Outline

- Introduction
- Part 1: Basics of Statistics
- Part 2: Regression
- Part 3: Probabilistic System Analysis using Monte Carlo Methods

Introduction of example model

Inputs, $\mathbf{X}$ → **Model** $\mathcal{M}$ → Outputs, $\mathbf{Y}$

Inputs:

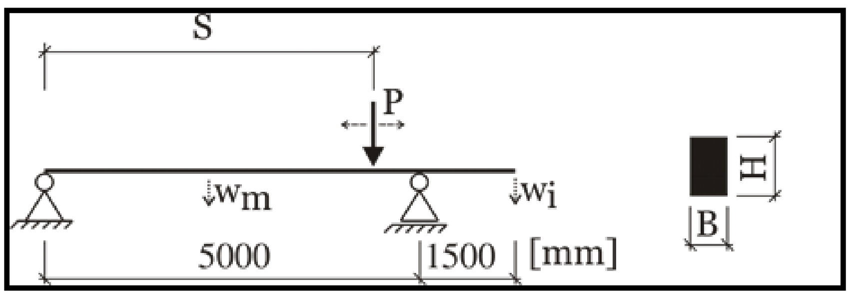
Young's modulus (E)
normal, N(210000, 10000)

Force (P)
normal, N(2500,300)

Position of force (S)
uniform, U(0,6500)

Height (H)
uniform, U(90,110)

Width(B)
uniform, U(45, 55)



beam with cantilever arm on bearings

Outputs:

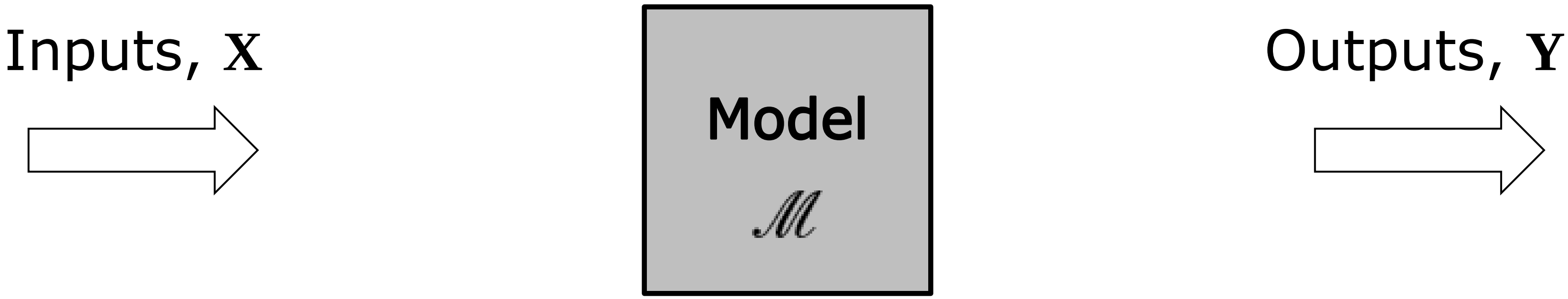
Deflection 1(w_m)

Deflection 2 (w_i)

Maximal deflection(w_{max})

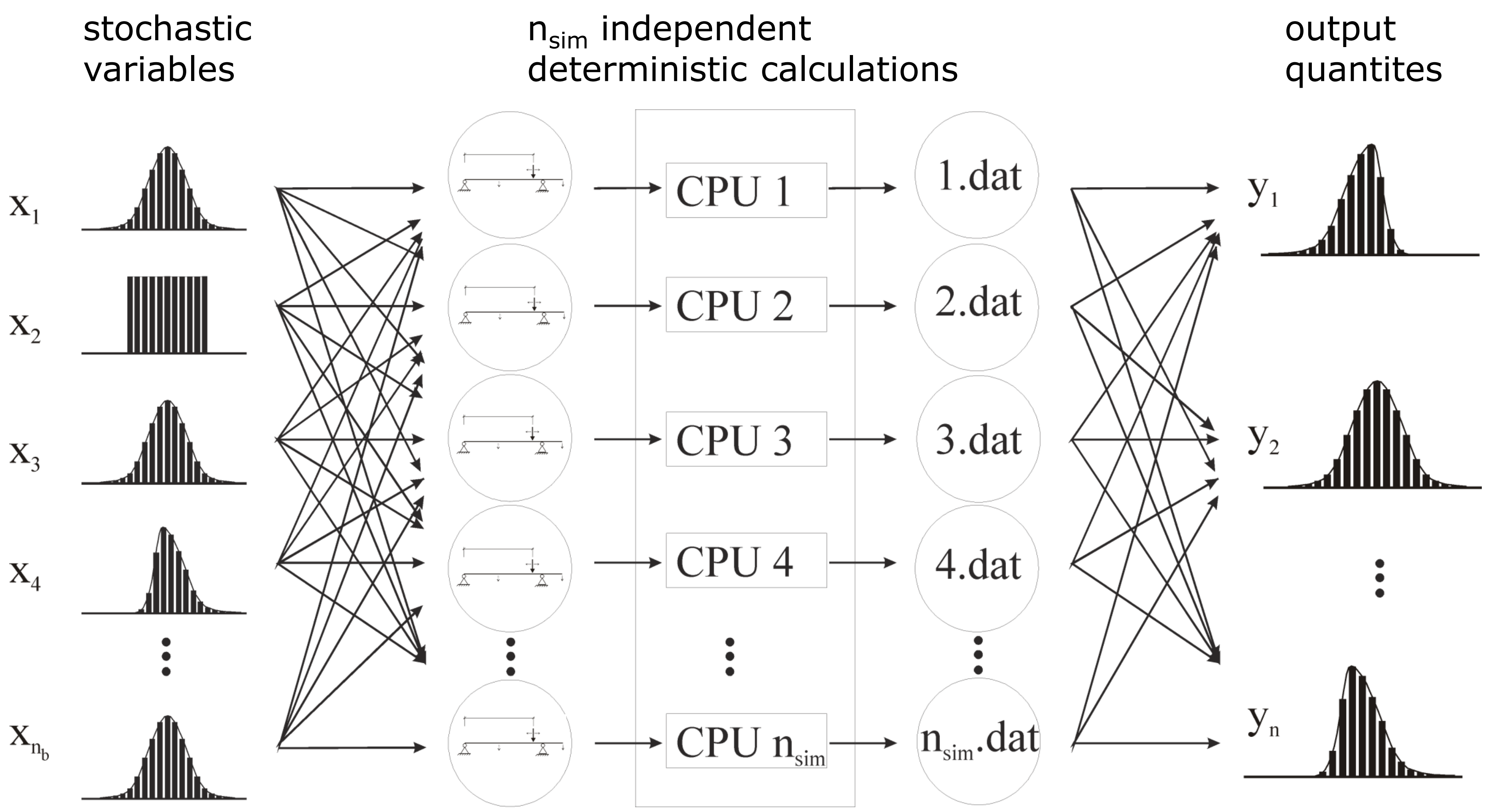
Deflection of maximum absolute deflection w for max($|w_m|$,$|w_i|$)

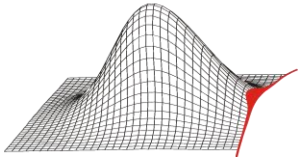
Monte Carlo Simulation (MCS)



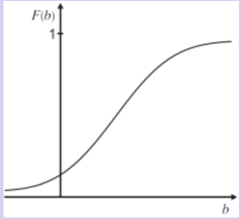
- Random number generator,
  web reference: http://random.mat.sbg.ac.at/
- Sampling method
- Correlation Control Algorithm/Joint Probability Distribution
- Statistics for the evaluation

Monte Carlo Simulation (MCS)



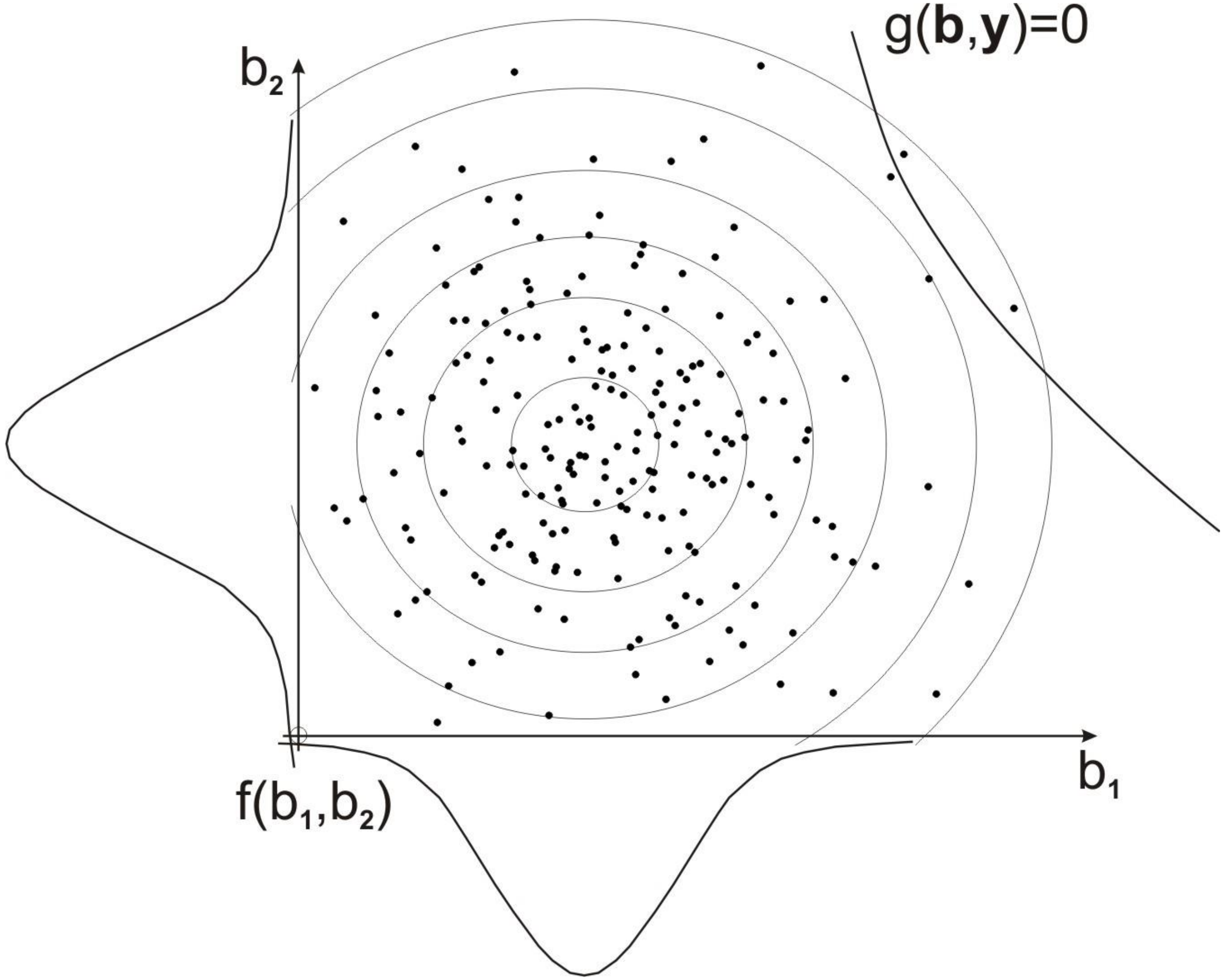


Probability of Failure

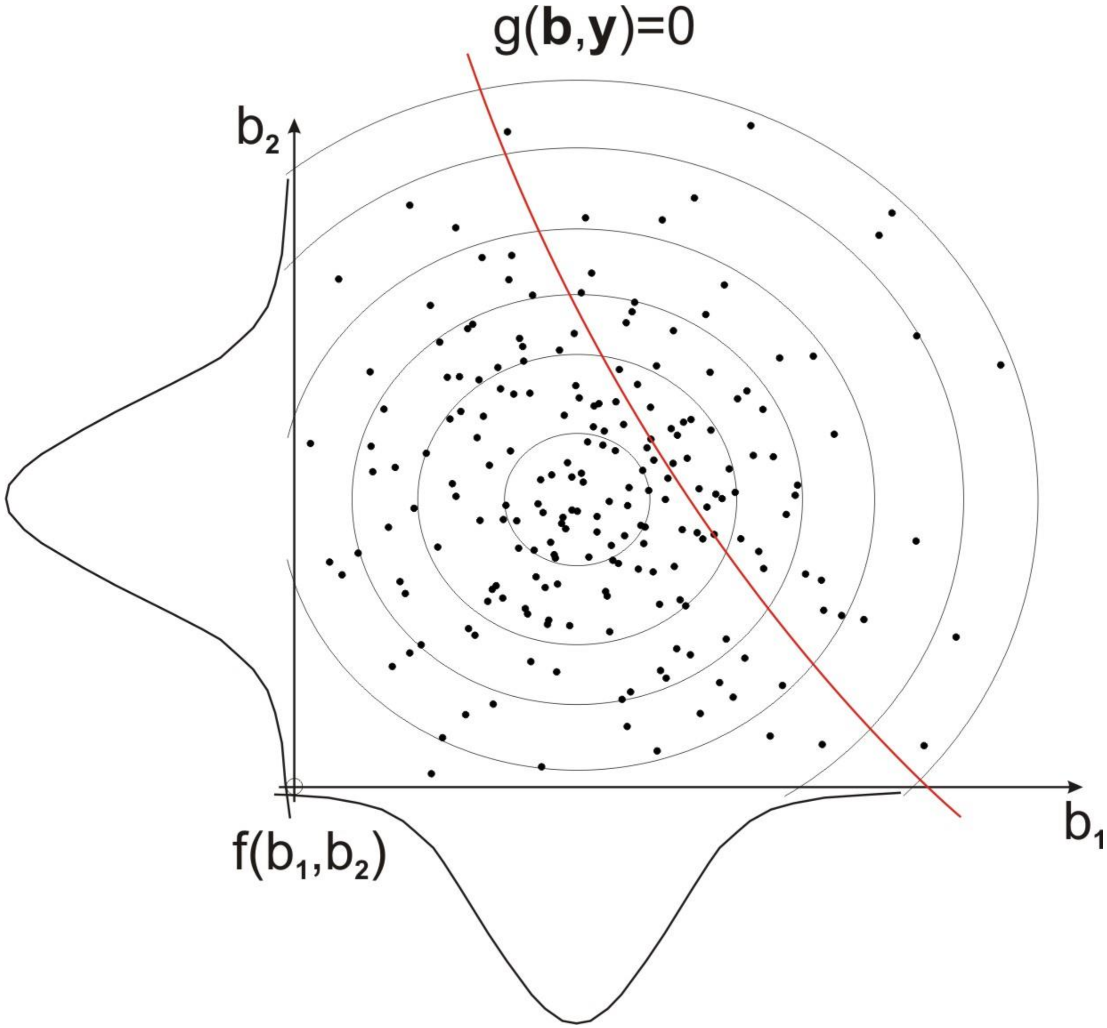
Result of probabilistic Simulation	Probability of failure
pdf of input variables -roughly known (as in industry)	
-precisely known (rarely)	
Required number of deterministic runs	[1] $n_{sim,LHS} \geq \frac{10}{\hat{P}_f}$
Output	

[1] *Dirk Roos et al. Design Reliability Analysis. 24th CAD-FEM Users' Meeting. International Congress on FEM Technology. Stuttgart. 2006*

Probability of Failure



Probability of Failure



Define criteria: $w_{\max} \geq 10\ \text{mm}$

$$\hat{P}_o = \frac{1}{n_{sim}} \sum_{k=1}^{n_{sim}} 1D_o\left(X^{(k)}\right) = \frac{n_o}{n_{sim}}$$

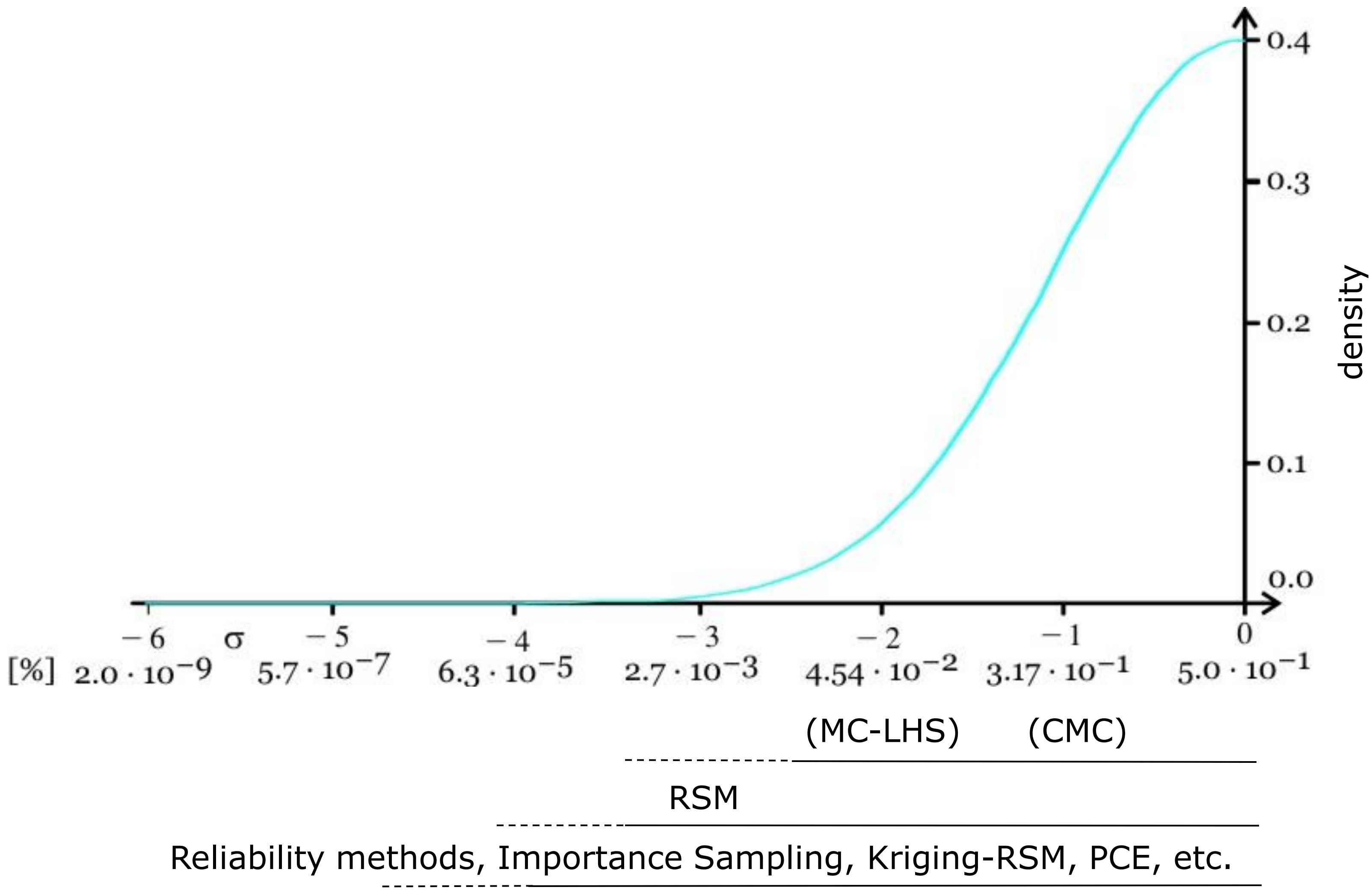
D_o – occurrence domain

	$\hat{P}_o$	95% confidence interval
SRS[1] (CMC)	0.132	0.104 - 0.165*

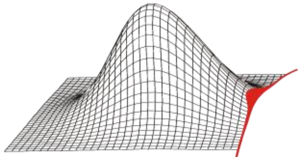
[1] Simple Random Sampling, $n_{sim} = 500$

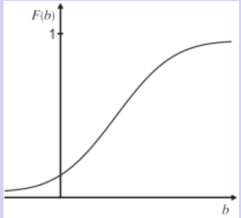
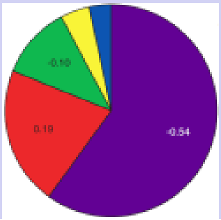
* Clopper Pearson interval from *L. Sachs, J. Hedderich. Angewandte Statistik. Springer. 2009 pp. 293*

Probability of Failure



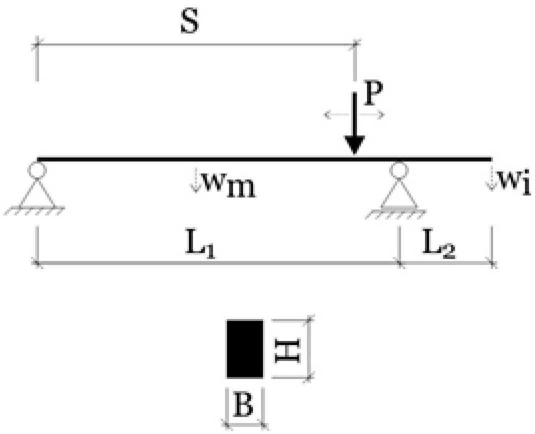
Sensitivity analysis

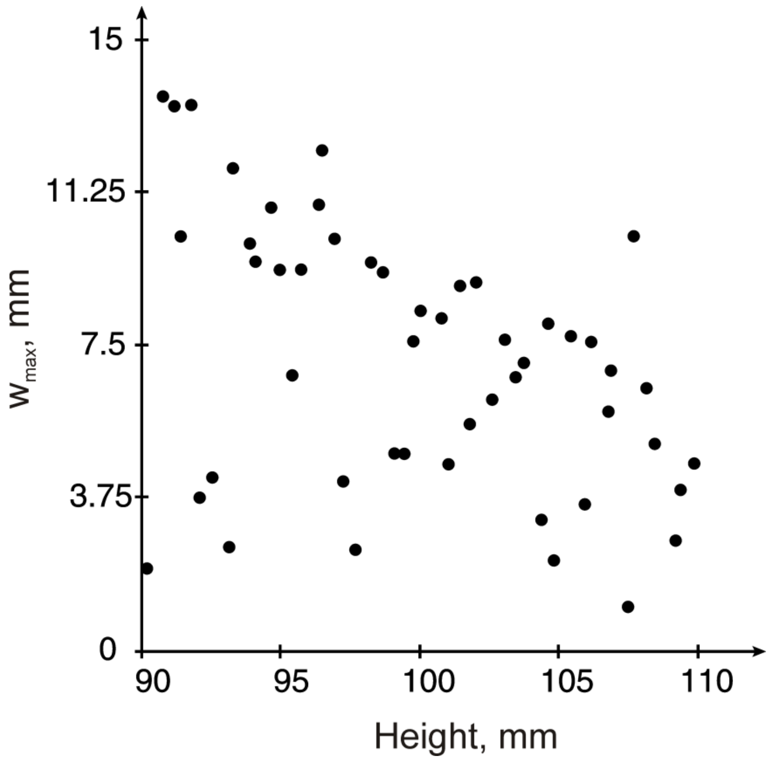


Result of probabilistic Simulation	Probability of failure	Sensitivities
pdf of input variables -roughly known (as in industry)		
-precisely known (rarely)		
Required number of deterministic runs	$n_{sim,LHS} \geq \frac{10}{\hat{P}_f}$	
Output		

Latin Hypercube Sampling (LHS), n_{sim} = 50 and uncorrelated inputs

Anthill plots and rank correlation coefficient

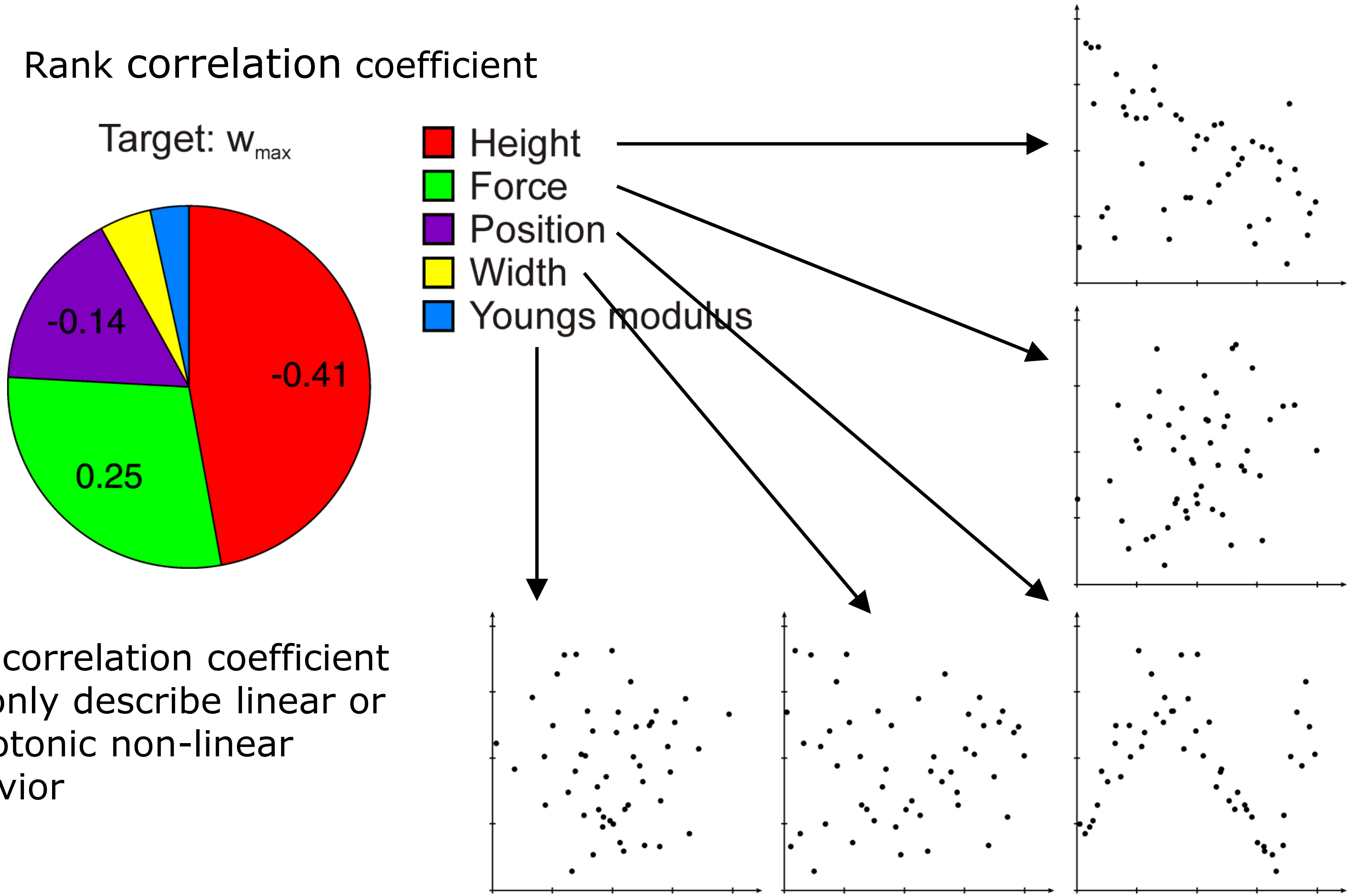




$\tilde{r} = -0.41$

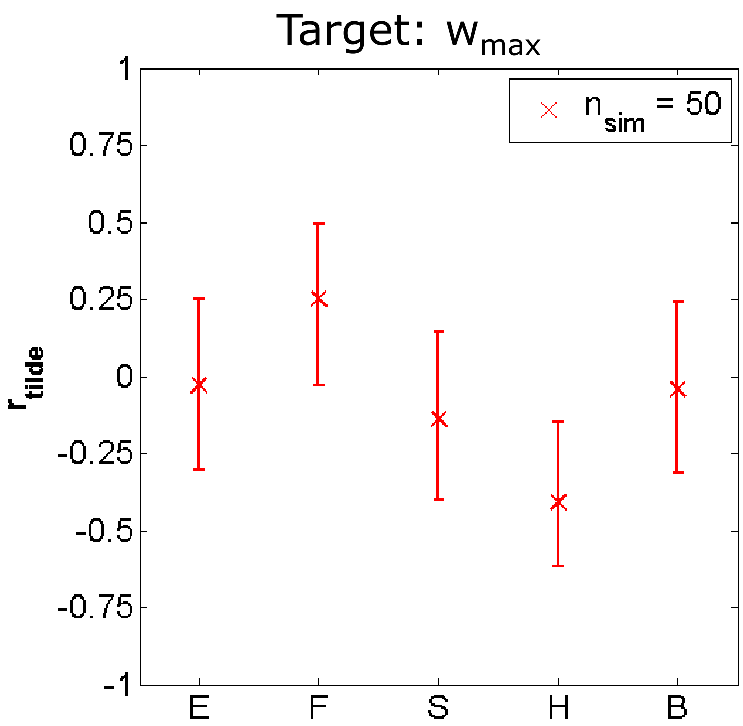
Sensitivity analysis

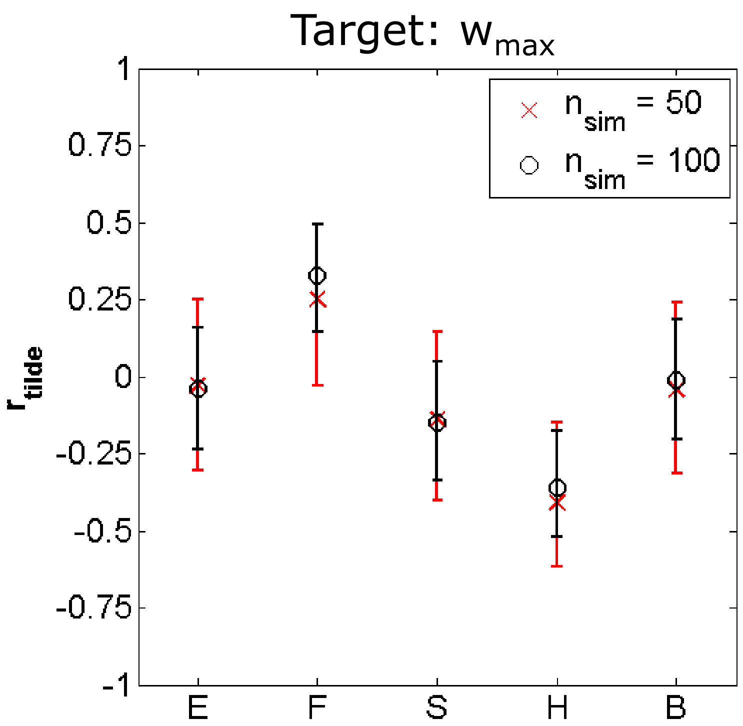
Rank correlation coefficient



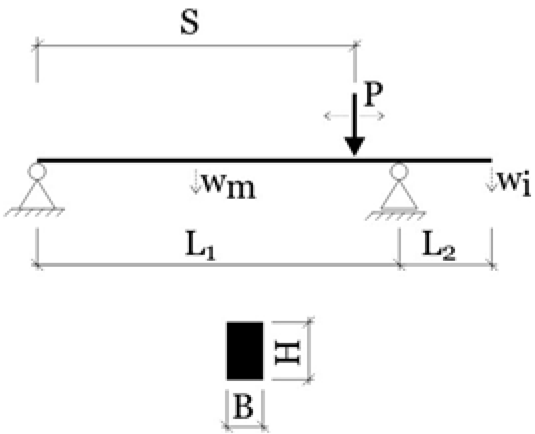
rank correlation coefficient can only describe linear or monotonic non-linear behavior

Rank correlation coefficient and confidence interval





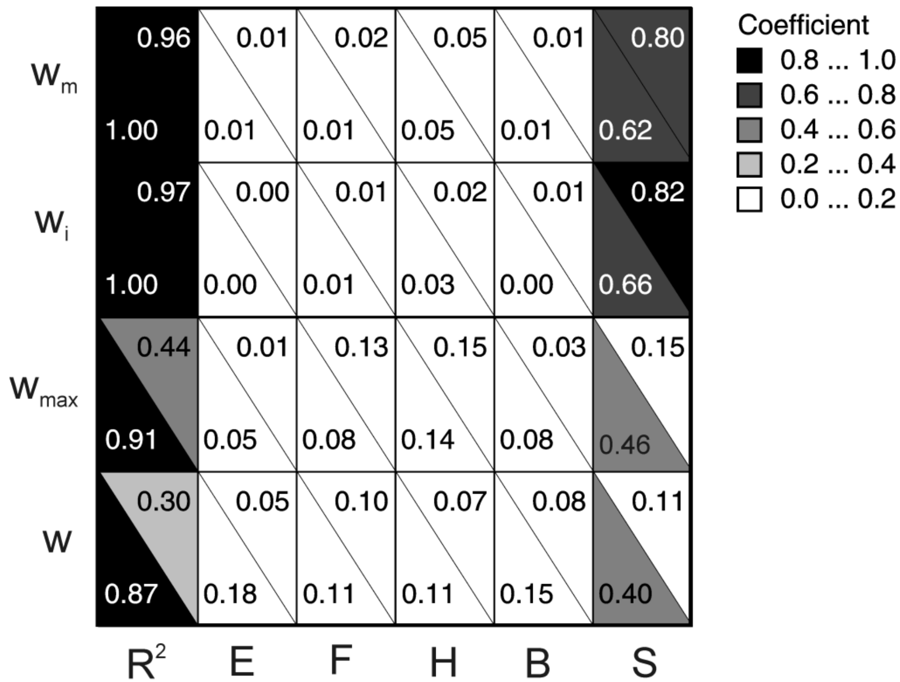
RSM and Coefficient of Importance [[2]Bucher, 2009]



Basis: LHS, n_{sim} = 100

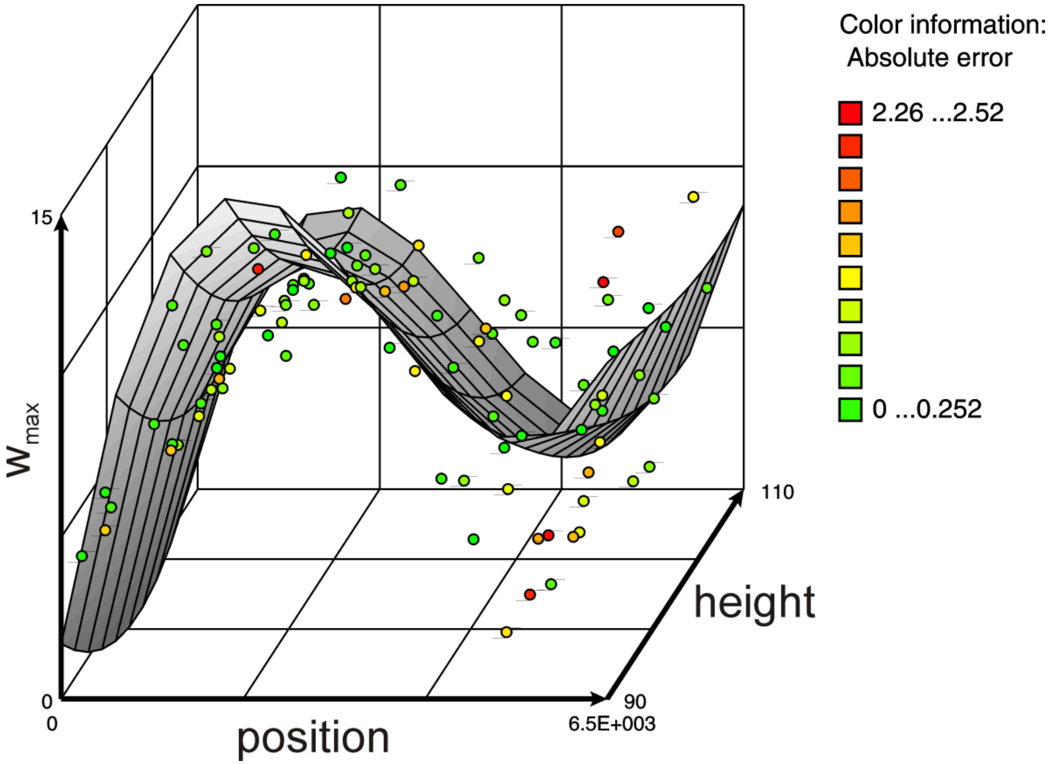
RSM 1: polynomial second order (21)

RSM 2: polynomial third order (56)



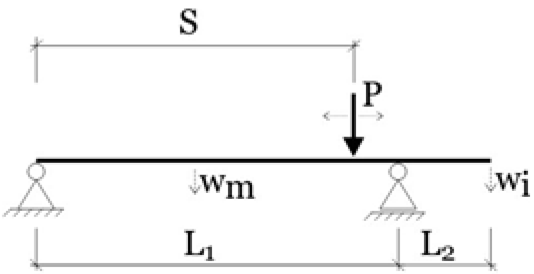
RSM and Coefficient of Importance

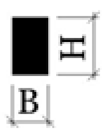
Absolut error



$Var(w_{max}) = 3.3$

RSM and Coefficient of Importance [2Bucher, 2009]



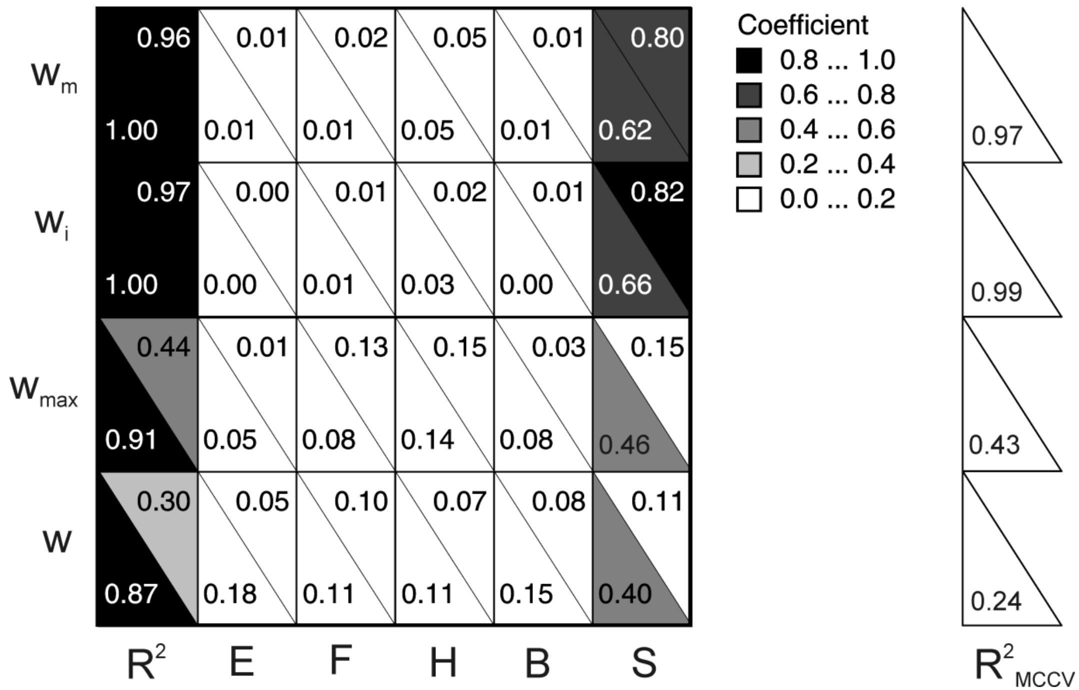


Basis: LHS, n_{sim} = 100

RSM 1: polynomial second order (21)

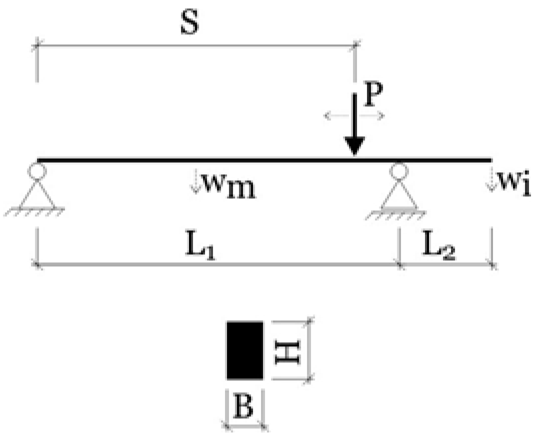
RSM 2: polynomial third order (56)

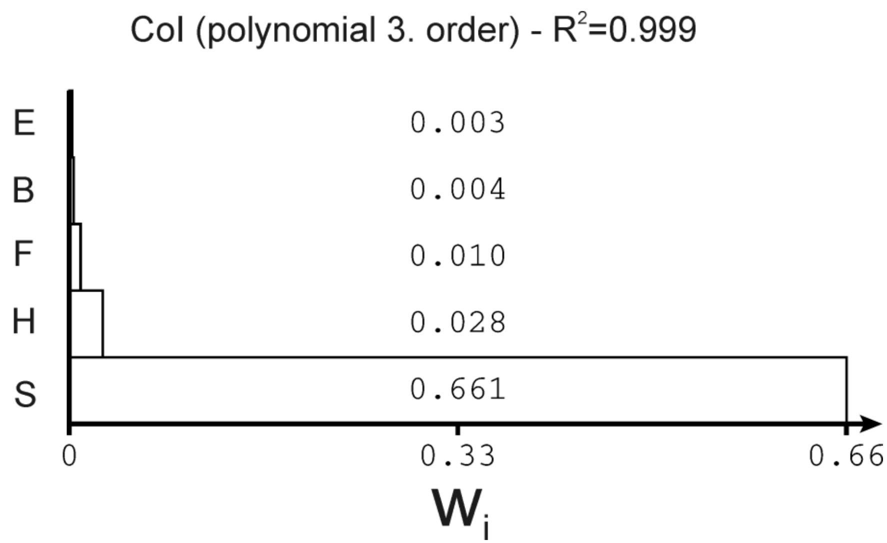
Cross validation:
R^2_{MCCV}
splitting ratio: 0.85
runs: 1000
[Beschorner et al., 2014]



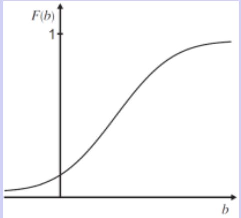
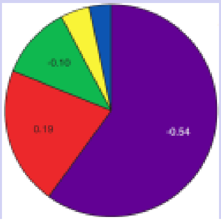
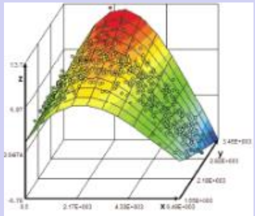
RSM and Coefficient of Importance

Basis: LHS, n_{sim} = 100

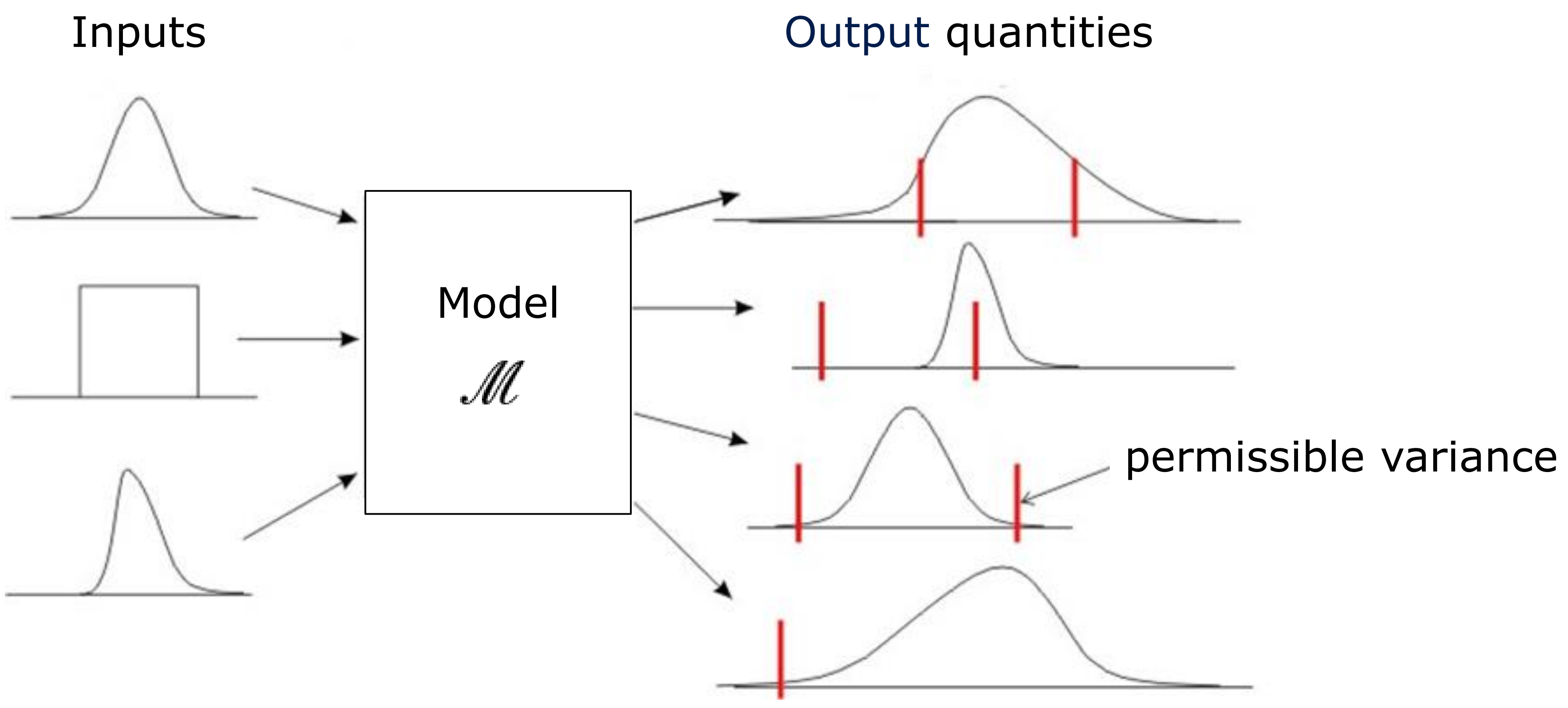




Robustness estimation

Result of probabilistic Simulation	Probability of failure	Sensitivities	Robustness
pdf of input variables -roughly known (as in industry)	⛔	✓	✓
-precisely known (rarely)	✓	✓	✓
Required number of deterministic runs	$n_{sim,LHS} \geq \frac{10}{\hat{P}_f}$		
Output			

Robustness estimation



What is a robust Design?

engineering measures	statistical measures
▪ exceedance of thresholds ▪ Occurrence of undesirable sudden changes in the result values (e.g. local maximum of result quantity) ▪ Response of system instabilities (e.g. buckling)	▪ Position of the mean values of the output quantities ▪ Magnitude of the coefficient of variation of the output quantities [[3]Will et al., 2006]

Required number of simulations (n_{sim}) with Monte Carlo methods:

- Depends on the probability of the event
- Verification by confidence interval of the statistic measures

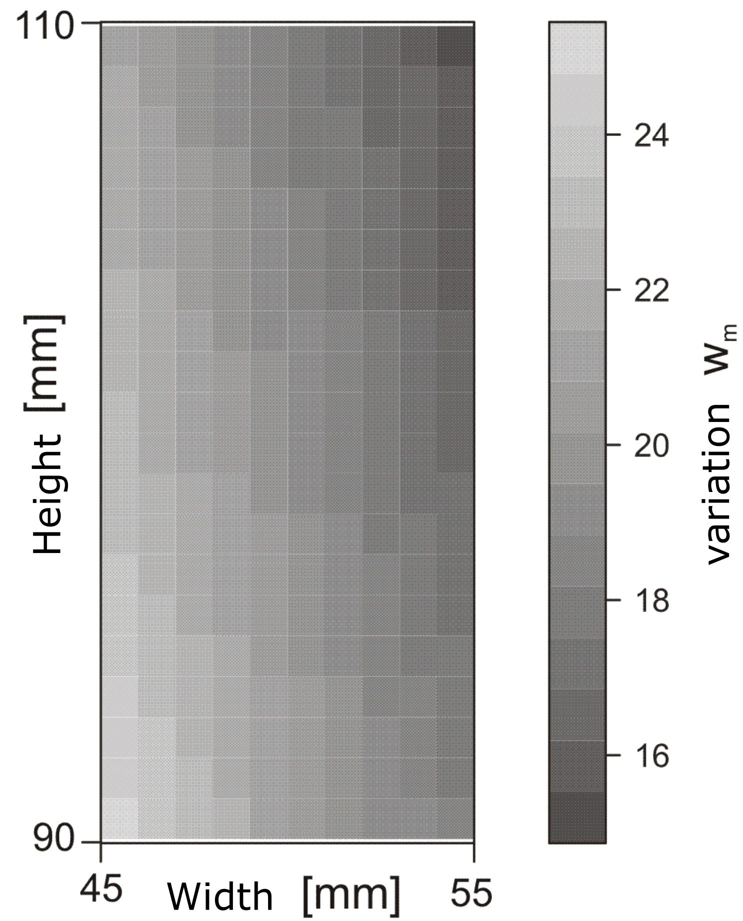
Robustness estimation on RSM

- Divide design space in fields of the manufacturing tolerances
- Conduct a MCS in each field
- Plot the variation of the output quantity over the design space

✕ design point

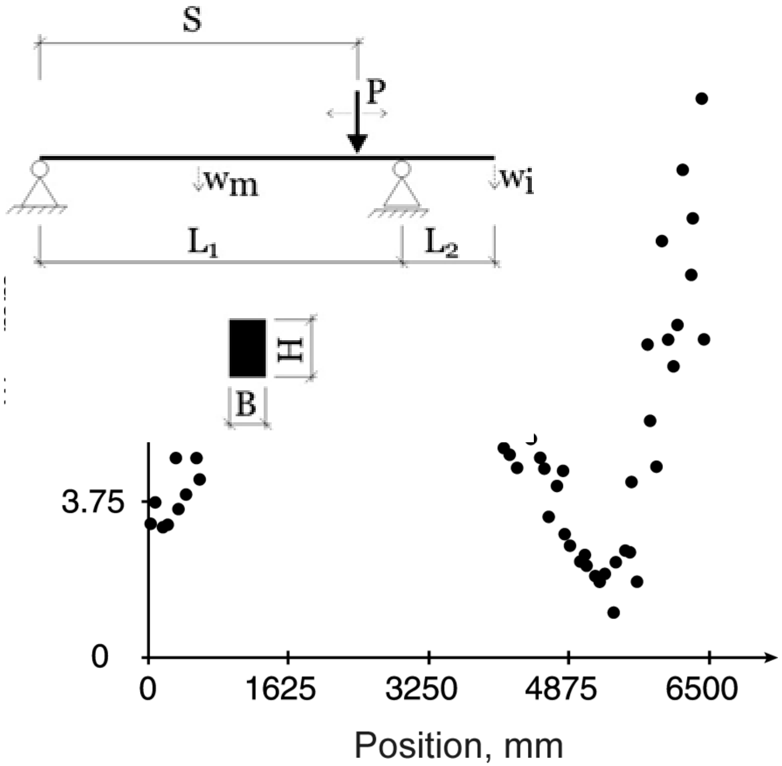
manufacturing tolerances

simulation space



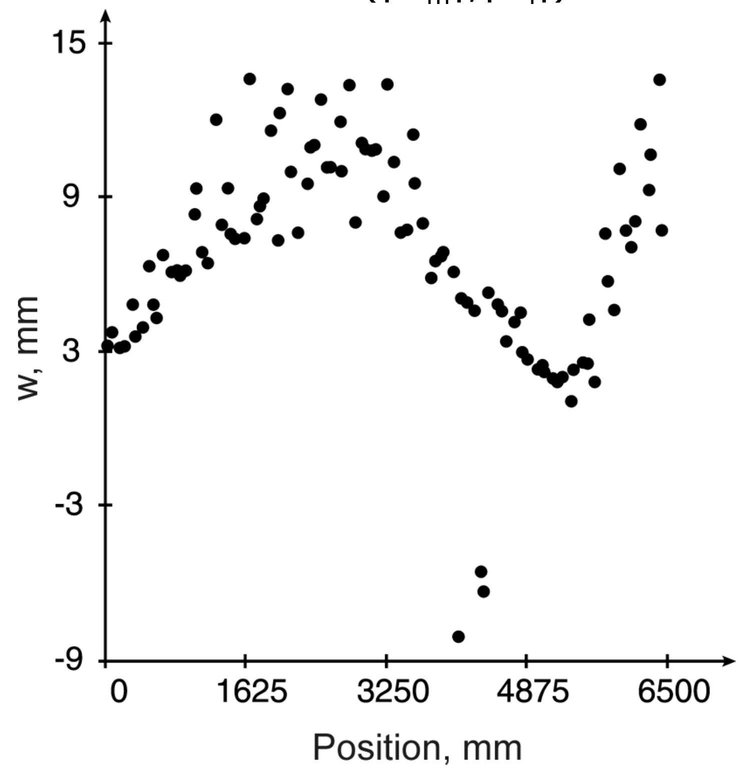
Undesirable sudden changes in the result values

LHS, n_{sim} = 100



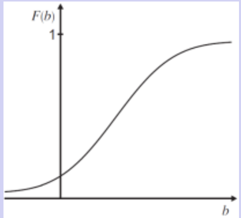
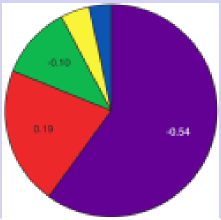
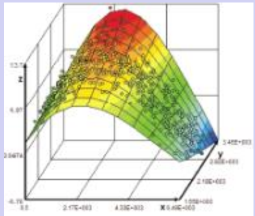
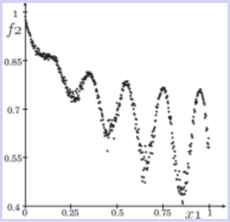
branching system

w for max($|w_m|$,$|w_i|$)



Monte Carlo Simulation

Application of Monte-Carlo methods for probabilistic investigations using optimized LHS under consideration of input parameter correlation

Result of probabilistic Simulation	Probability of failure	Sensitivities	Robustness	System improvement
pdf of input variables -roughly known (as in industry)	⛔	✔	✔	✔
-precisely known (rarely)	✔	✔	✔	✔
Required number of deterministic runs	$n_{sim,LHS} \geq \frac{10}{\hat{P}_f}$	- Verification by confidence interval - Position of the mean values of the output quantities $n_{sim} \approx 50$; minimum: n_{sim} = no. inputs + 10…20		
Output - one single MC Simulation provides all result quantities				

Monte Carlo Simulation

Advantages*	Disadvantages*
▪ Accuracy of the output quantities is almost independent of the dimension of the input space (convergence rate is independent of the dimension of the input space - CMC) ▪ Consideration of all result variables within one MCS ▪ Working with deterministic "black box" models possible	▪ Dependence of the quality of the stat. measures on the number of realizations n_{sim} ▪ rate of convergence is of order $n_{sim}^{-1/2}$ (CMC)

* compared to other probabilistic methods

References

(1) Dirk Roos et al. Design Reliability Analysis. 24th CAD-FEM Users' Meeting. International Congress on FEM Technology. Stuttgart. 2006

(2) Christian Bucher. Computational Analysis of Randomness in Structural Mechanics, Volume 3 of Structures and Infrastructures Series. CRC Press, May 2009.

(3) J. Will, Christian Bucher. Statistische Maße für rechnerische Robustheitsbewertungen CAE gestützter Berechnungsmodelle. Weimarer Optimierungs- und Stochastiktage 3.0. 2006

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Tutorial

Introduction into probabilistic methods and their application in engineering sciences with focus on monte carlo and response surface methods

David Pusch
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