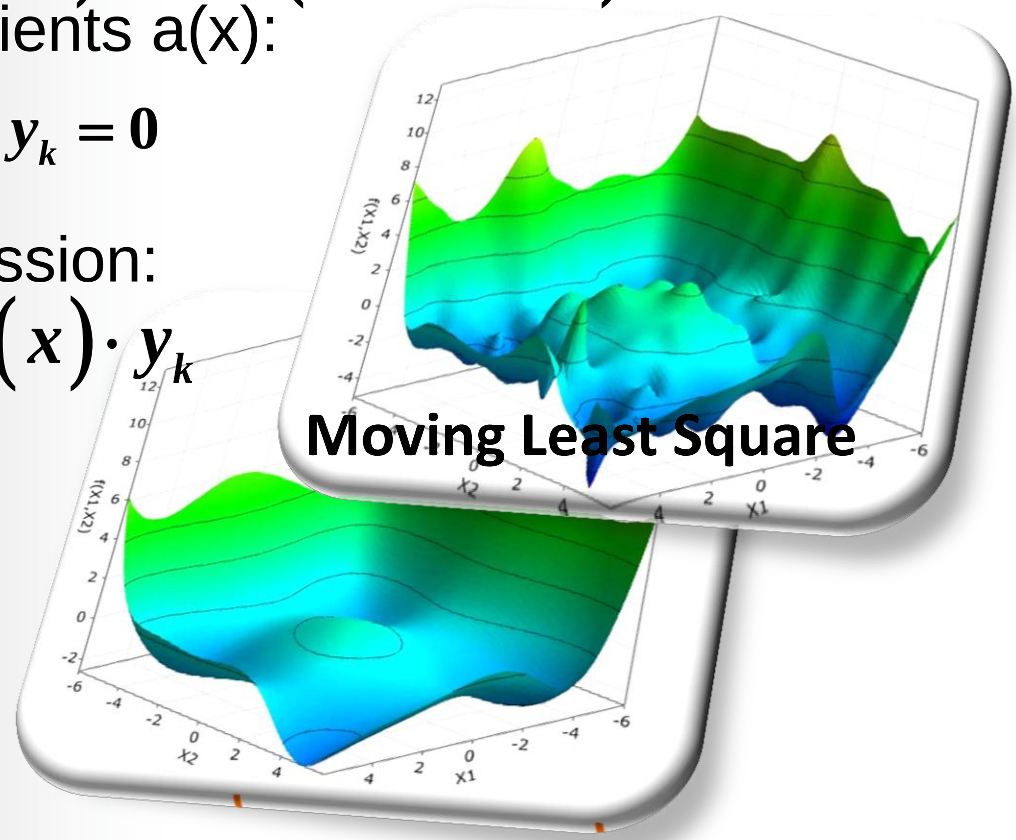
Moving Least Square Regression:

$$\hat{y}(x) = h^T(x) \cdot A(x)^{-1} \cdot B(x) \cdot y_k$$

$$A(x) = H_k \cdot W(x) \cdot H_k^T$$

$$B(x) = H_k \cdot W(x)$$

$$W(x) = diag\left[w(x)\right]$$



Coefficient of Prognosis, CoP

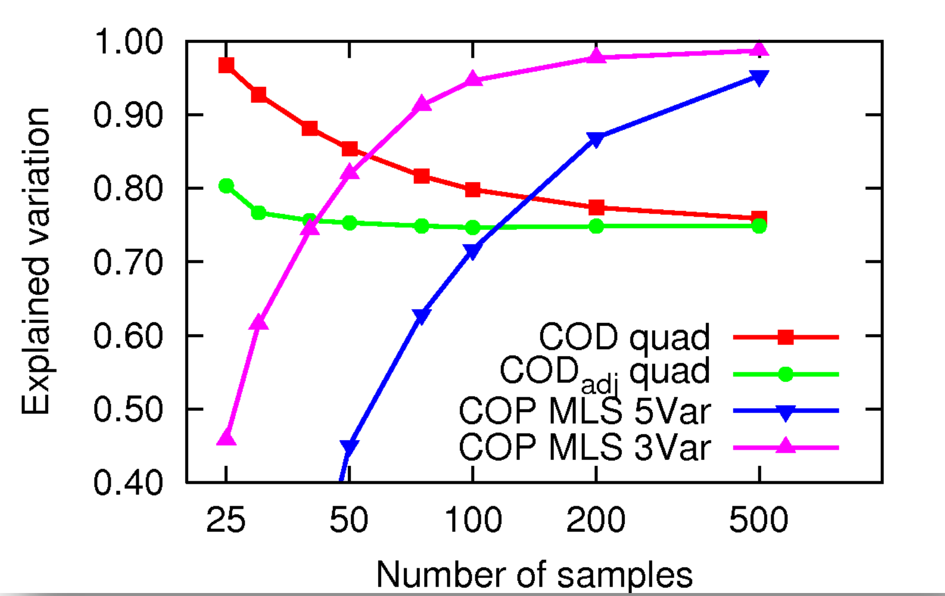
- Fraction of explained variation of prediction

$$CoP = 1 - \frac{S_E}{S_T}$$

$$S_T = \sum (Y_i - \mu_{Yi})^2$$

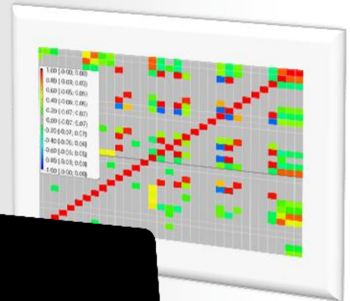
$$S_E = \sum \left(Y_i - \hat{Y}_i\right)^2$$

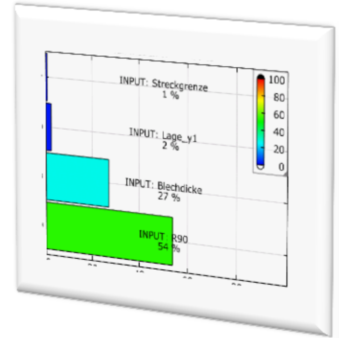
$$CoP_i = CoP \cdot S_{Ti}$$

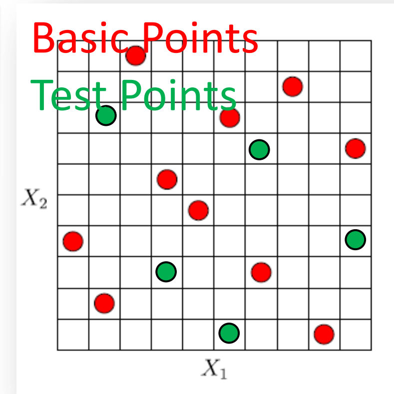


- Estimation of CoP by cross validation using a partitioning of available the samples
- CoP increases with increasing number of samples
- CoP is suitable for interpolation and regression models
- With MLS continuous functions also including coupling terms can be represented with a certain number of samples
- Prediction quality is better if unimportant variables are removed from the approximation model

Meta-Model of Optimal Prognosis, MoP







Significance Filer

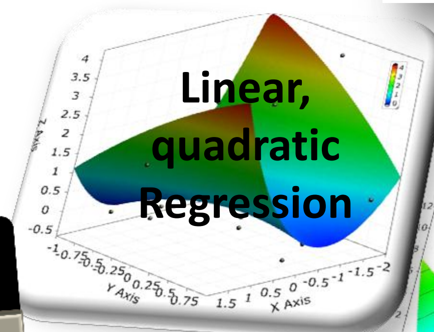
Importance Filter

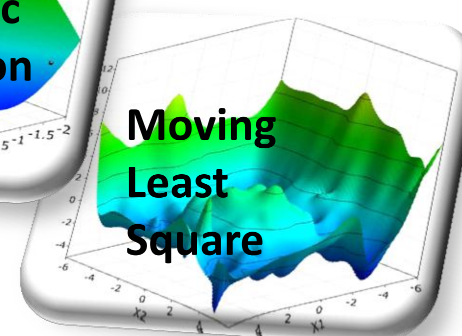
Test-Data Point Split

Response Surface

Coefficient of Prognosis

Variation of Filter Limits…

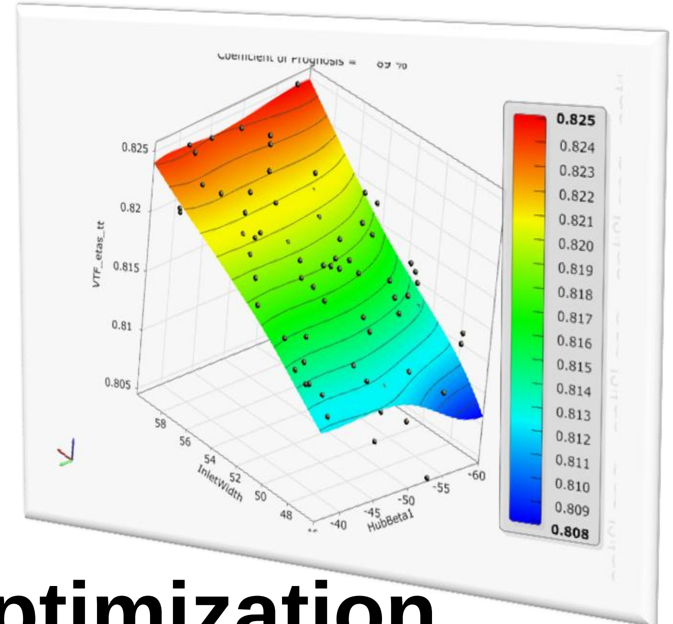


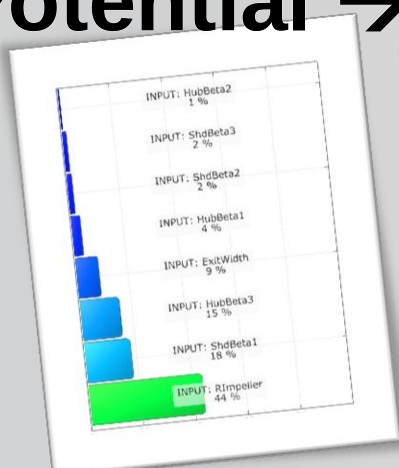


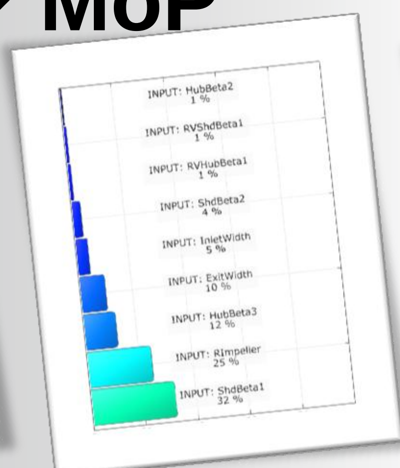
$$CoP = \left(\frac{E\left(Y \cdot \hat{Y}\right)}{\sigma_Y \cdot \sigma_{\hat{Y}}} \right)^2 = \left(\frac{\sum_{k=1}^{N} \left(y^{(k)} - \mu_y\right) \cdot \left(\hat{y}^{(k)} - \mu_{\hat{y}}\right)}{\left(N-1\right) \cdot \sigma_Y \cdot \sigma_{\hat{Y}}} \right)^2$$

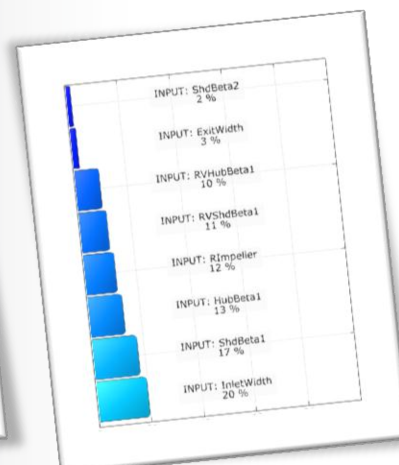
Value of CoP and MoP

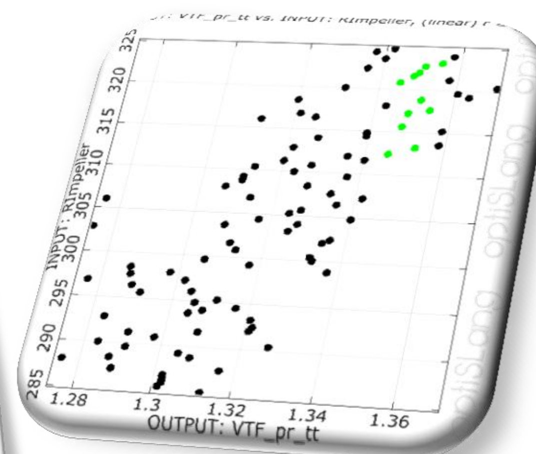
- **Statistical Reliability → CoP**
- **Parameter Reduction**
  - Number of Parameters
  - Min/Max Parameter Bounds
- **Response Surface shows Optimization Potential → MoP**





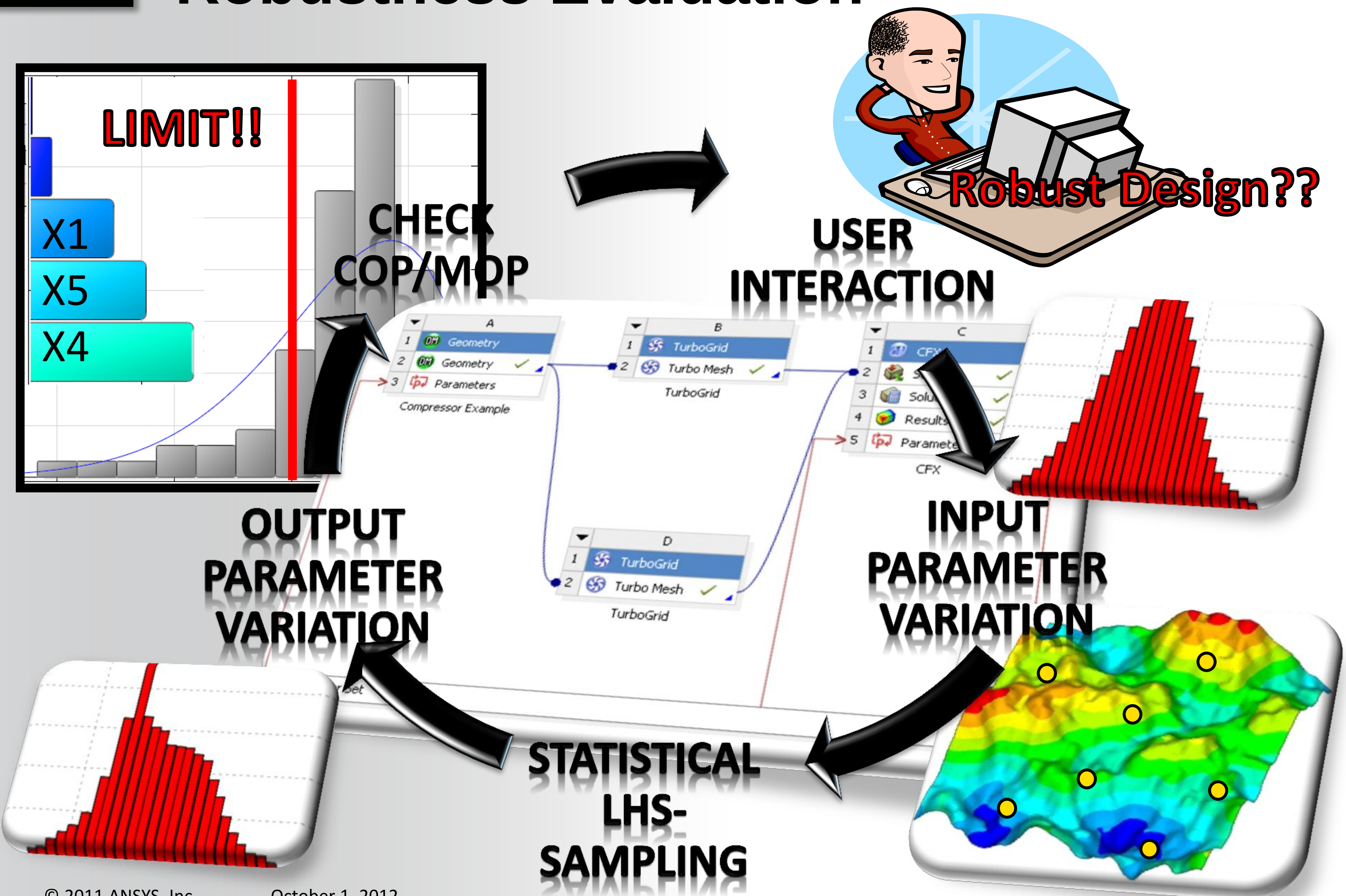




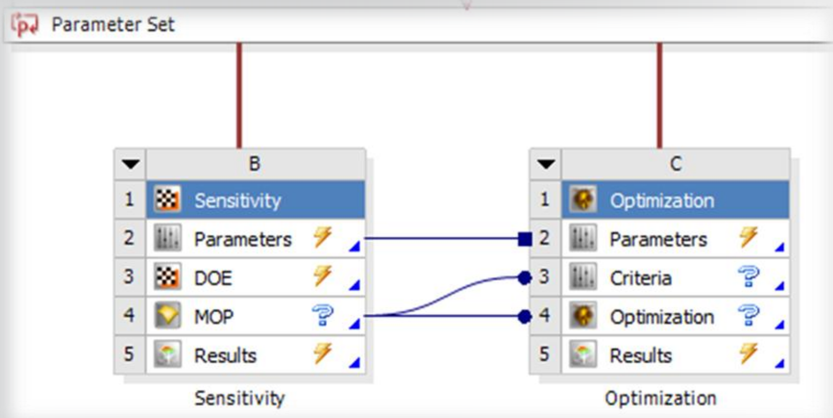


  October 1, 2012

Robustness Evaluation

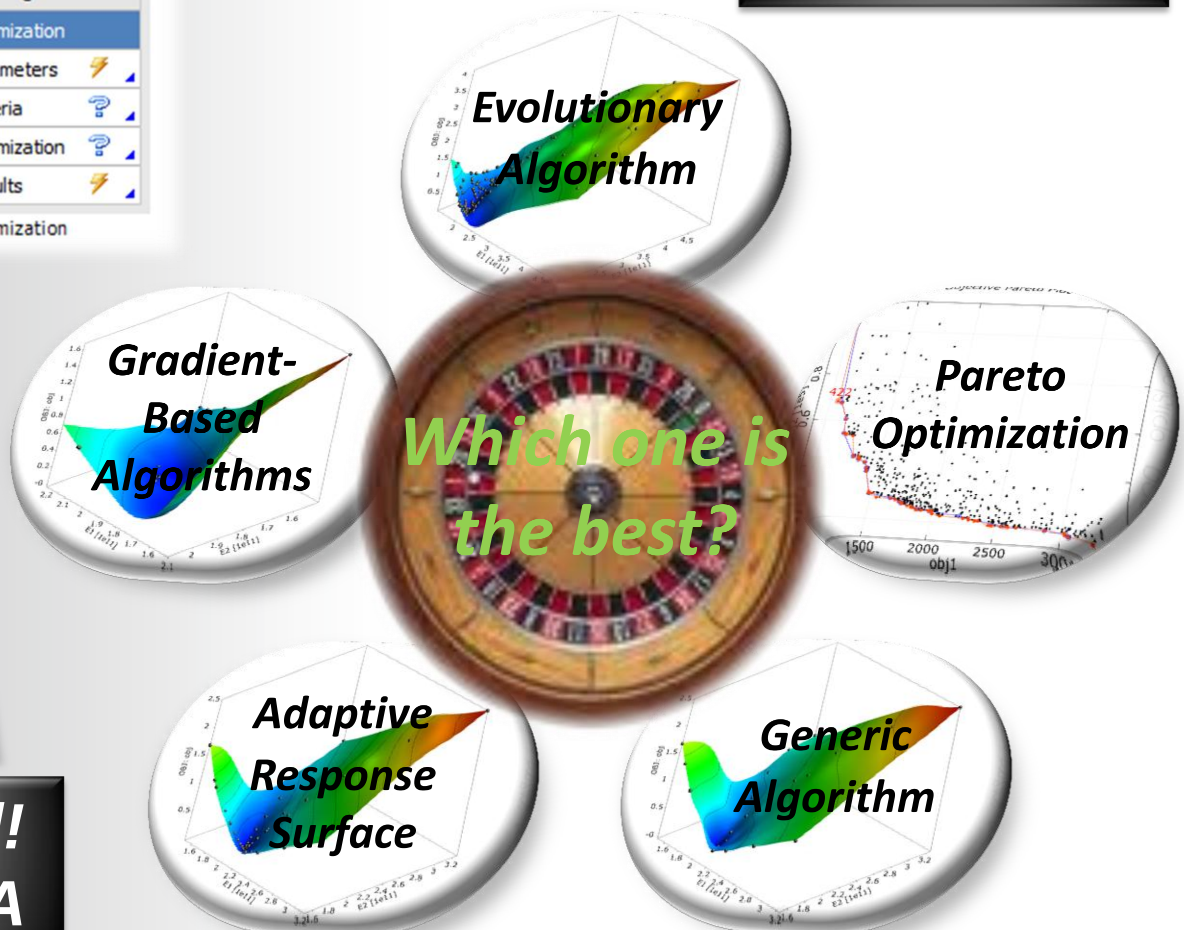


Design Optimization



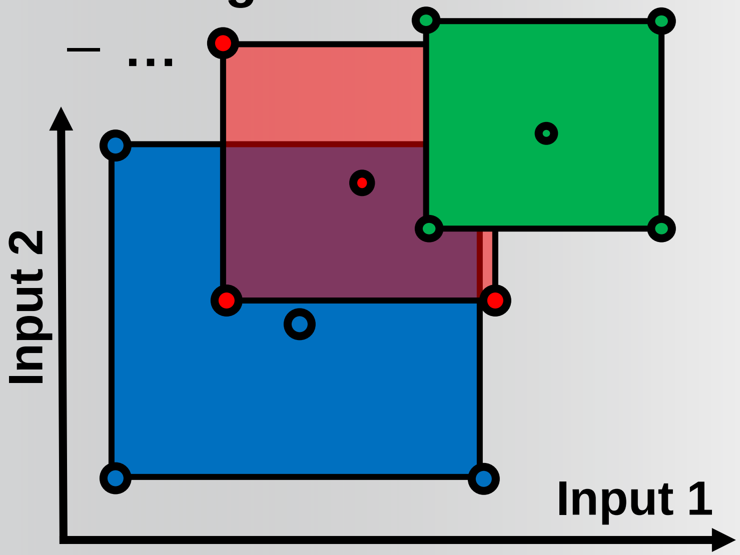
Strategy is required! and derived from SA

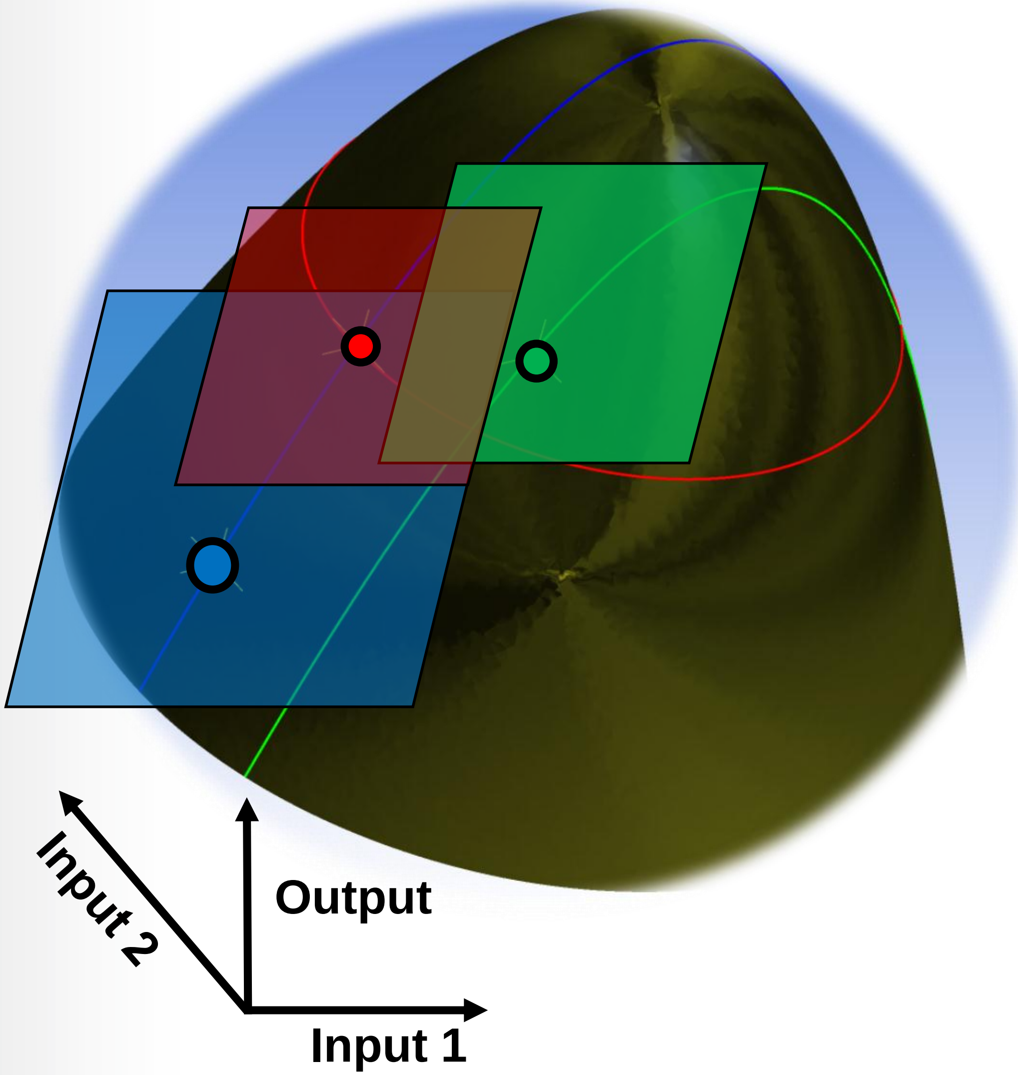
Optimization Algorithms:



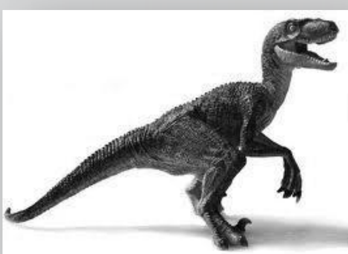
Adaptive Response Surface Method

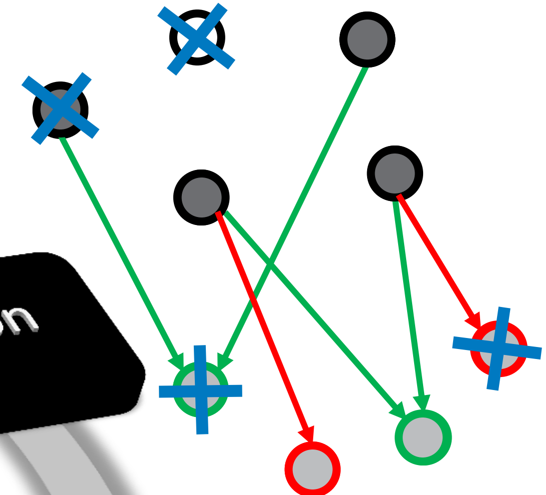
- Start Point
- Initial Sample
  - Approximated Response Surface
  - Best Point
  - New Sample with smaller Range
  - …

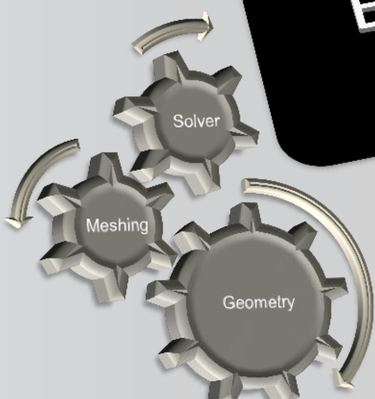




Evolutionary Algorithms







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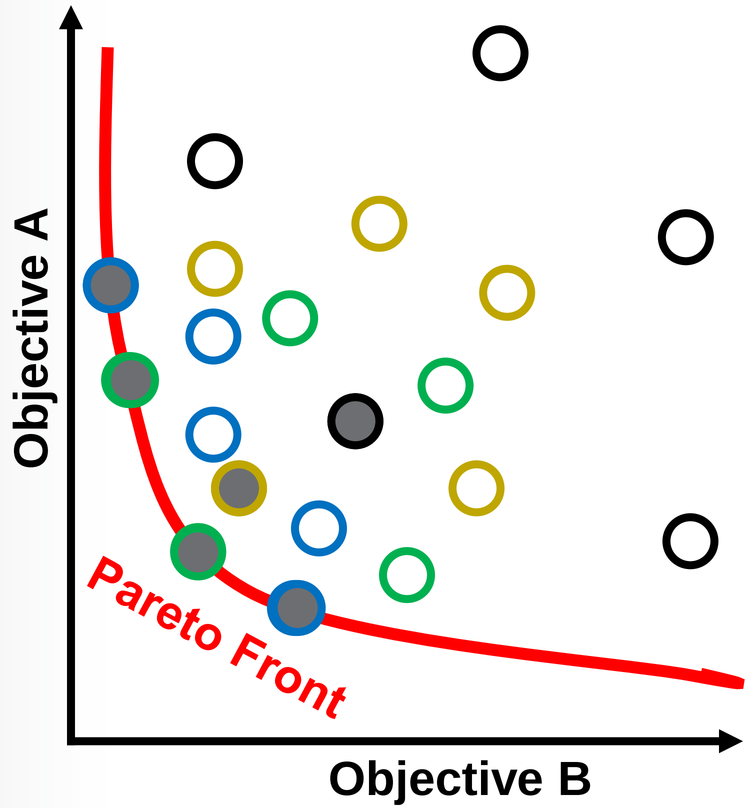
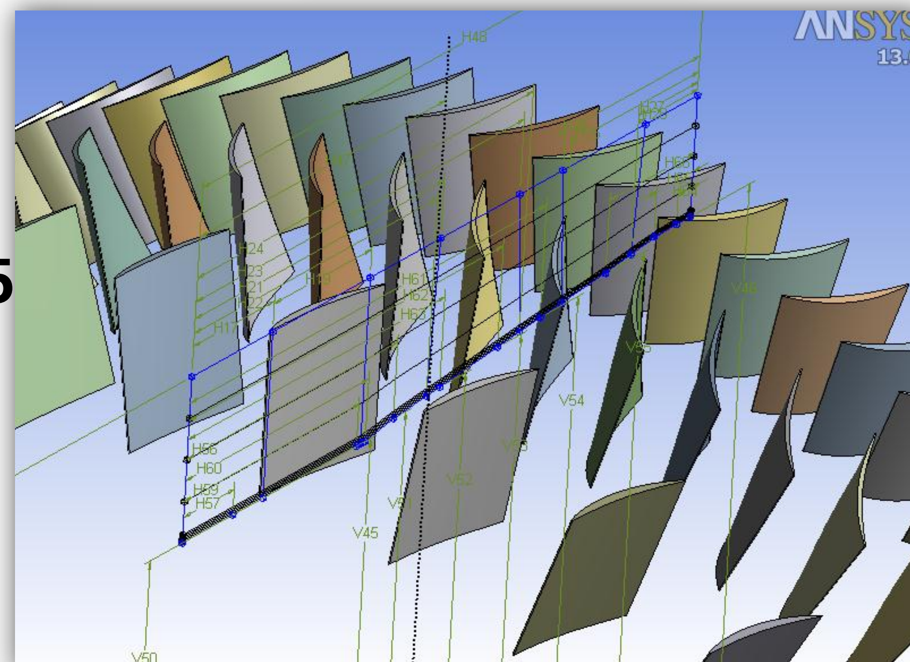
Pareto Optimization

- **Initial Generation**
  - Select best
- **Second Generation**
  - Select best
- **Third Generation**
  - Select best
- **Fourth Generation**
  - Select best
- …

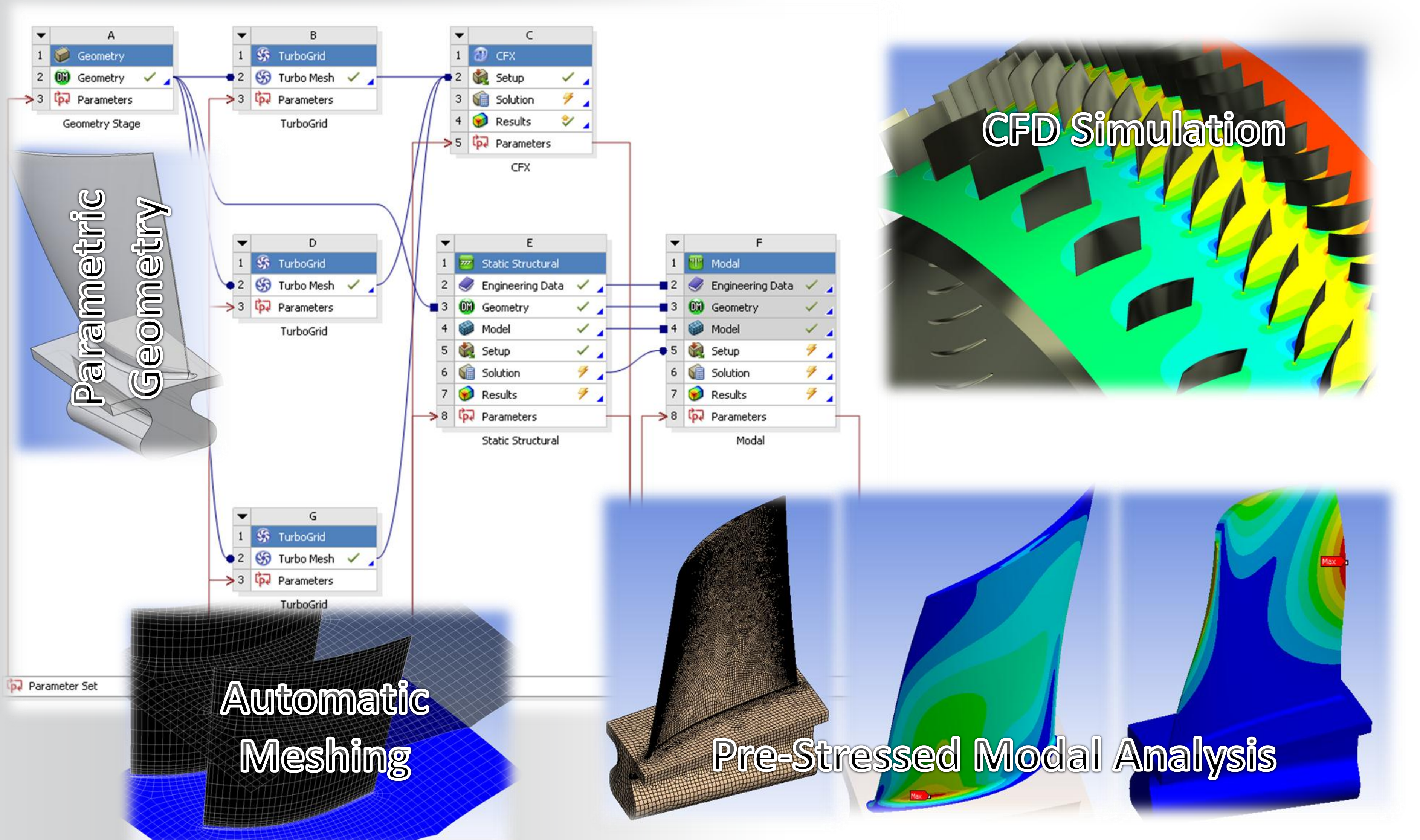


Primary Design, PCA Ltd.

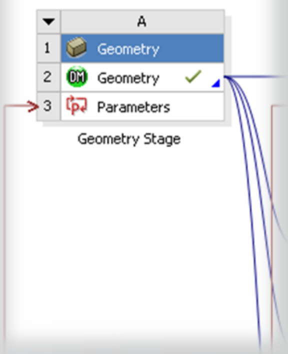
- **1.5 Stage Axial Compressor**
- **IGV(n=37)**
- **R1 (n=71, Gap @ Shroud 2% Span)**
- **S1 (n=91, Gap @ Hub 2% Span)**
- **Pressure Ratio Π=1.4**
- **Mass Flow Rate 10.6 [kg/s]**
- **Diameter d = 0.525 [m]**
- **Rot. Vel. Ω = 9300 [rpm]**
- **Blade Mach Number M_u=0.75**
- **Specific Speed n_s= 1.3**
- **Specific Diameter d_s=2.3**
- **Load Coefficient Ψ=0.45**



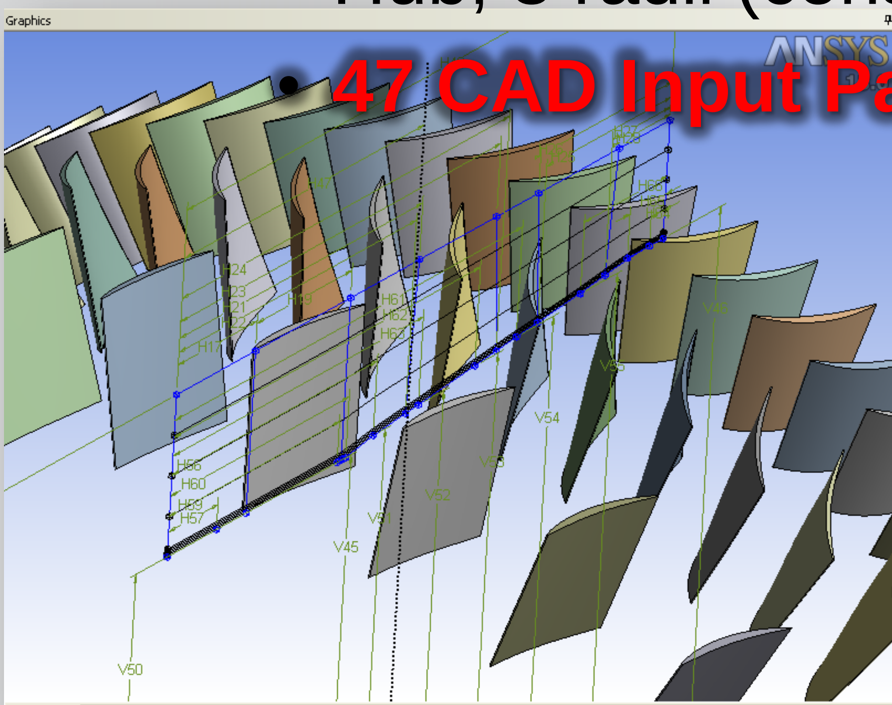
Application Overview

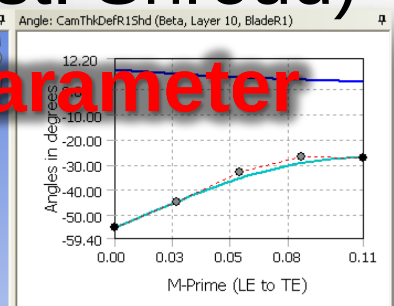


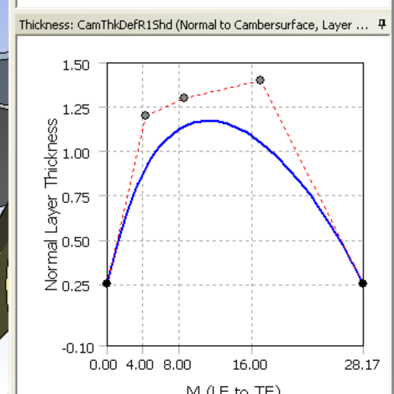
Geometry Parameterization

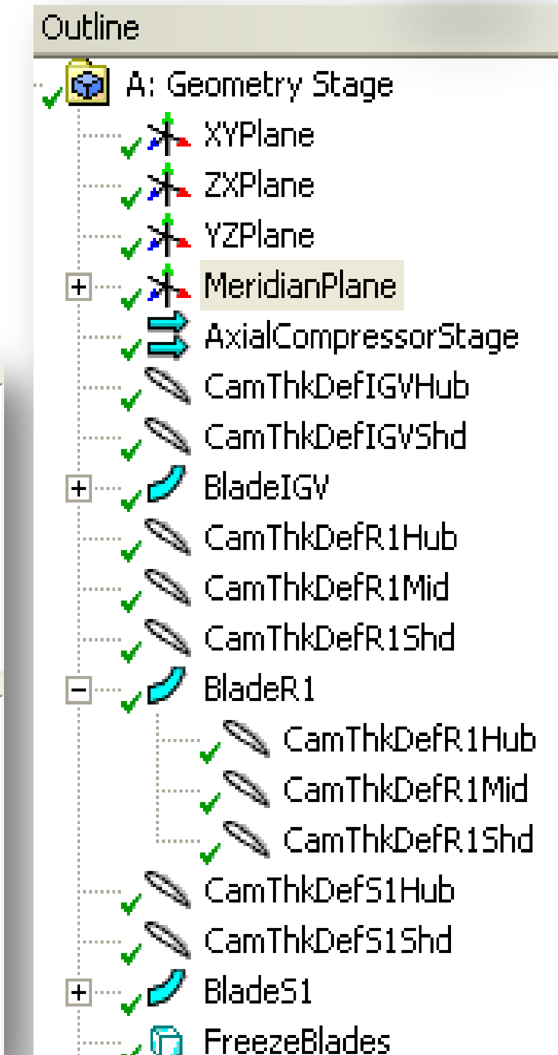


- Camber/Thickness for
  - IGV, R1, S1; 2-3 Layers
  - 5 β_i per Layer, 3xThk
- Hub, 8 radii (const. Shroud)
- **47 CAD Input Parameter**

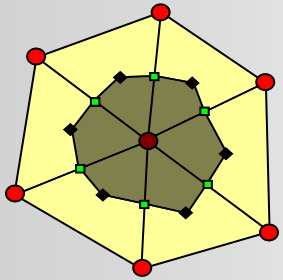






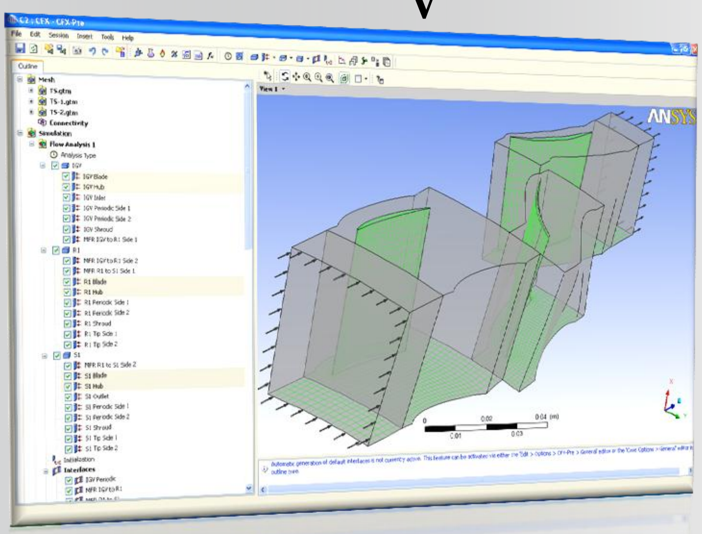


CFD Simulation



- **CFD Solver: CFX**
- **Nodal based FVM**

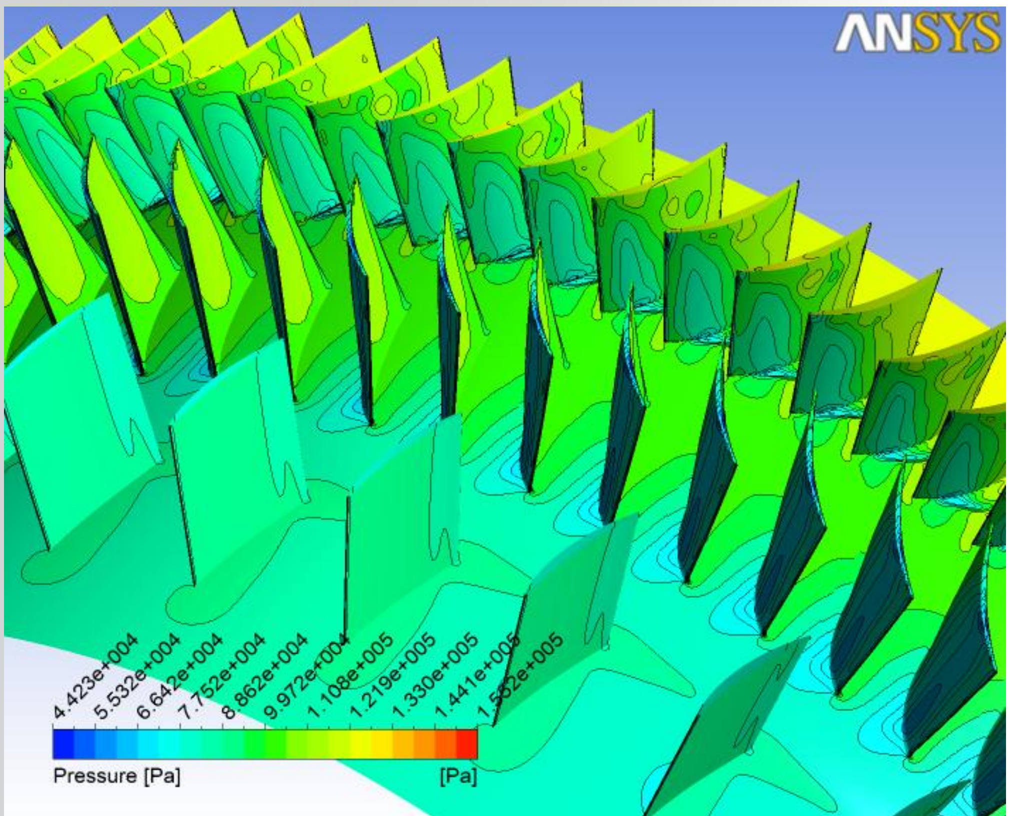
$$\frac{\partial}{\partial t}\int_V \rho\varphi\, dV + \oint_A \rho\varphi\, \mathbf{V}\cdot d\mathbf{A} = \oint_A \Gamma\nabla\varphi\cdot d\mathbf{A} + \int_V S_\varphi\, dV$$

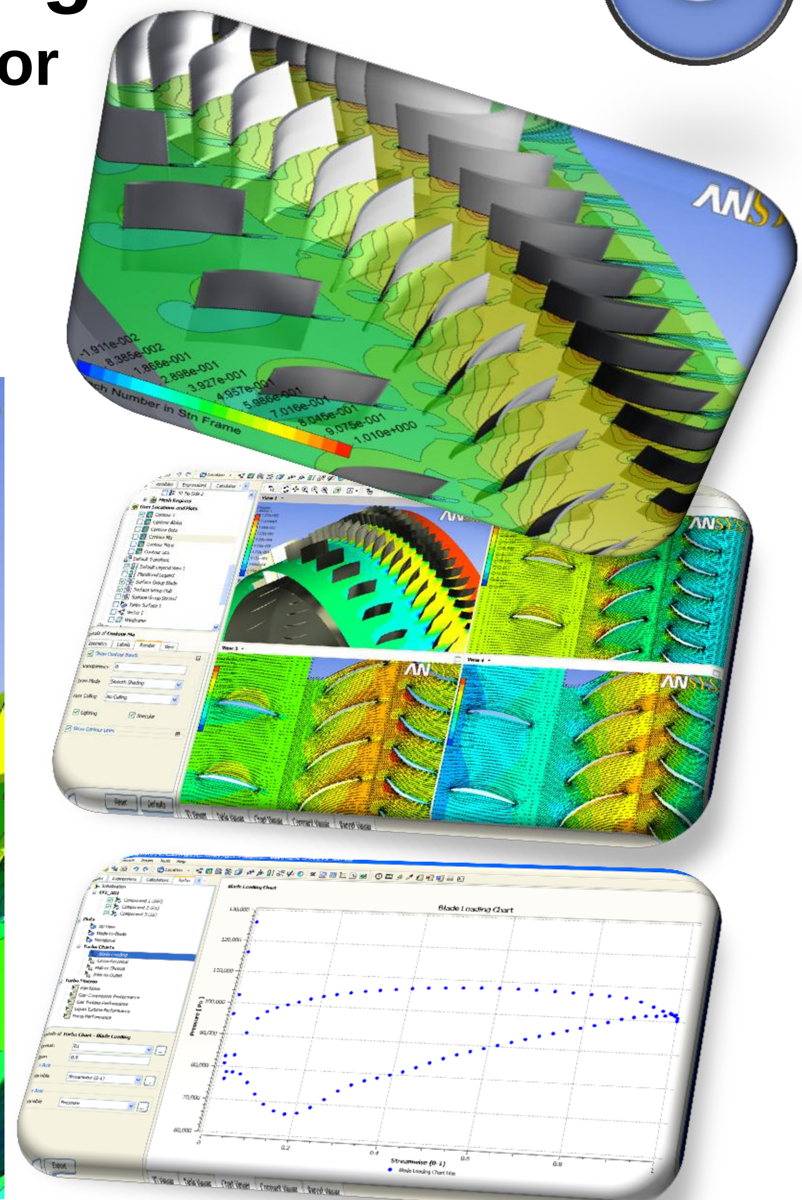


- **Coupled Solution + AMG**
  - Mass & Momentum, Energy…
- **Turbulence Model:**
  - Shear Stress Transport
- – **Two sector by passage, MFR:**
  - Profile-/Time Transformation
  - Periodic Interface

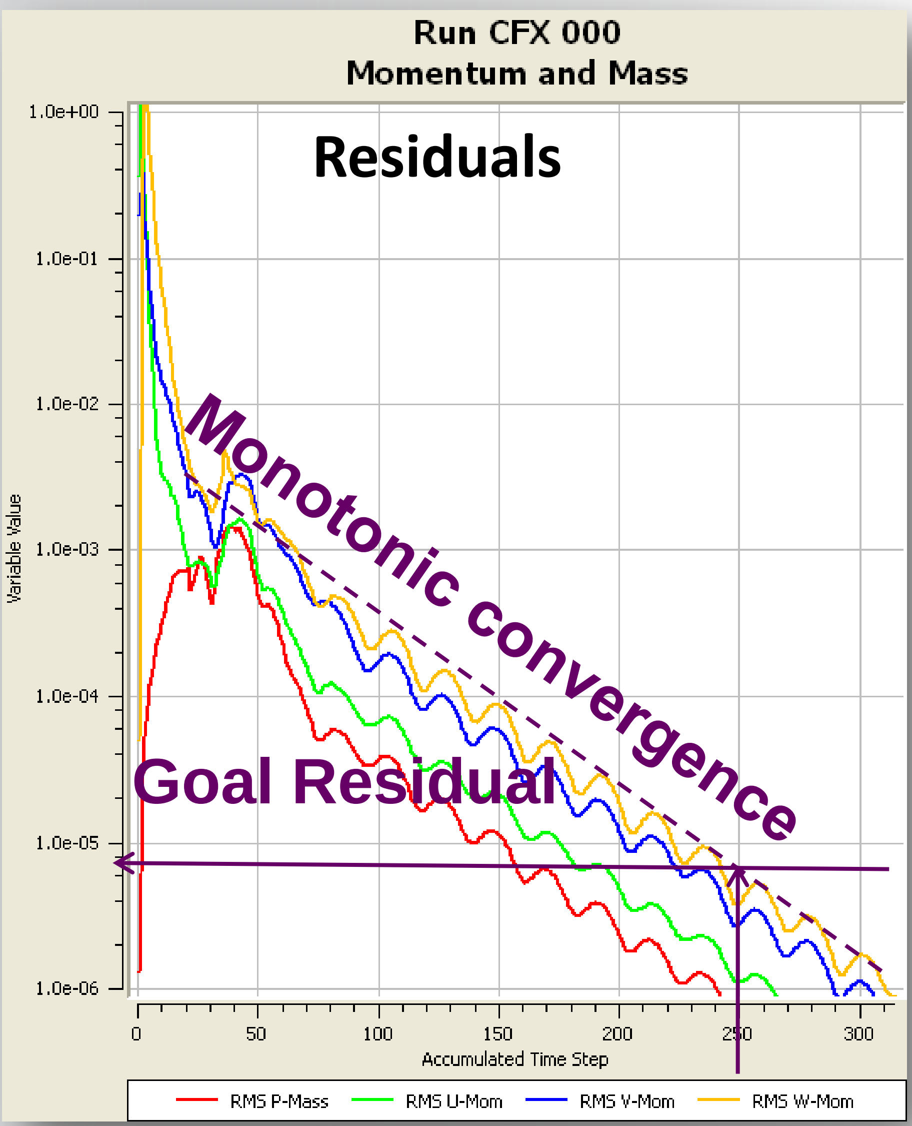
CFD Post Processing

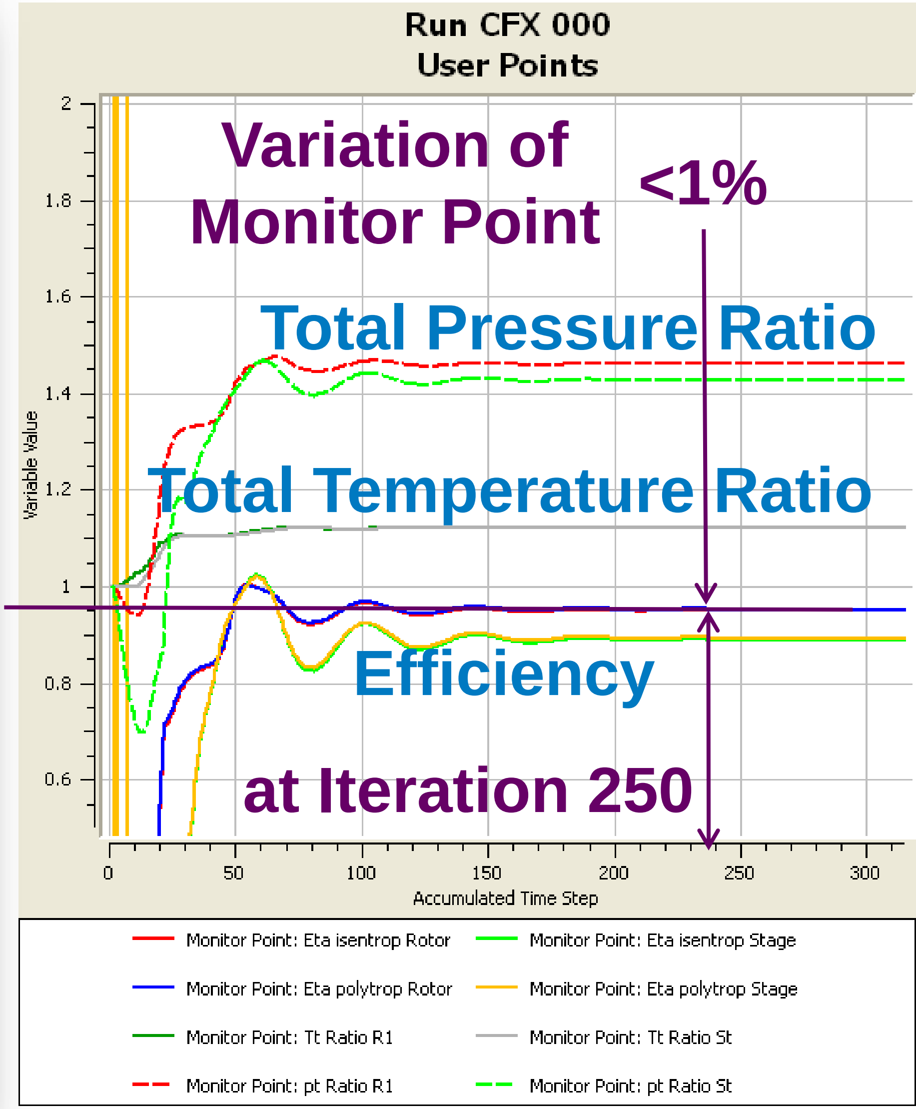
- General Post-Processor
- Turbo Mode
- Highly Automated
- Customize able



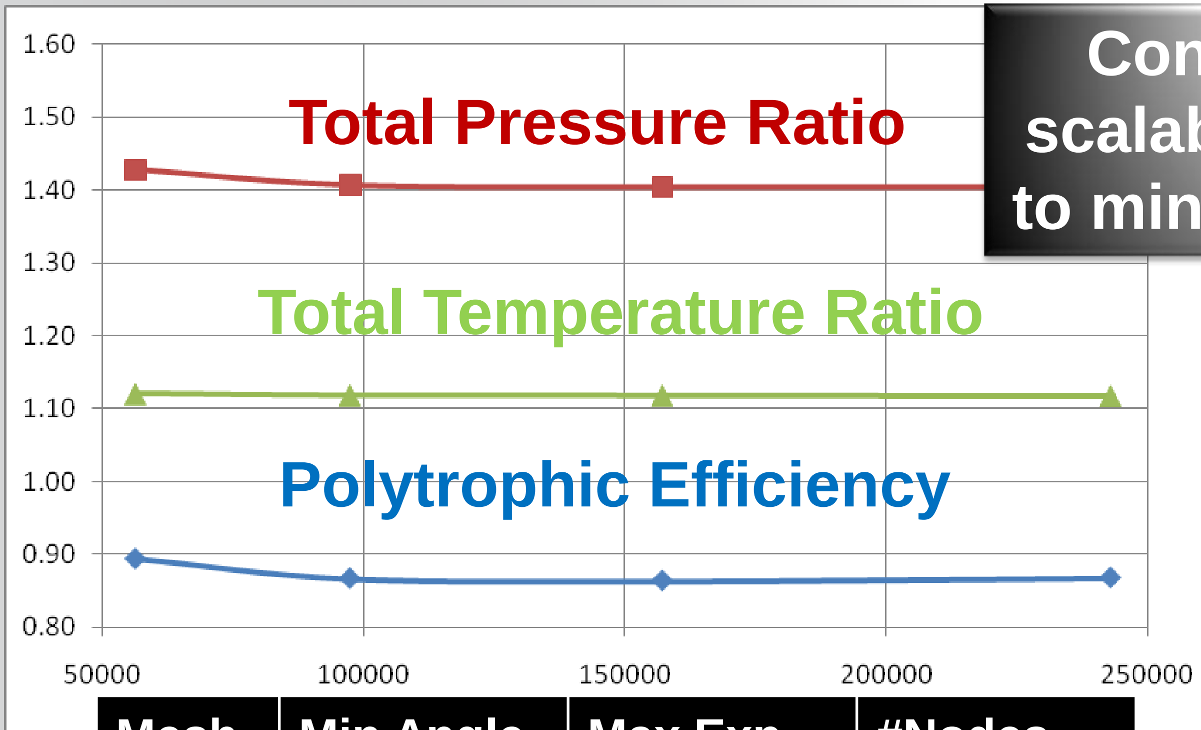


Quality Assurance Iteration Error



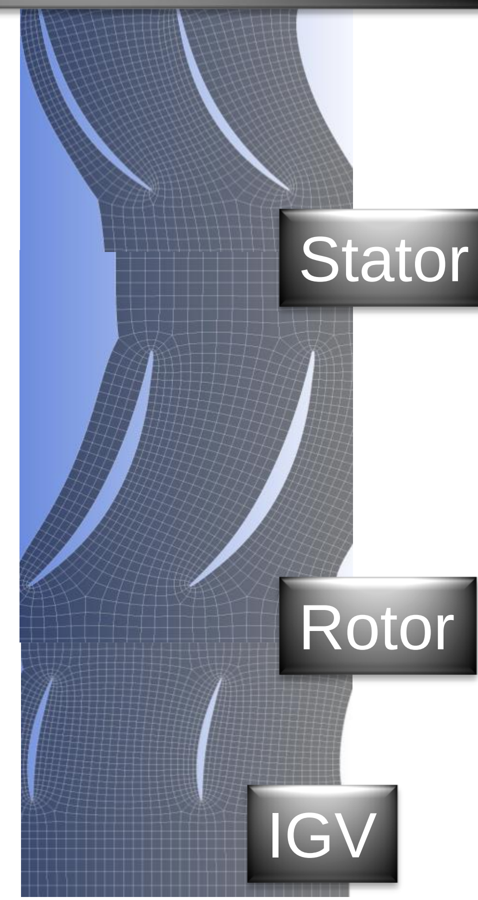


Quality Assurance Discretization Error



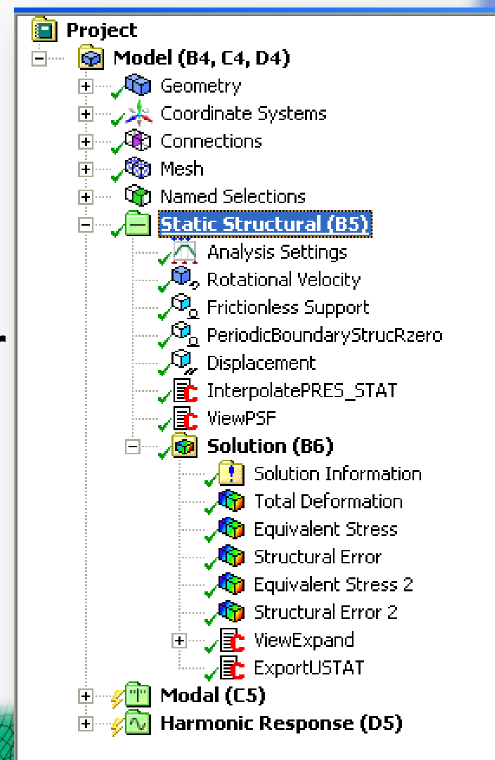
Convergence study on scalable high quality mesh to minimize numerical error

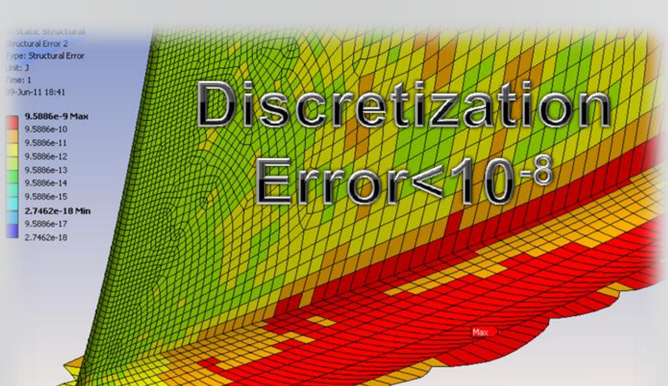
Mesh	Min Angle	Max Exp.	#Nodes
1	29.59	15.2	~50000
2	30.85	6.69	~100000
3	31.52	4.90	~150000
4	35.73	4.60	~250000

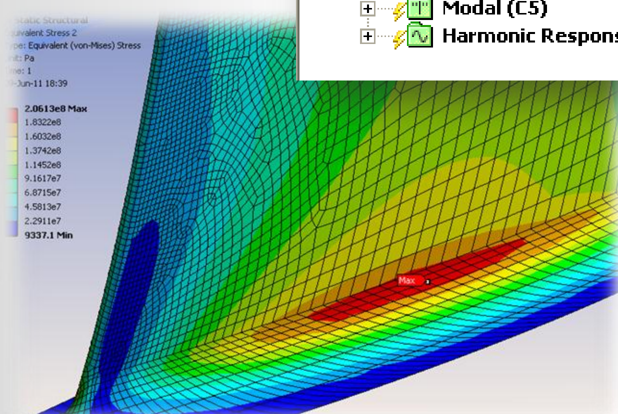


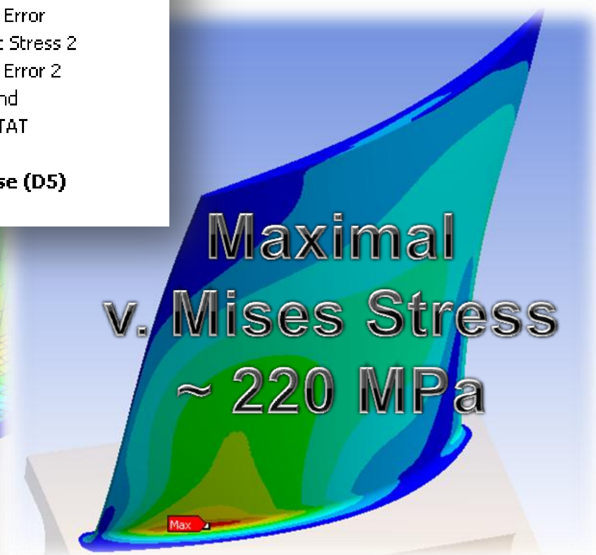
Static Structural (Pre-Stress)

- Static Solution:
  - Displacement
  - Strain & Stress
  - Numerical Error
  - Pre-Stress for further Analysis

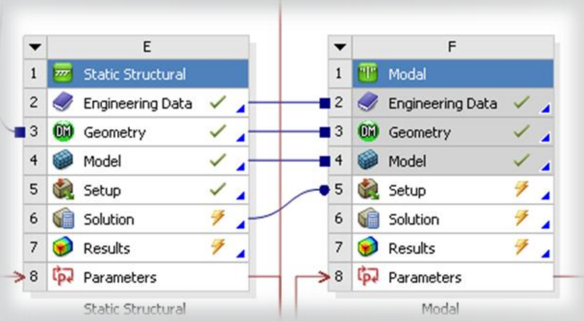






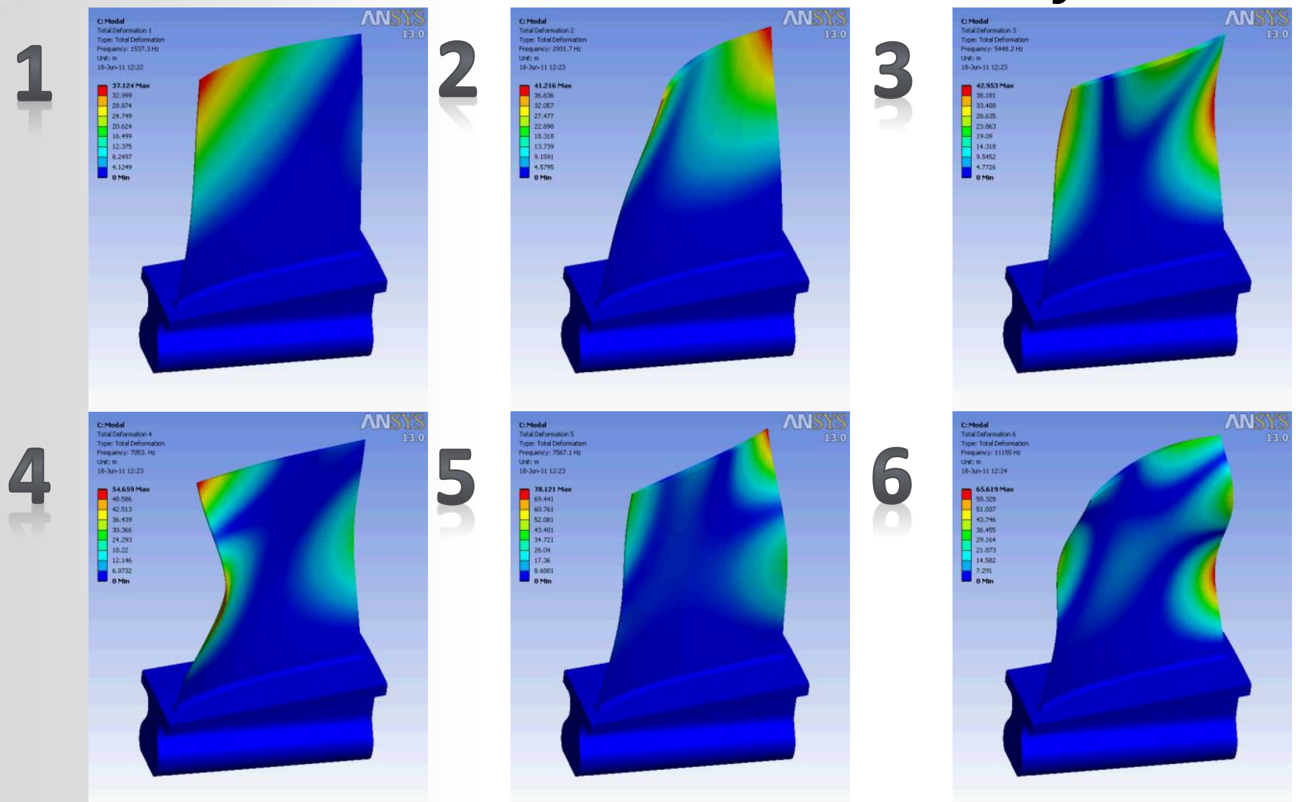


Modal Analysis

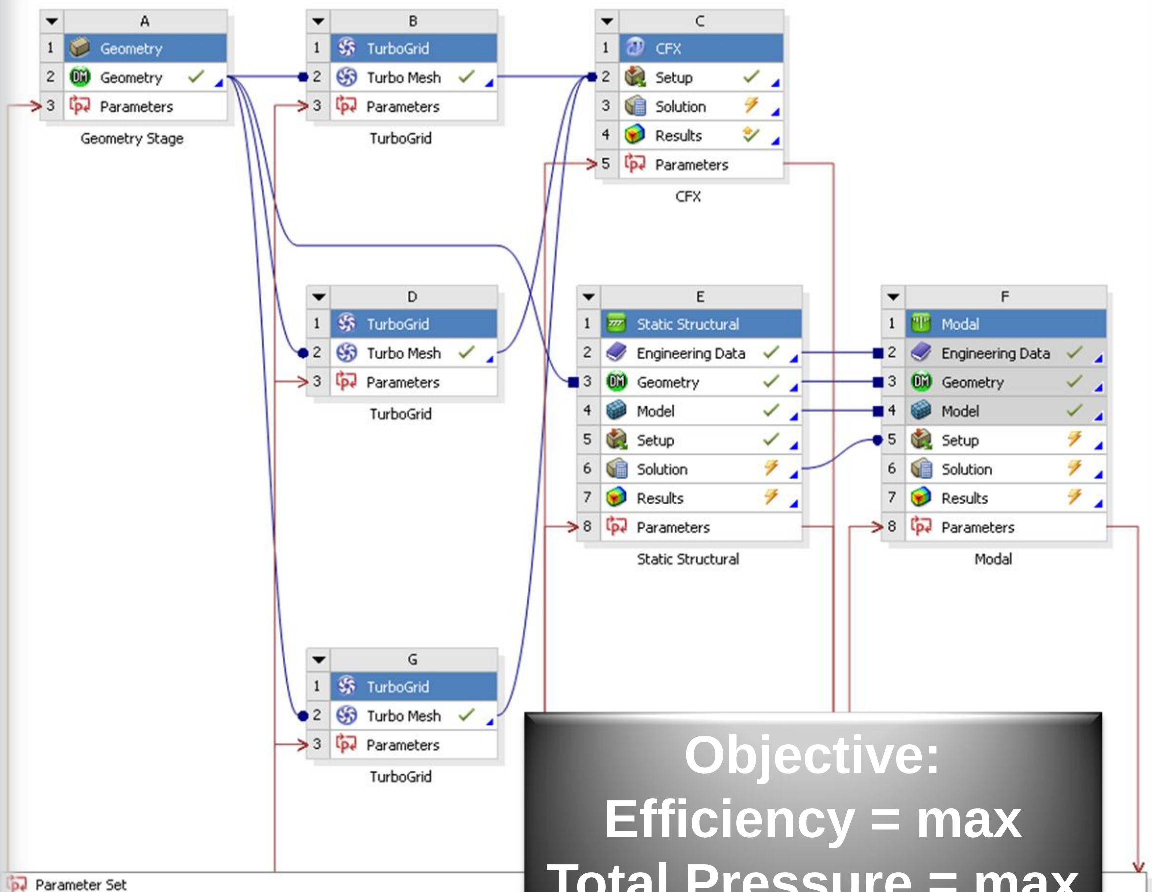


- Pre-Stressed Modal Analysis:
  - Eigen Frequencies and Vectors
  - Data for further MOR-Analysis

	Mode	Frequency [Hz]
1	1.	1537.3
2	2.	2931.7
3	3.	5448.2
4	4.	7053.
5	5.	7567.1
6	6.	11155



Process and Objectives



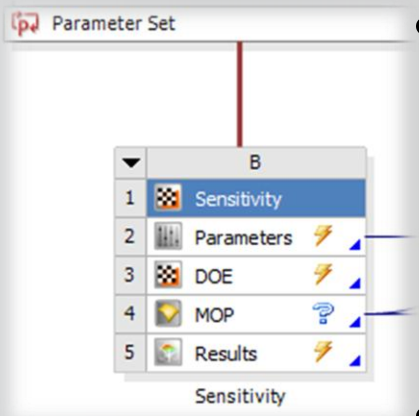
Objective:
Efficiency = max
Total Pressure = max
Stress < Limit
No Resonance

Input Parameters	**47 (59) Input Parameter**
Geometry Stage (A1)	
TurboGrid (B1)	
TurboGrid (D1)	
TurboGrid (G1)	
CFX (C1)	
P16	nPitchS1
P15	nPitchR1
P14	nPitchIGV
P17	myAirCP
P18	myAirR
P19	myomega
P20	mymass
P21	Ttin
P22	ptin
Static Structural (E1)	
P89	Face Sizing Element Size
P90	Mesh Max Size
P91	Mesh Min Size
P92	Mesh Max Face Size
P93	Rotational Velocity Z Component
P94	ViewExpand ARG1
P111	Density
P112	Young's Modulus
P113	Poisson's Ratio
Modal (F1)	
New input parameter	New name
Output Parameters	
Charts	

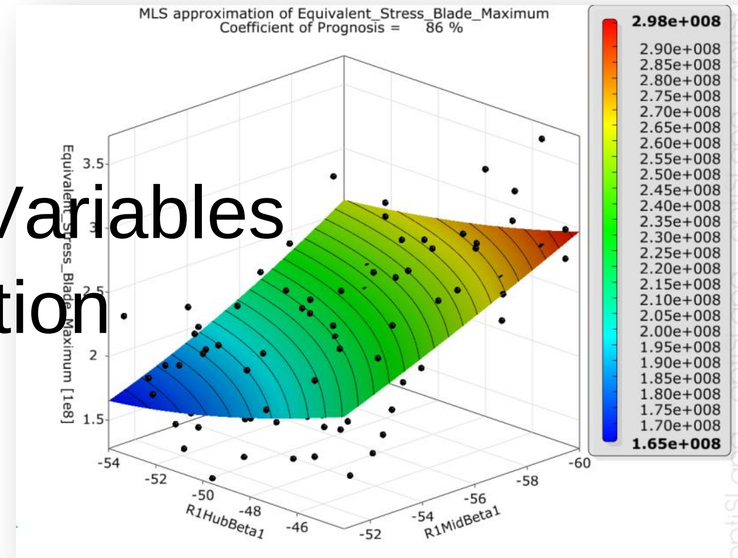
11 Input Constraints

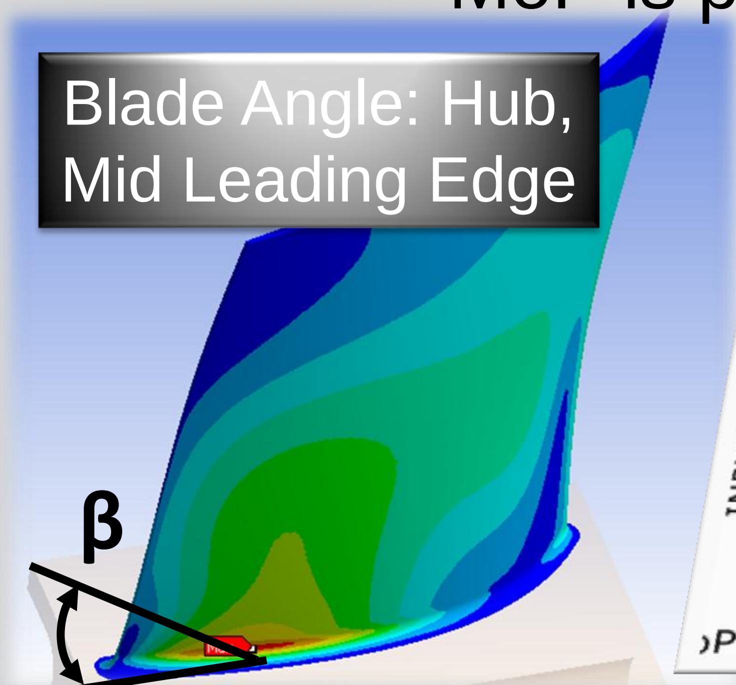
24 Output Parameter

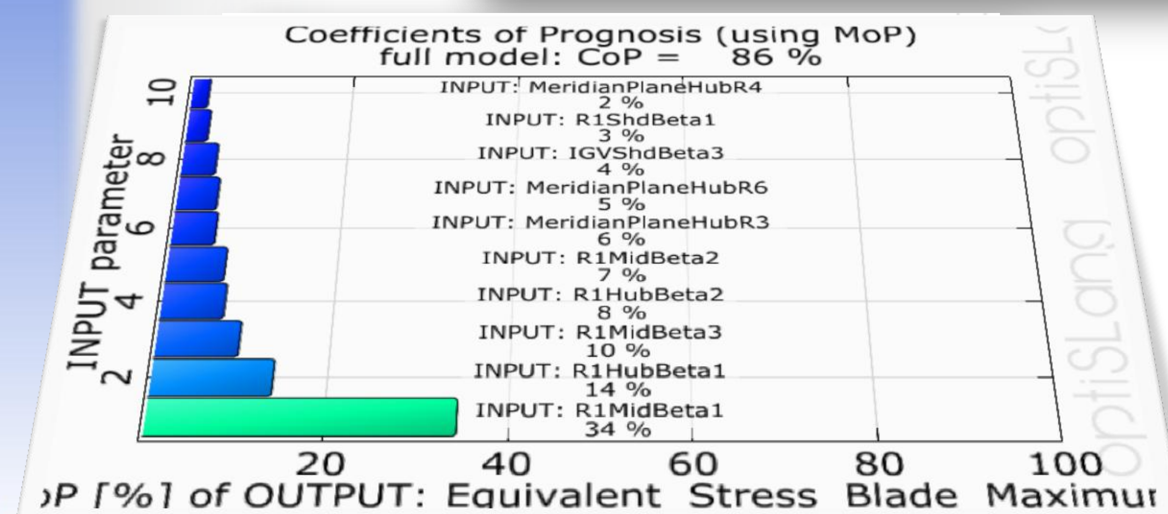
Maximal Stress



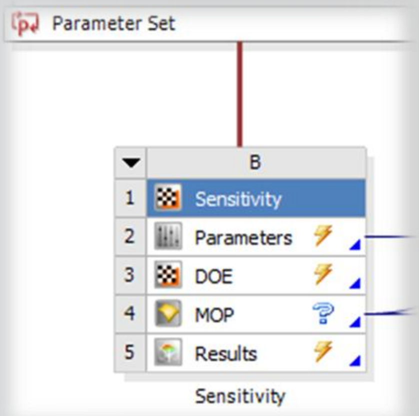
- CoP=86%
  - Statistic is reliable
  - Detect important Variables
  - Parameter Reduction
- MoP is plausible

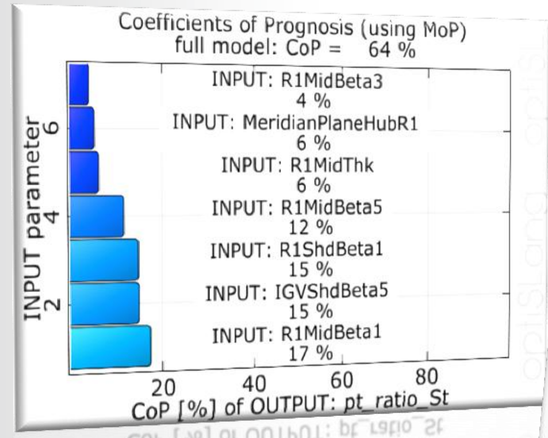


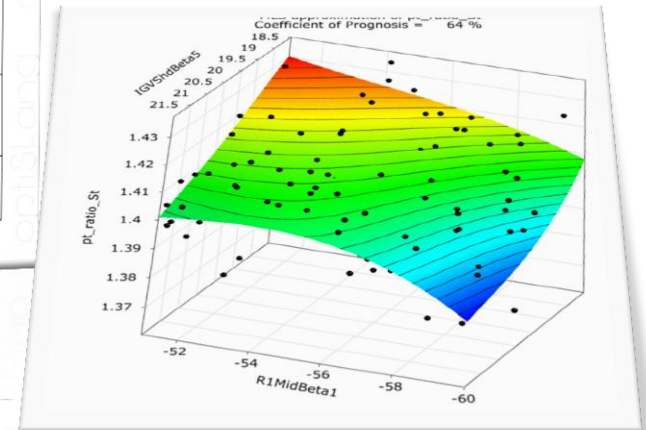




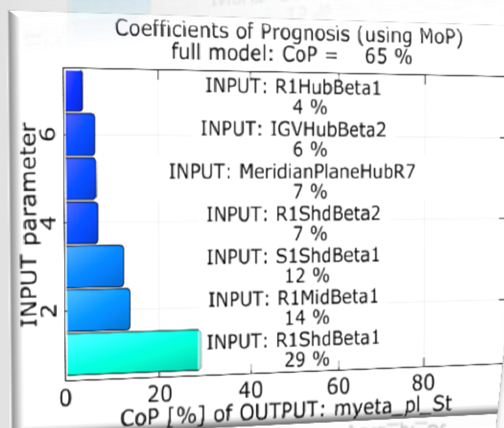
Aero Dynamic

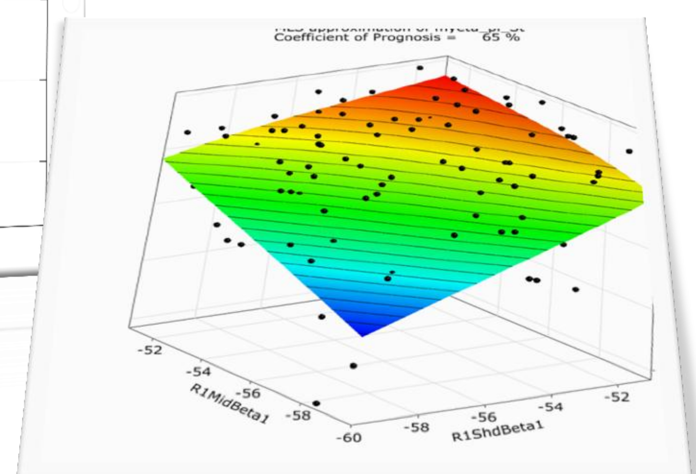




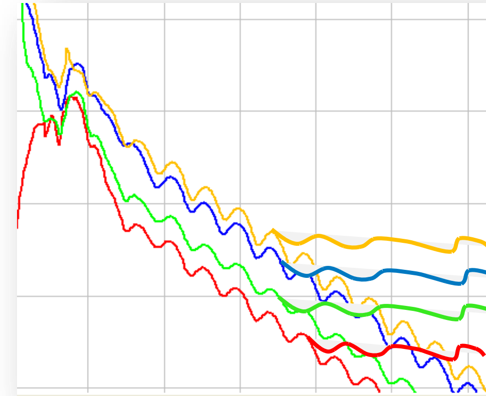


- CoP=64% and 65%
  – small value
  – Numerical error?
  – Model error?
- Important Variables
  – Parameter Reduction
- MoP is plausible

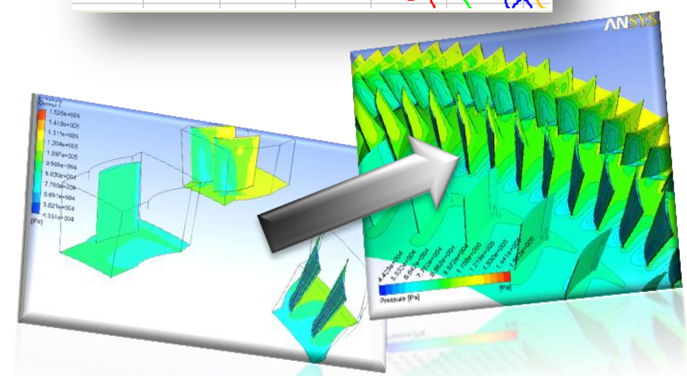


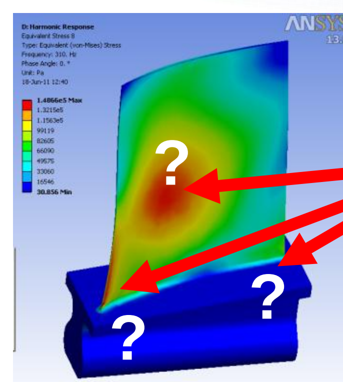


Trouble Shooting for small CoP



- Number of Evaluated Designs?
  - Check CoP(80)~CoP(150)
- Numerical Error?
  - Best-Practice!
- Model Error?
- Multiple-Mechanisms
  - Use alternative Output
- Options:
  - Design Optimization
  - Meta-Model in Subspace



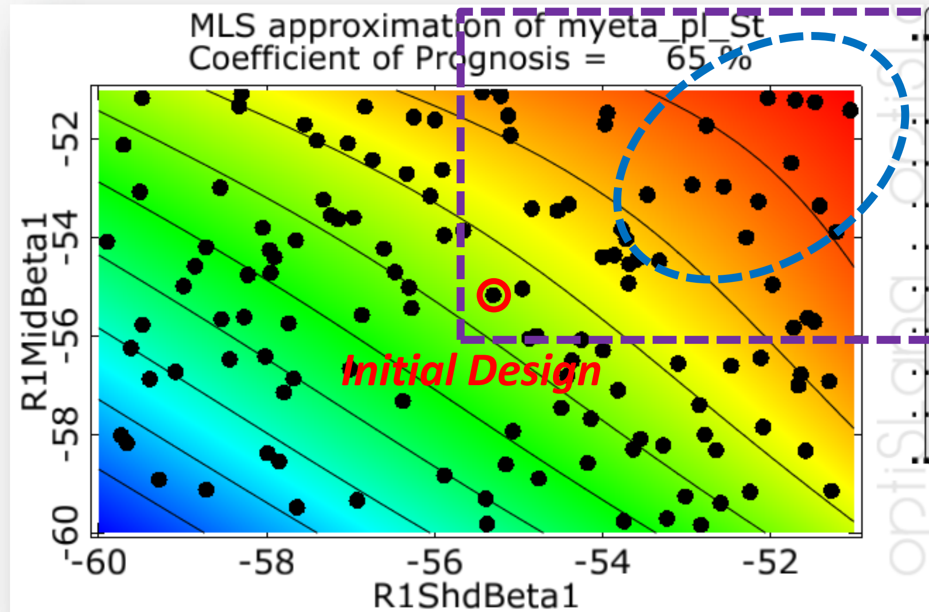


Where is the maximal Stress?

Design Optimization, Strategy

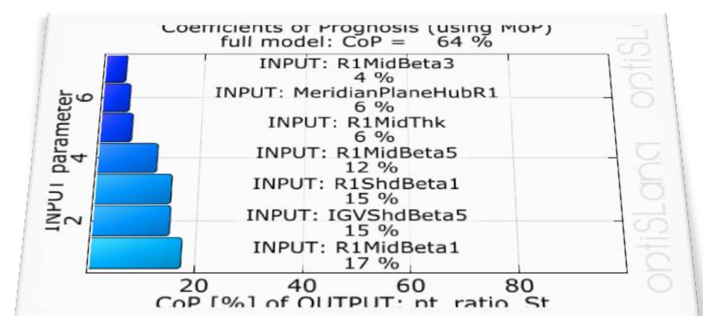
Sensitivity Analysis:

- Shows potential
- Indicates global optimum
- Parameter reduction
- Modify parameter space

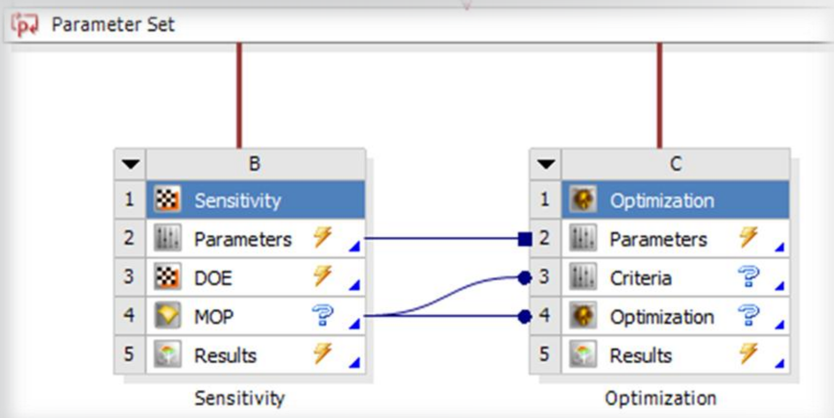


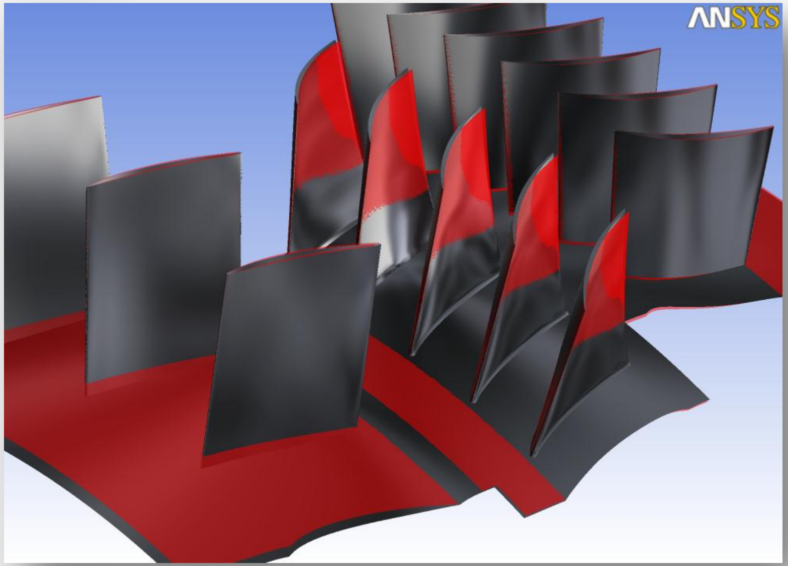
Strategy:

- Get best Design from SA/MoP
- Evaluate this Design and get initial for:
- Optimization in sub space: ARSM
    - Small Number of Parameter
    - Global Optimum



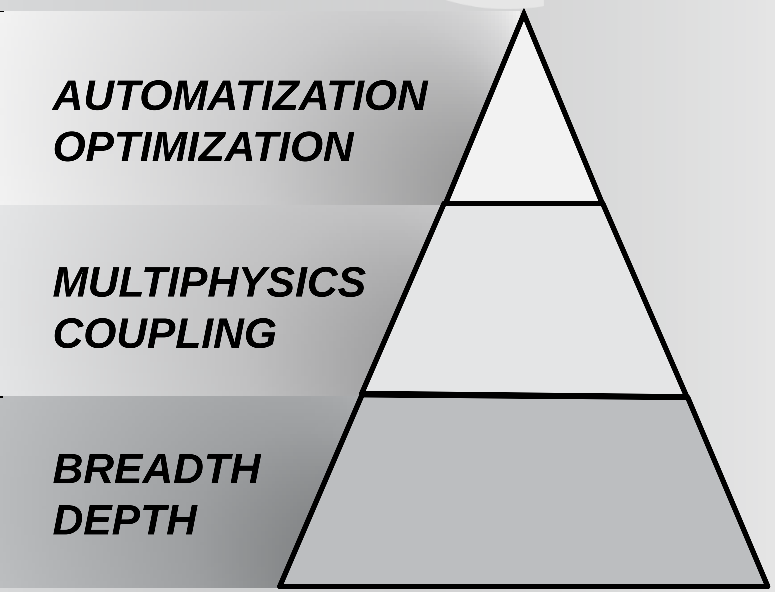
Design Optimization, Summary



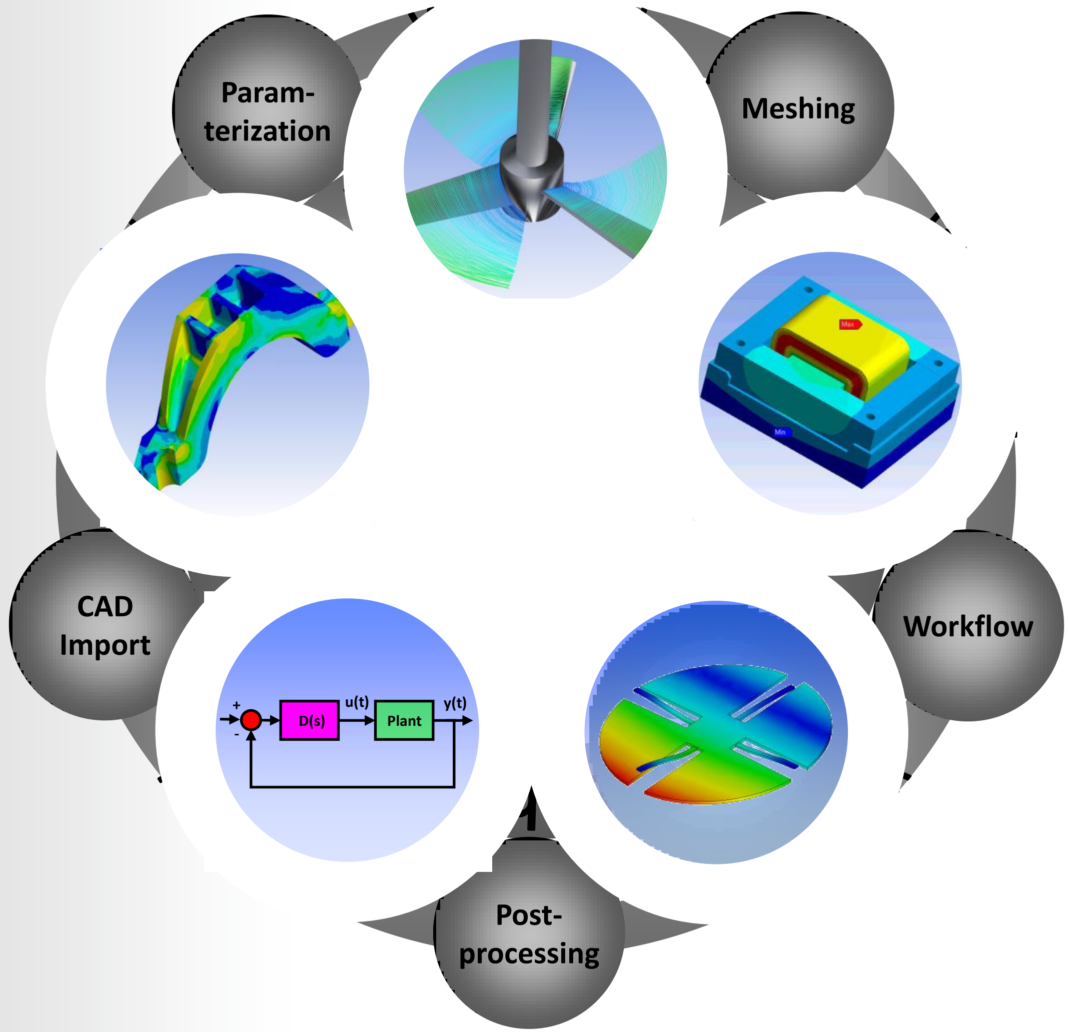


	Initial Design	Best Design SA	Best Design Solved (MoP)	Best Design ARSM
Efficiency [%]	87.0	88.0	88.9 (91.0)	88.9
p_{tot} Ratio [-]	1.41	1.41	1.41 (1.44)	1.41
Max. Stress [MPa]	219	235	232 (230)	239
#Designs	1	150	1 (0)	100

Summary



AUTOMATIZATION
OPTIMIZATION
MULTIPHYSICS
COUPLING
BREADTH
DEPTH



Param-
terization
Meshing
CAD
Import
Workflow
Post-
processing
D(s)
u(t)
Plant
y(t)