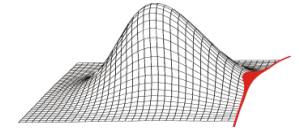
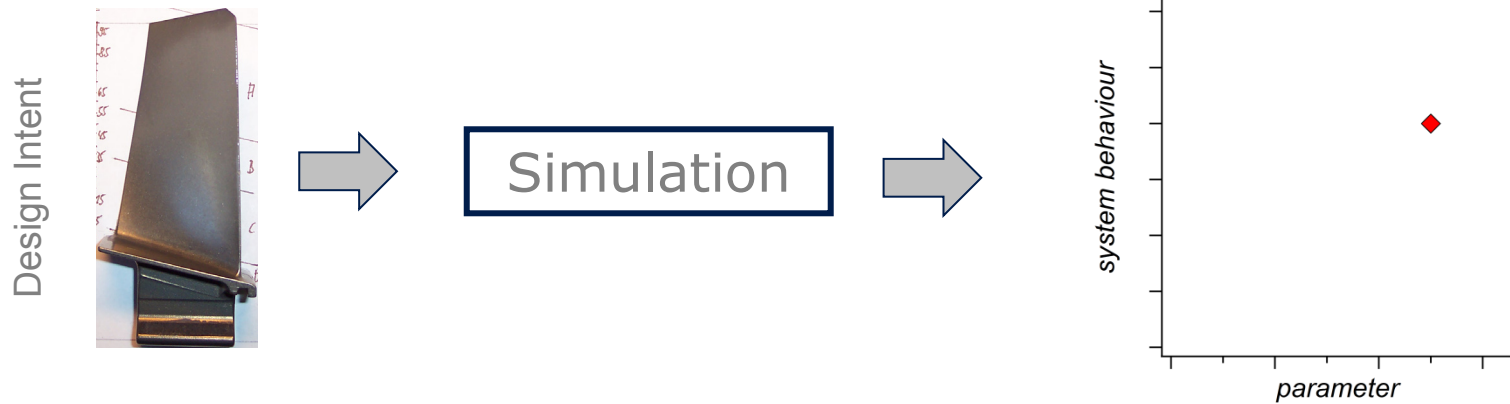


A Guideline to enable Probabilistic Profile Design based on Optical Measurement Data

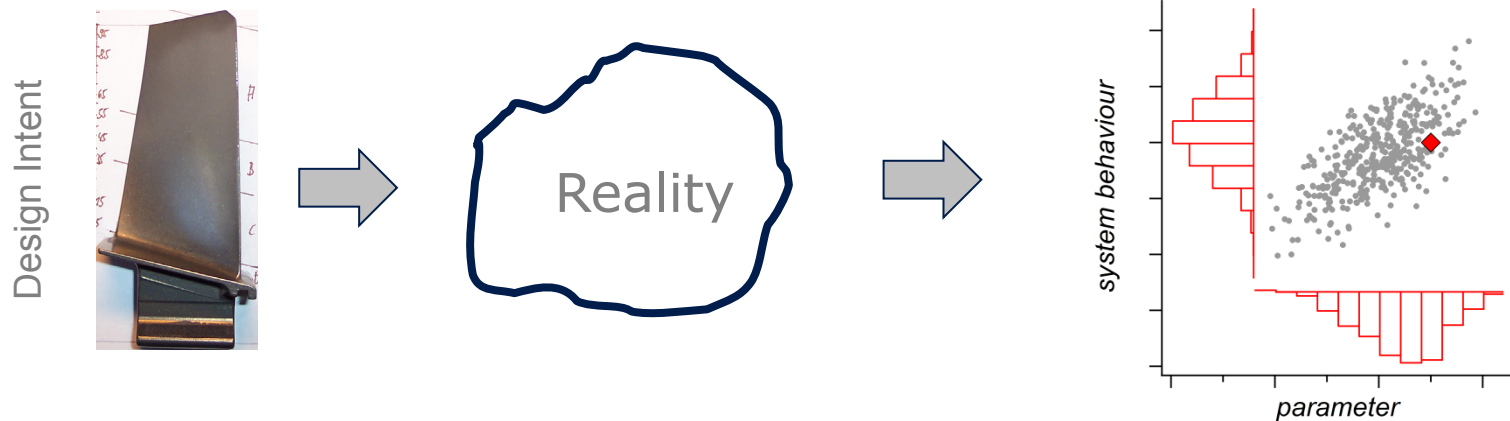
Lars Högner, Thomas Backhaus, Oliver Baum, Sebastian Knebel, Paul Voigt, Robin Schmidt

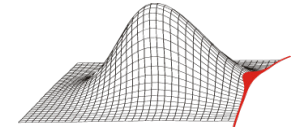


- Classic simulation-based approach used in design



- Real behavior of physical system

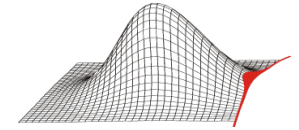




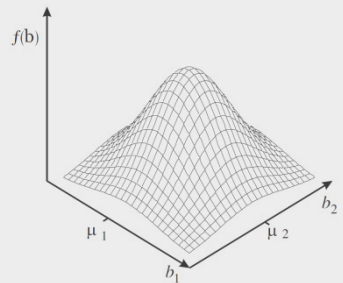
Two individual turbine blades from the same engine



Massachusetts Institute of Technology, Prof. David L. Darmofal



1) Introduction



$$\frac{1}{m-1} \sum_{k=1}^{n_{sim}} (b_{ri,k} - \bar{b})$$

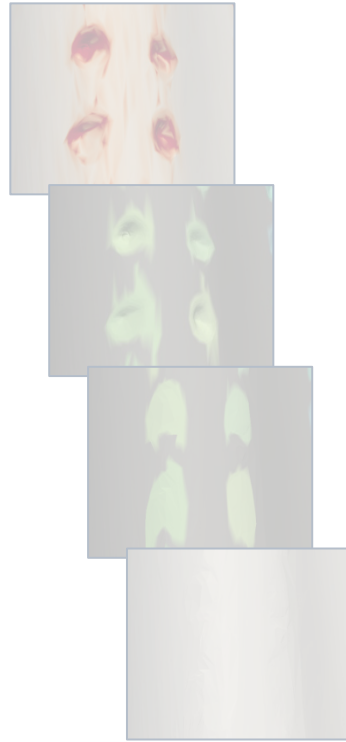
$r = 0.70$
 $\bar{r} = 0.68$

$r = 0.48$
 $\bar{r} = 0.48$

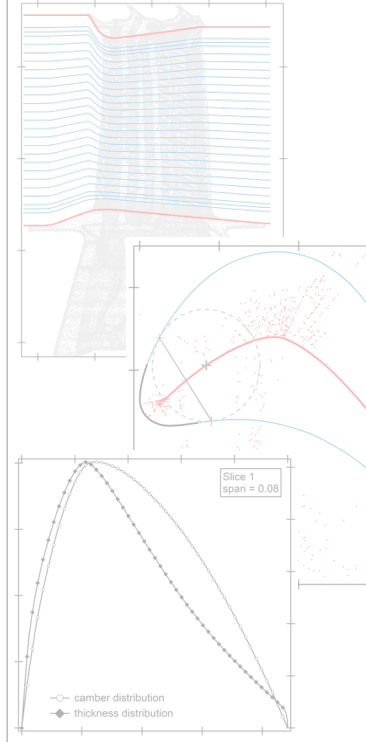
2) Optical Measurement



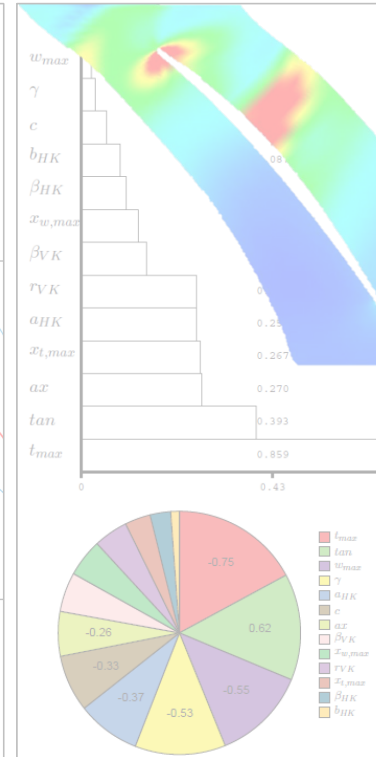
3) Mesh Pre-Processing

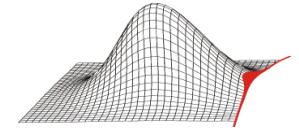


4) Parameterisation and Rebuild



5) Applications





1) Introduction

1.1 Motivation

1.2 Stochastic theory

1.2.1 Random variables

1.2.2 Distributions

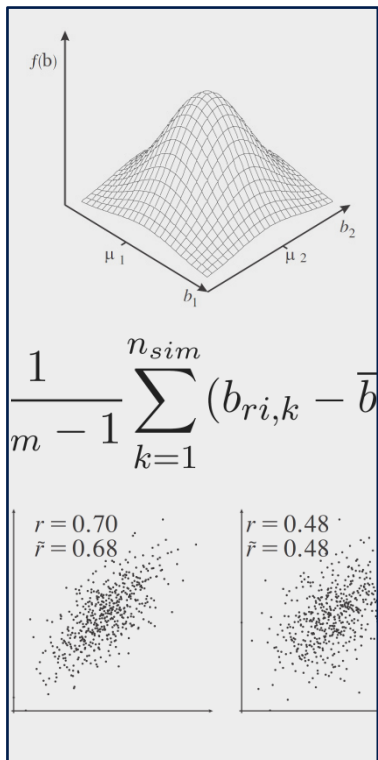
1.2.3 Statistical measures

1.2.4 Goodness-of-fit test

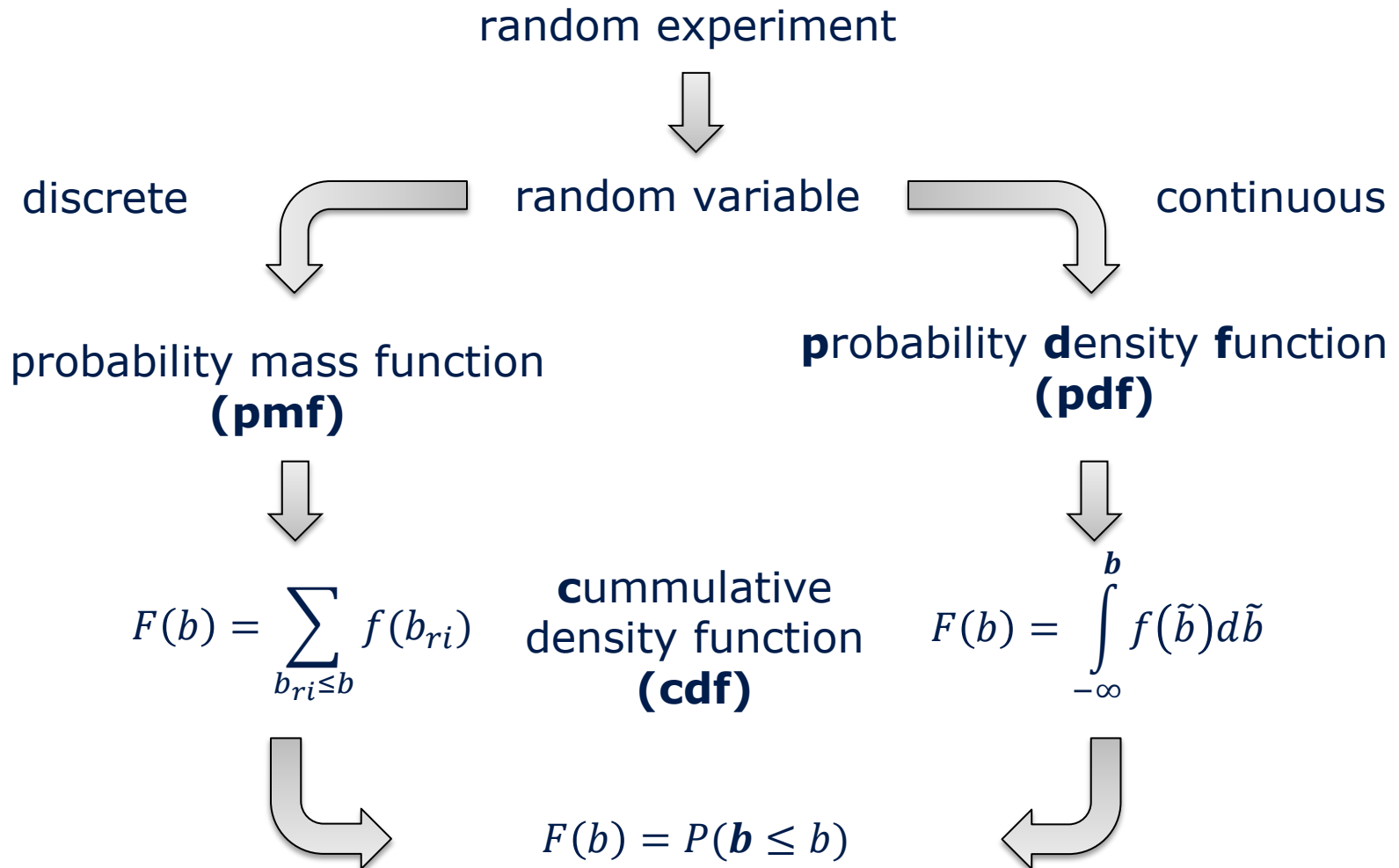
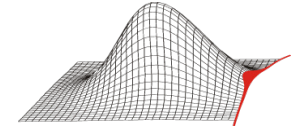
1.2.5 Correlation

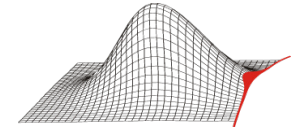
1.2.6 Confidence interval

1.2.7 Exemplary results



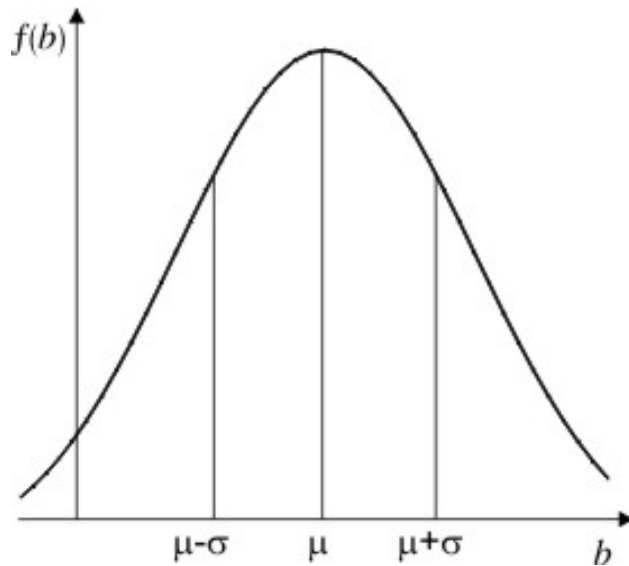
1.2.1 Random variables



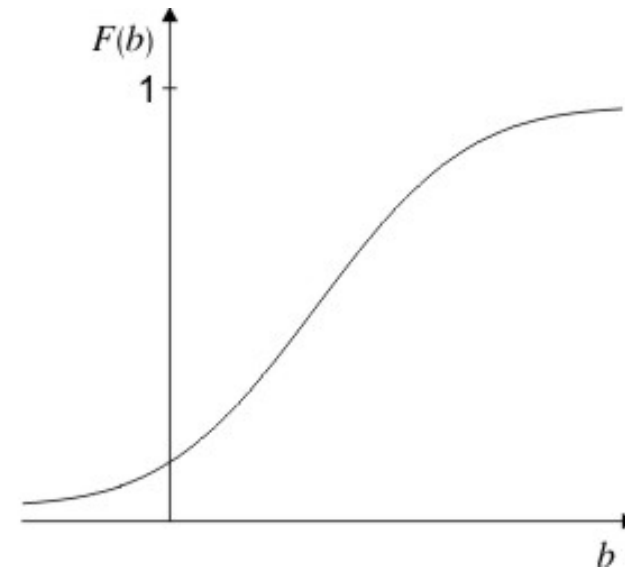


Gaussian Distribution (Normal Distribution)

[1]

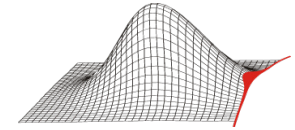


$$f(b) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(b - \mu)^2}{2\sigma^2}\right]$$



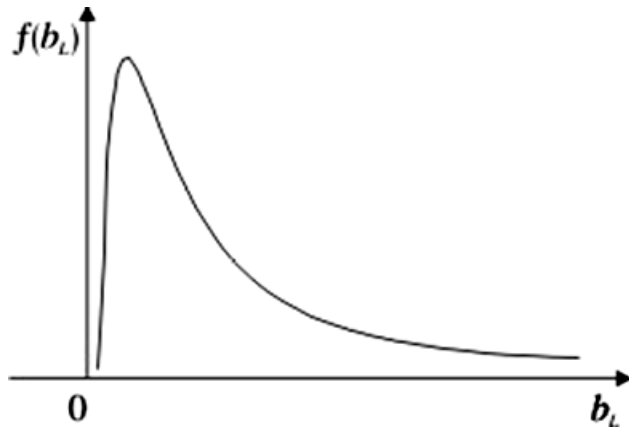
$$F(b) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^b \exp\left[-\frac{(\tilde{b} - \mu)^2}{2\sigma^2}\right] d\tilde{b}$$

[1] Sachs, L., 2004, *Angewandte Statistik, Anwendung statistischer Methoden*

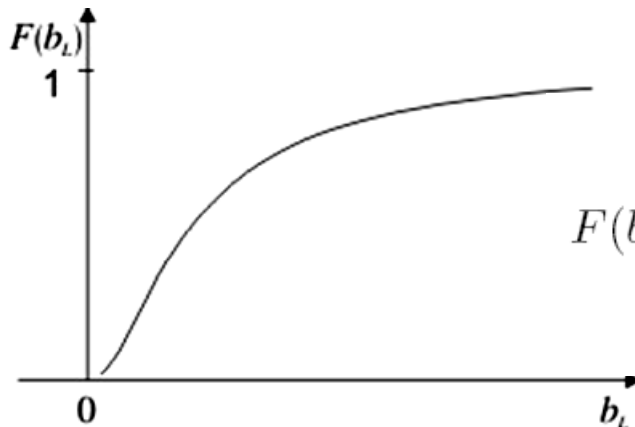


[1]

log-Normal Distribution

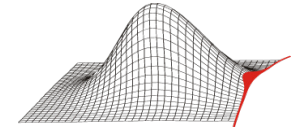


$$f(b_L) = \begin{cases} \frac{1}{\zeta \sqrt{2\pi} b_L} \exp \left\{ -\frac{(\ln b_L - \lambda)^2}{2 \zeta^2} \right\} & \text{für } b_L > 0 \\ 0 & \text{sonst} \end{cases}$$



$$F(b_L) = \begin{cases} \frac{1}{\zeta \sqrt{2\pi}} \int_0^{b_L} \frac{1}{\tilde{b}_L} \exp \left\{ -\frac{(\ln \tilde{b}_L - \lambda)^2}{2 \zeta^2} \right\} d\tilde{b}_L & \text{für } \tilde{b}_L > 0 \\ 0 & \text{sonst} \end{cases}$$

[1] Sachs, L., 2004, *Angewandte Statistik, Anwendung statistischer Methoden*



- **arithmetic mean**

$$\bar{b}_{ri} = \frac{1}{n_{sim}} \sum_{k=1}^{n_{sim}} b_{ri,k}$$

- centroid of the area underneath the density function
- sensitive towards outliers

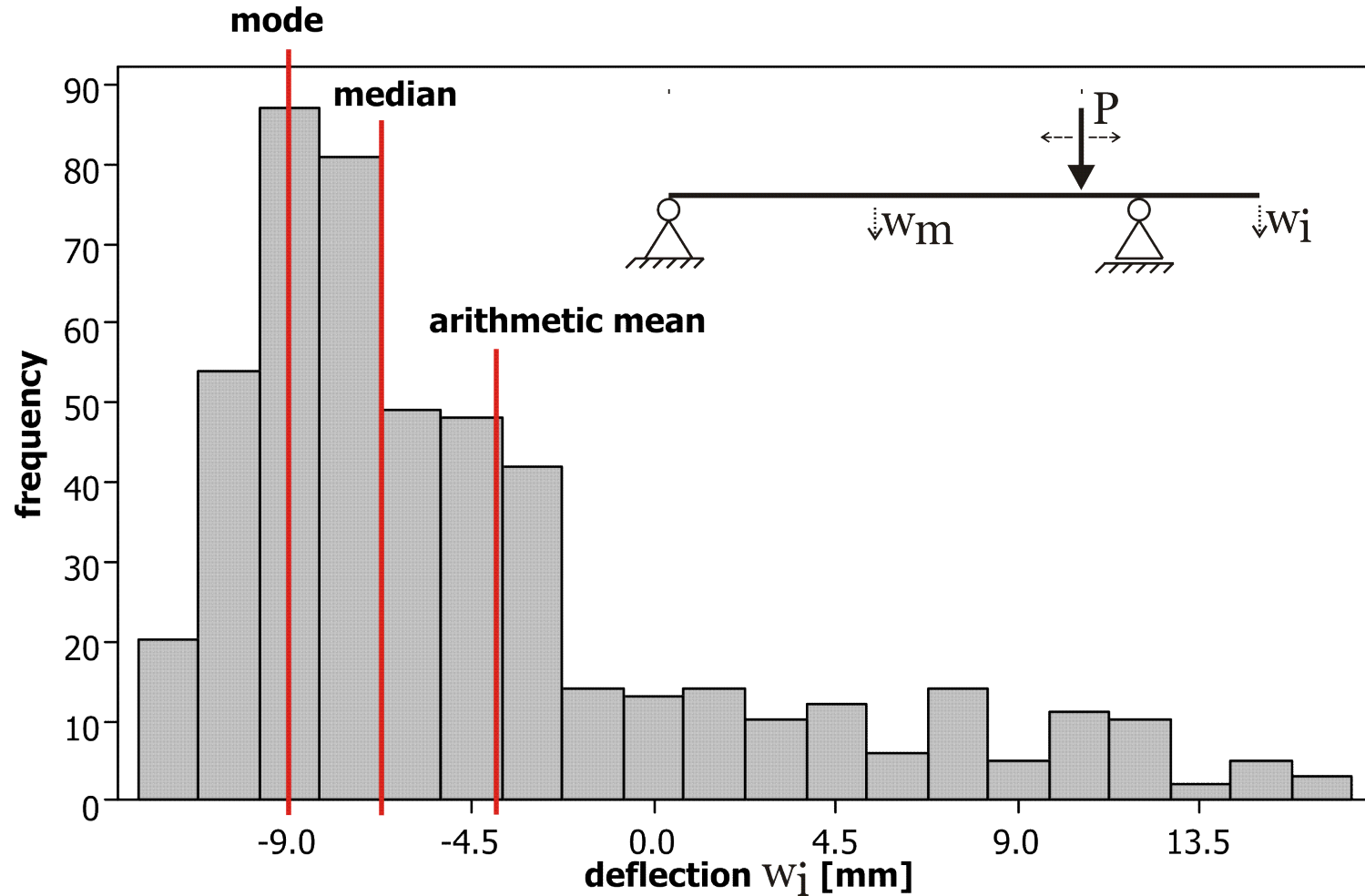
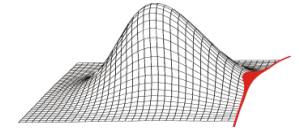
- **median**

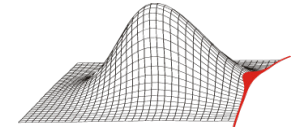
- divides the area below the probability density function into two pieces of equal size
- robust towards outliers

- **modal value or mode**

- value of the data set, that occurs with the greatest frequency
- not necessarily unique

1.2.3 Statistical measures





- **standard deviation**

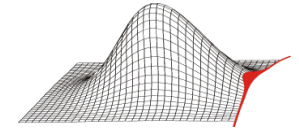
$$\sigma(b_{ri}) = \sqrt{\text{Var}(\mathbf{b}_{ri})} = \sqrt{\frac{1}{n_{sim} - 1} \sum_{k=1}^{n_{sim}} (b_{ri,k} - \bar{b}_{ri})^2}$$

- Variation towards the expected value \bar{b}_{ri}

- **coefficient of variation**

$$\delta(\mathbf{b}_{ri}) = \frac{\sigma(\mathbf{b}_{ri})}{\bar{b}_{ri}}$$

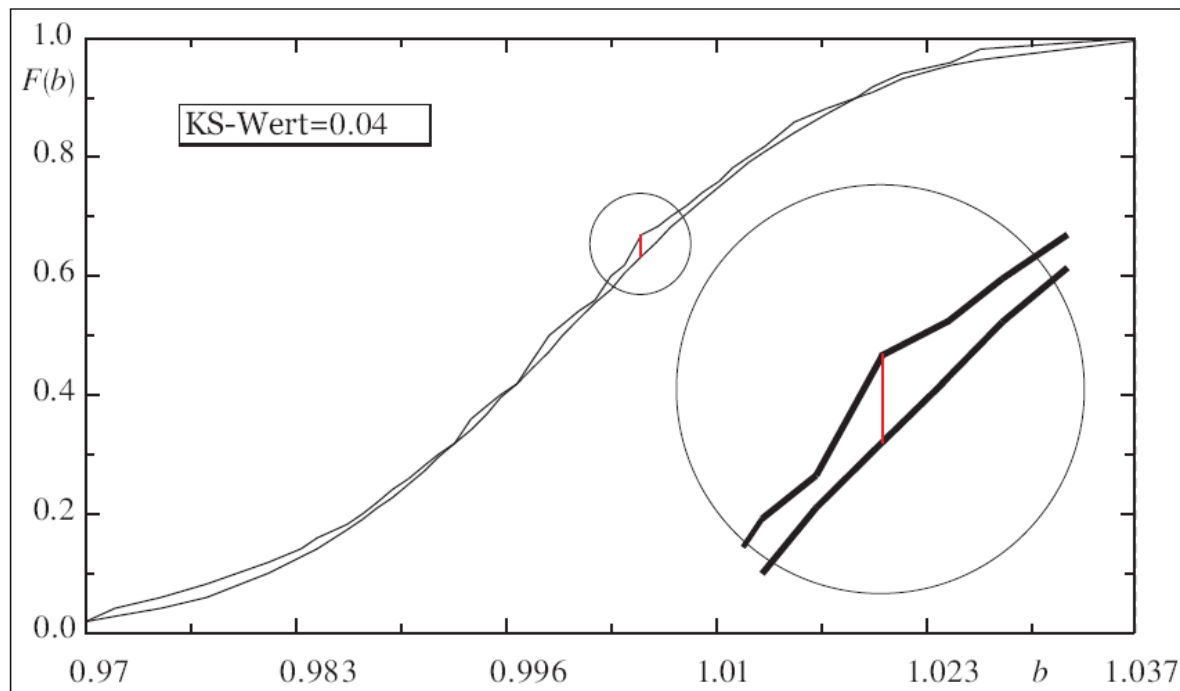
- Normalisation of standard deviation using expected value
- Enables the comparison of different stochastic variables \mathbf{b}

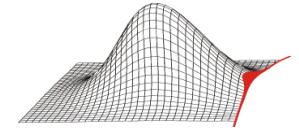


Kolmogorow-Smirnow-Test

- Direct comparison of discrete density function F_d and continuous density function F_c

$$KS = \max_{-\infty < b < \infty} |F_d(b) - F_c(b)|$$





Kolmogorow-Smirnow-Test

- Direct comparison of discrete density function F_d and continuous density function F_c

$$KS = \max_{-\infty < b < \infty} |F_d(b) - F_c(b)|$$

Anderson-Darling-Test

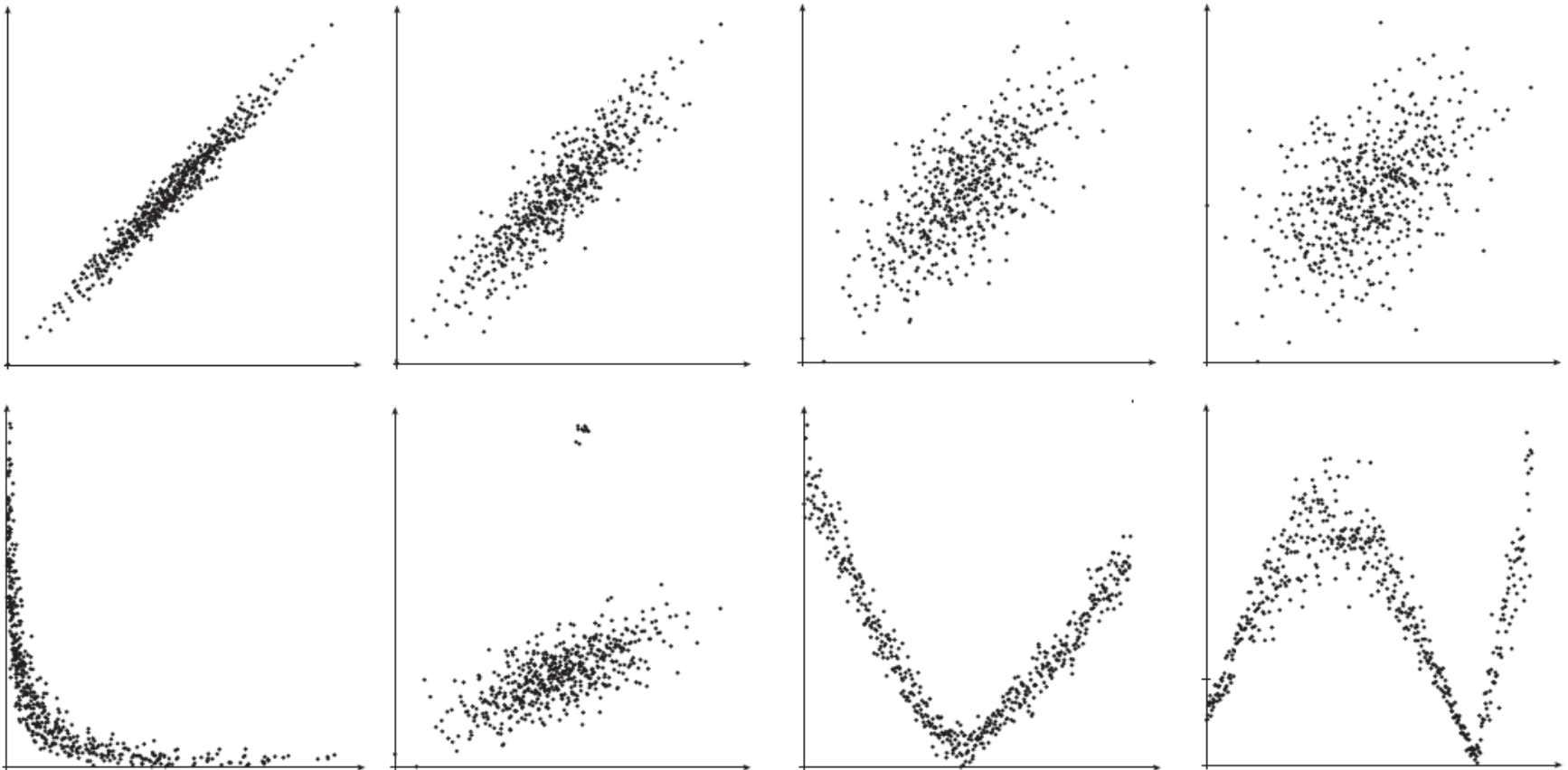
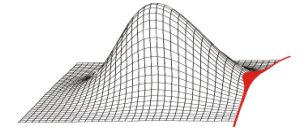
- Modification of Kolmogorow-Smirnow-Test
- Stronger weighting of deviations F_d and F_c in the edge region

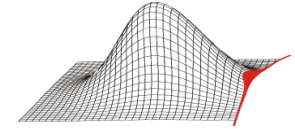
$$A^2 = -n_{sim} - \frac{1}{n_{sim}} \sum_k^{n_{sim}} (2k - 1) (\ln F_s(b_k) + \ln [1 - F_s(b_{n_{sim}+1-k})])$$

- Tables of critical values of A for different distributions are available in [3]

- [2] Anderson, T. W., Darling, D.A., 1952, Asymptotic Theory of Certain “Goodness of Fit” Criteria Based on Stochastic Processes, Annals of Mathematical Statistics 23, Pages 193-212
- [3] Stephens, M.A., 1974, EDF Statistics for Goodness of Fit and some Comparisons, Journal of the American Statistical Association, Vol. 69, Pages 730-737

1.2.5 Correlation





Pearson's Correlation Coefficient

$$r_{b_{ri}b_{rj}} = \frac{\text{Cov}(\mathbf{b}_{ri}, \mathbf{b}_{rj})}{\sqrt{\text{Var}(\mathbf{b}_{ri})} \sqrt{\text{Var}(\mathbf{b}_{rj})}}$$

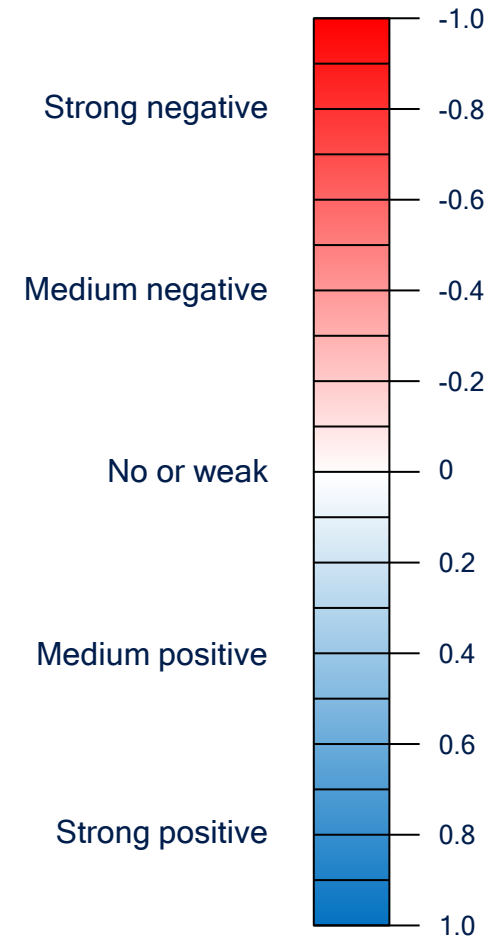
- Identification of linear relationships

Spearman's Rank Correlation Coefficient

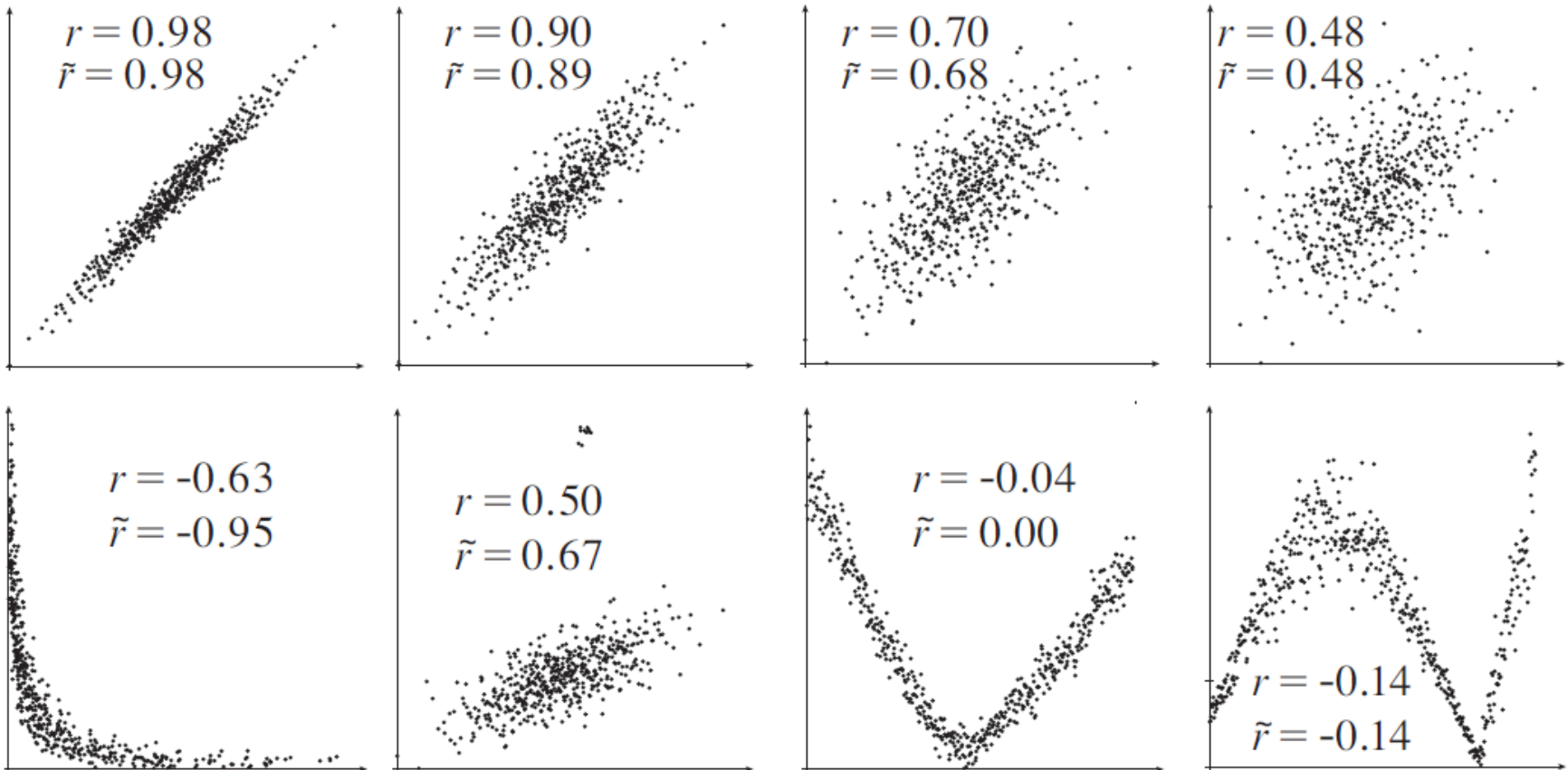
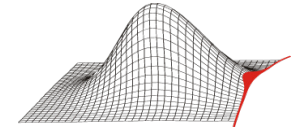
$$\mathbf{b}_{ri} = \begin{bmatrix} b_{ri,1} \\ \vdots \\ b_{ri,n_{sim}} \end{bmatrix} \Rightarrow \text{Rank}(\mathbf{b}_{ri}) = \begin{bmatrix} R_{b_{ri,1}} = \text{Rank of } b_{ri,1} \text{ in } \mathbf{b}_{ri} \\ \vdots \\ R_{b_{ri,n_{sim}}} = \text{Rank of } b_{ri,n_{sim}} \text{ in } \mathbf{b}_{ri} \end{bmatrix}$$

$$\tilde{r}_{b_{ri}b_{rj}} = \frac{\sum_{k=1}^{n_{sim}} (R_{b_{ri,k}} - \bar{R}_{b_{ri}})(R_{b_{rj,k}} - \bar{R}_{b_{rj}})}{\sqrt{\sum_{k=1}^{n_{sim}} (R_{b_{ri,k}} - \bar{R}_{b_{ri}})^2} \sqrt{\sum_{k=1}^{n_{sim}} (R_{b_{rj,k}} - \bar{R}_{b_{rj}})^2}}$$

- Identification of linear or monotonic no-linear relationships



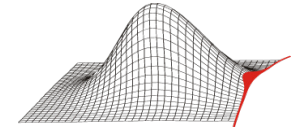
1.2.5 Correlation



r ... Pearson's Correlation Coefficient

\tilde{r} ... Spearman's Rank Correlation Coefficient

1.2.6 Confidence interval



- Statistical measures (mean, standard deviation, ...) are point estimations
- Confidence interval provides information about quality of estimation

γ ... confidence level

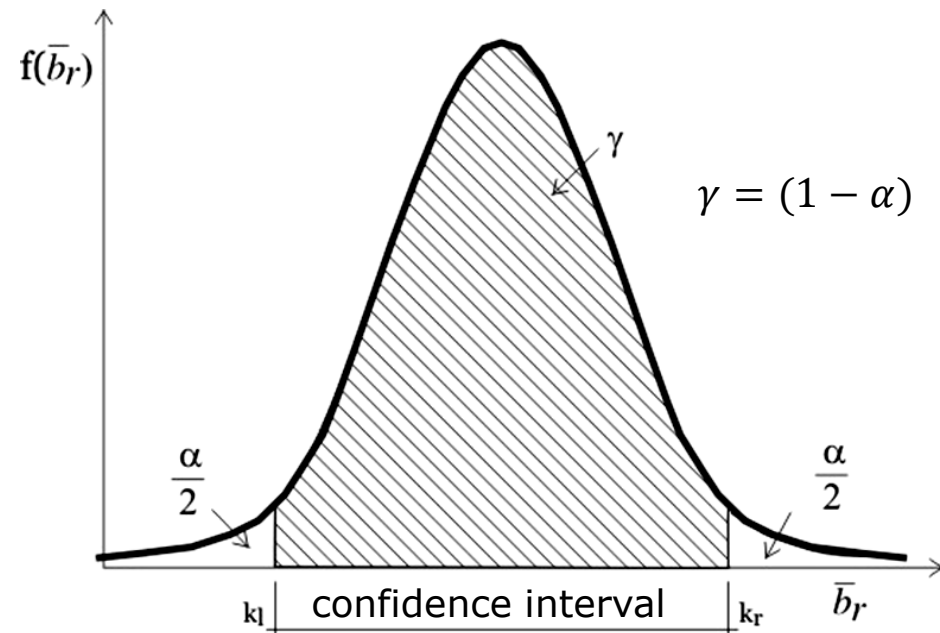
α ... statistical significance

$$\gamma = 1 - \alpha$$

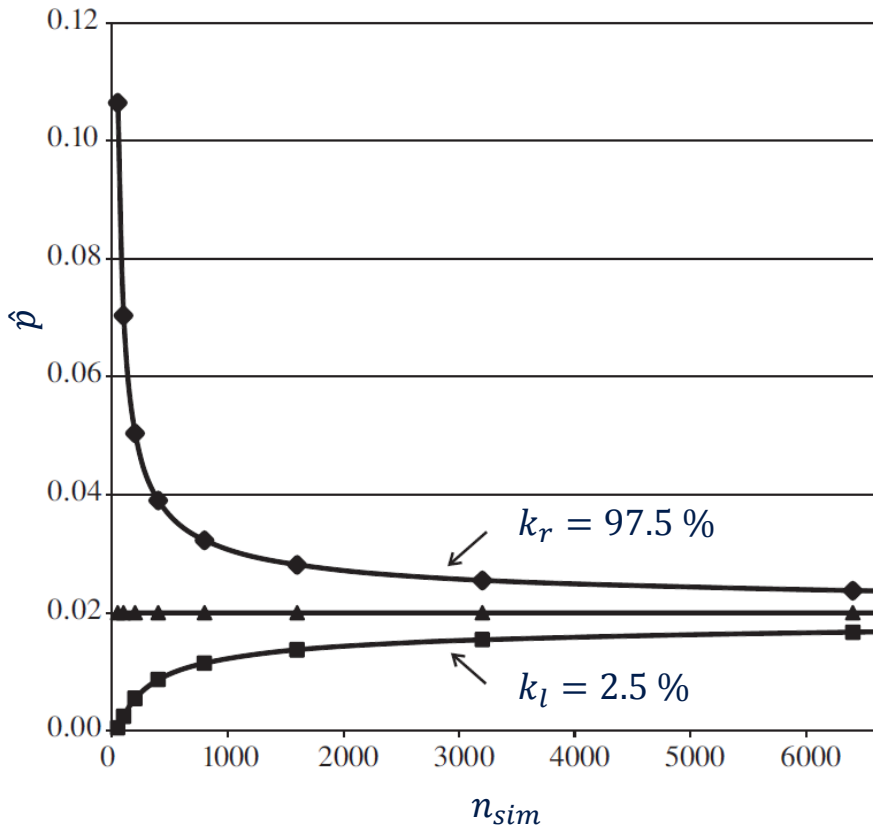
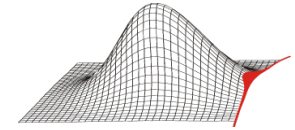
\hat{p} ... relative frequency $\hat{p} = \frac{n_f}{n_{sim}}$

$$k_l = \frac{n_f}{n_f + (n_{sim} - n_f + 1) F_{\frac{\alpha}{2}; 2(n_{sim} - n_f + 1); 2n_f}}$$

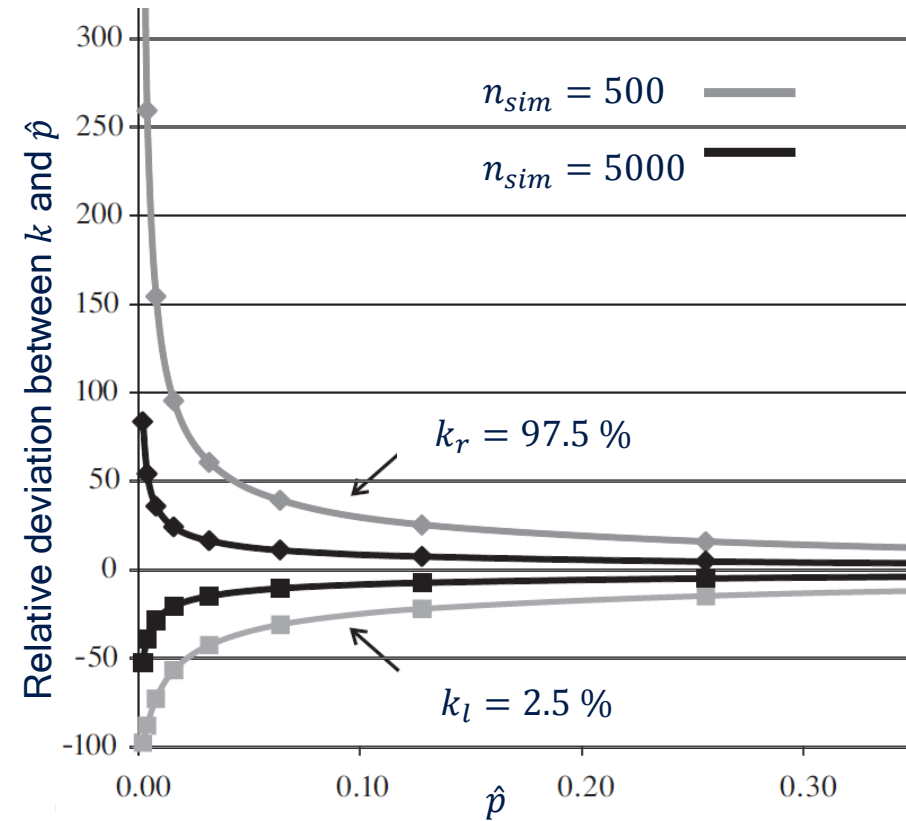
$$k_r = \frac{(n_f + 1) F_{\frac{\alpha}{2}; 2(n_f + 1); 2(n_{sim} - n_f)}}{n_{sim} - n_f + (n_f + 1) F_{\frac{\alpha}{2}; 2(n_f + 1); 2(n_{sim} - n_f)}}$$



1.2.6 Confidence interval

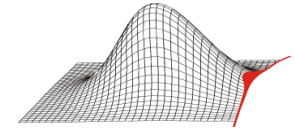










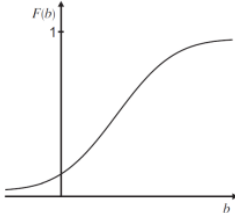
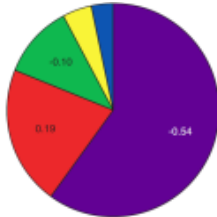
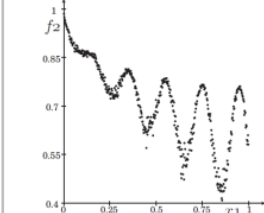
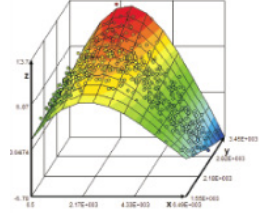
- Confidence interval depends on n_{sim}
- Increase of n_{sim} leads to improved statistical quality



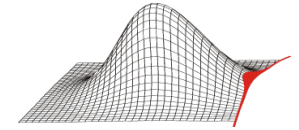
- Confidence interval depends on \hat{p}
- Estimation of rare events are more challenging

1.2.7 Exemplary results



Result of probabilistic Simulation	Probability of failure	Sensitivities	Robustness	System improvement
pdf and correlation of input variables <ul style="list-style-type: none"> roughly known (as in industry) 				
<ul style="list-style-type: none"> precisely known (rarely) 				
Required number of deterministic runs	$n_{sim,LHS} \geq \frac{10}{\hat{P}_f}$ [3]	<ul style="list-style-type: none"> Verification by confidence interval Position of the mean values of the output quantities $n_{sim} \approx 50$; minimum: $n_{sim} = \text{no. inputs} + 10 \dots 20$ 		
Output <ul style="list-style-type: none"> one single MC Simulation provides all result quantities 				

[3] Roos et al. Design Reliability Analysis. 24th CAD-FEM Users' Meeting. International Congress on FEM Technology. Stuttgart. 2006



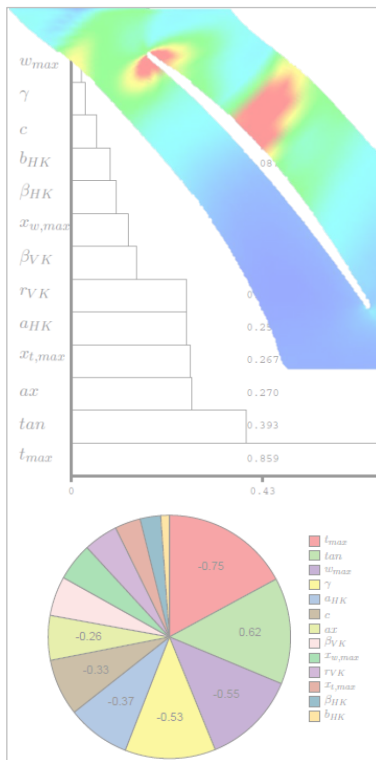
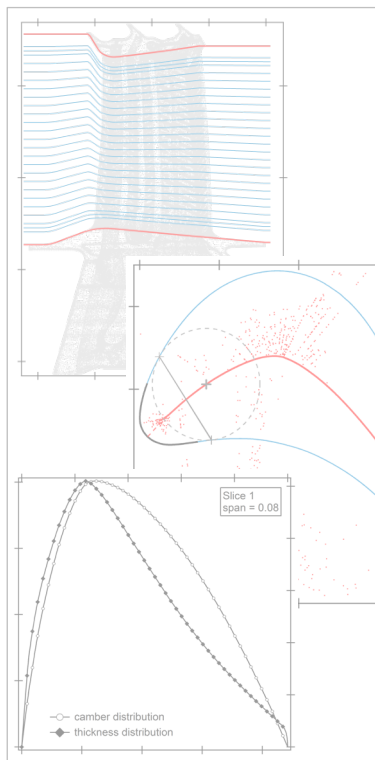
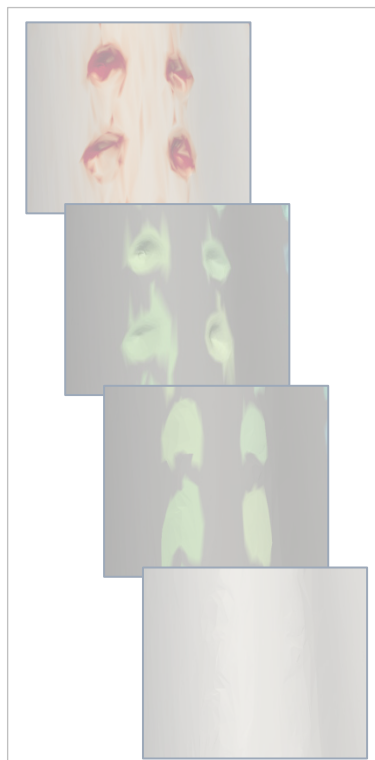
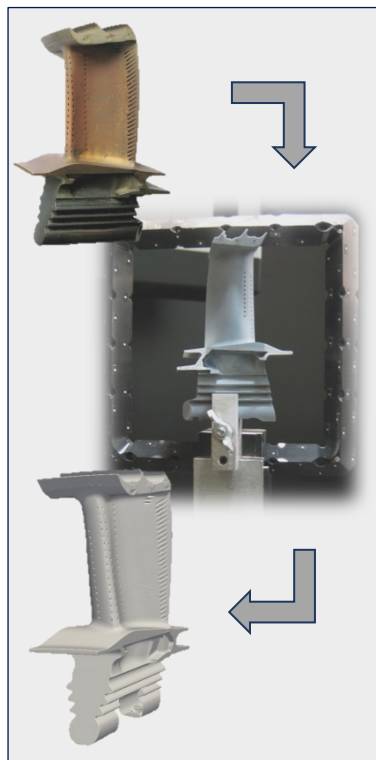
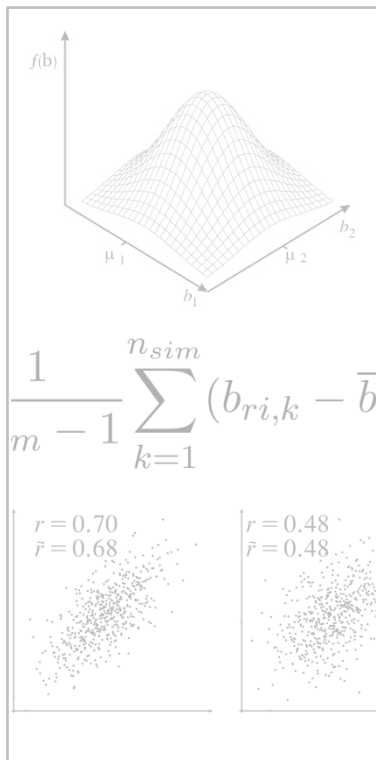
1) Introduction

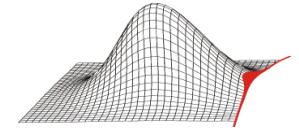
2) Optical Measurement

3) Mesh Pre-Processing

4) Parameterisation and Rebuild

5) Applications





Scatter of manufactured part

- Detection of selected features
- Expected geometric variation due to manufacturing process

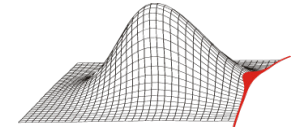
Reliability and Reproducibility

- Coating of components in order to avoid reflections
- Repeat measurements
- Automation



Accuracy

- Systematic errors (coating, geometry)
- Random error



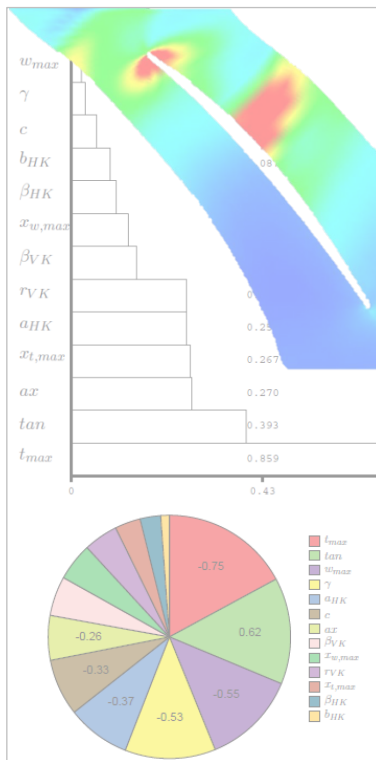
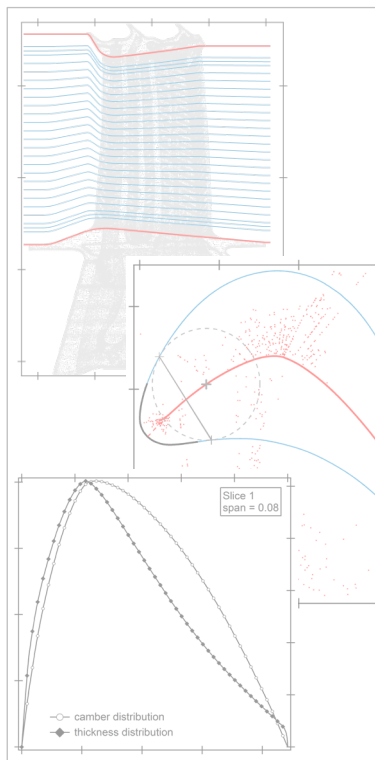
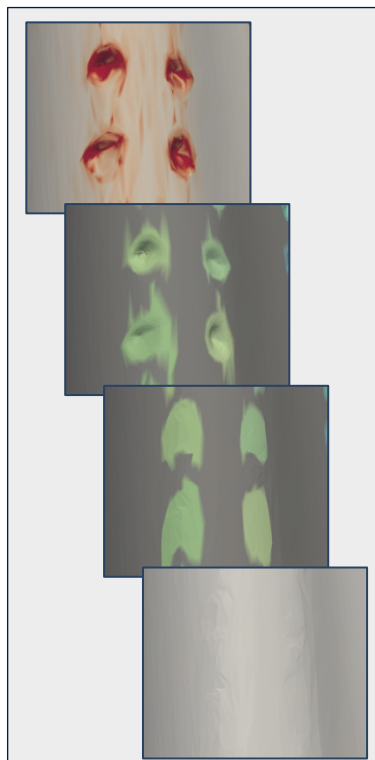
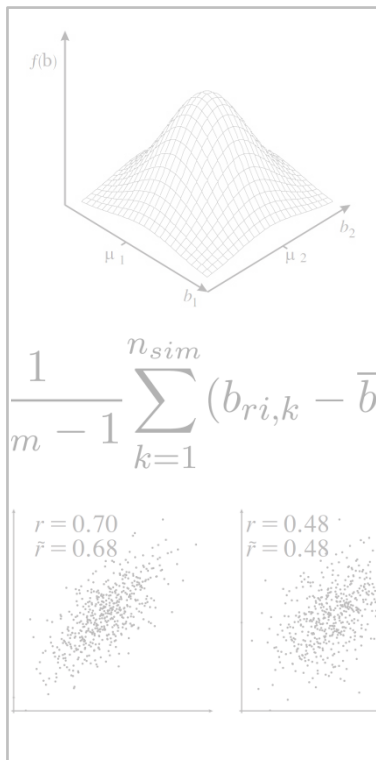
1) Introduction

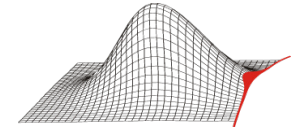
2) Optical Measurement

3) Mesh Pre-Processing

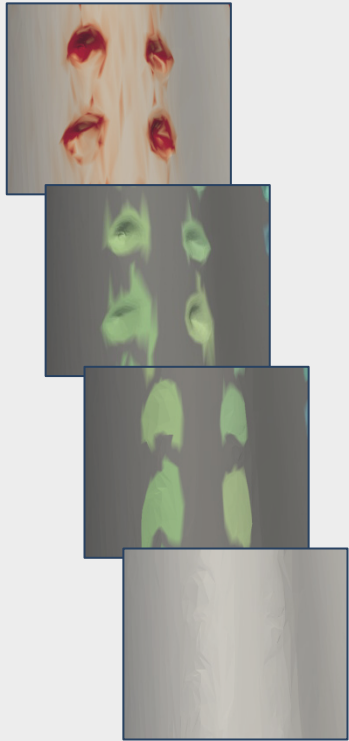
4) Parameterisation and Rebuild

5) Applications





3) Mesh Pre-Processing



3.1 Motivation & Aim

3.2 Feature Detection

3.2.1 Underlying Method

3.2.2 Procedure of Detection

3.2.3 Capability of Algorithm

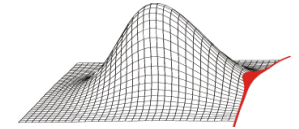
3.2.4 Idea of Application

3.4 Cleaning of the Mesh

3.4.1 Removal of Detected Features

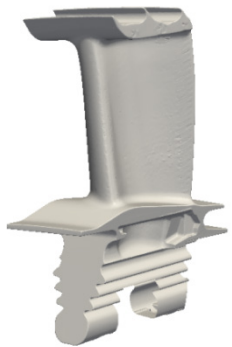
3.4.2 Closing of the Mesh Holes

3.5 Conclusion



Profile parameterisation

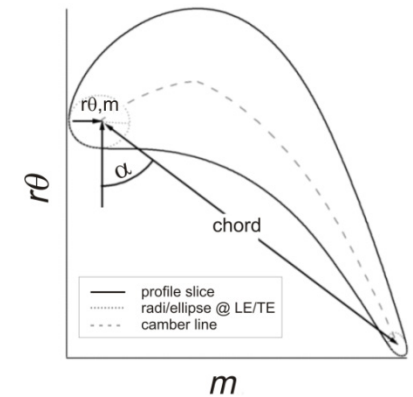
Polygon mesh



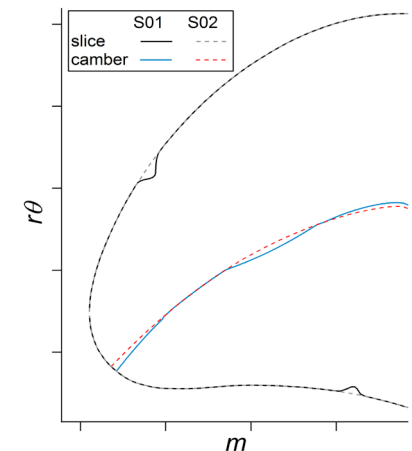
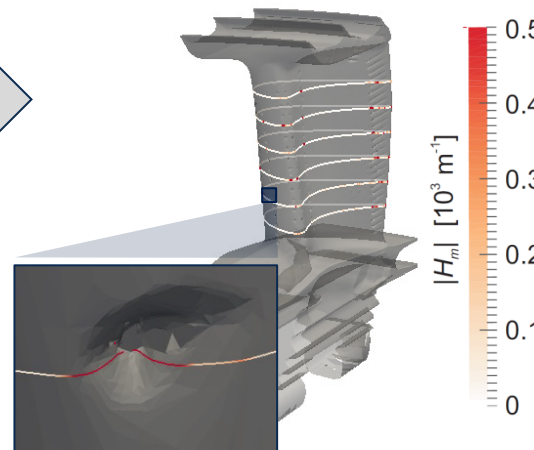
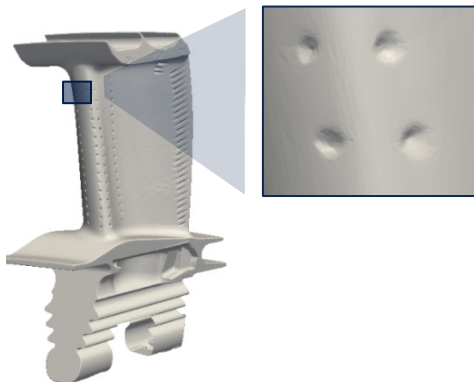
Slice extraction



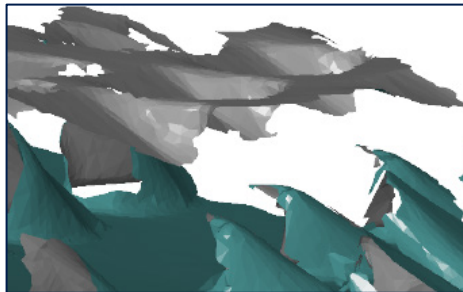
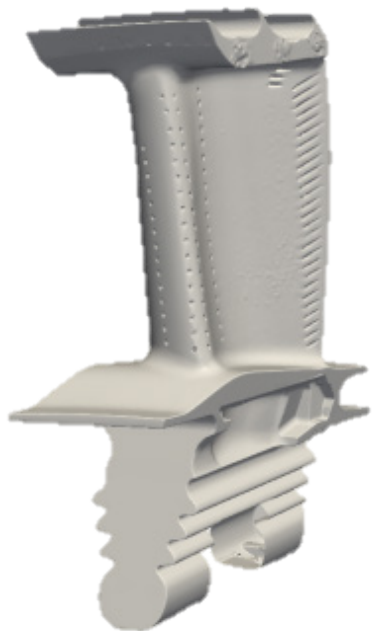
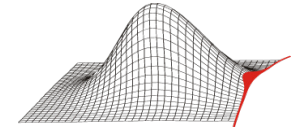
Parameterisation



[1] Lange et al. 2009
[2] Scharfenstein et al. 2013



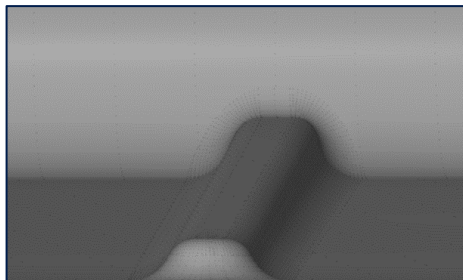
3.1 Motivation & Aim



Measurement artefacts
(holes, detached mesh snippets,
regions of bad resolution, etc.)

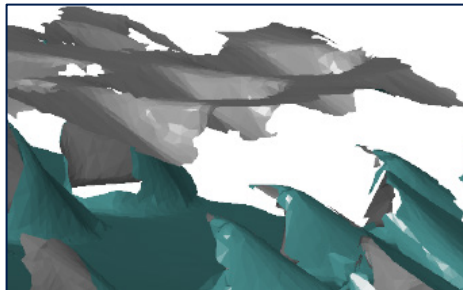
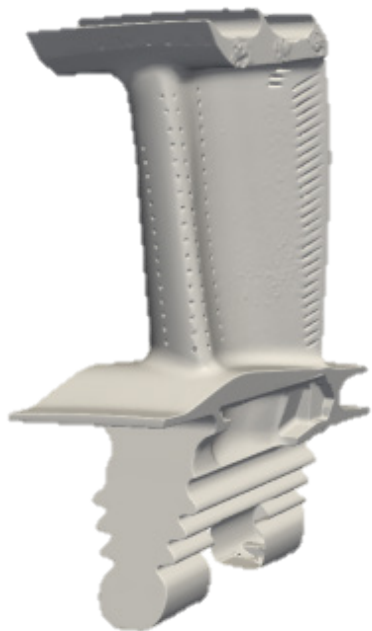
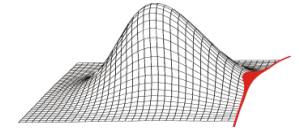


**Physical blade modifications
and damages**
(dents, humps, cracks, etc.)



Small blade features
(turbulators, cooling holes, etc.)

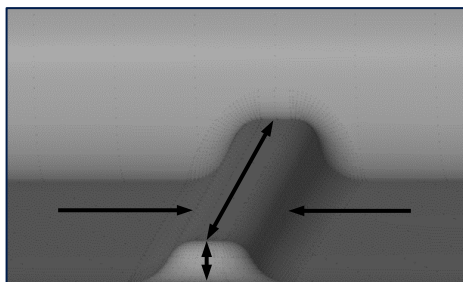
3.1 Motivation & Aim



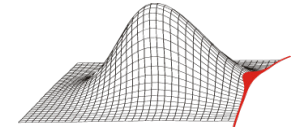
**Clean the mesh from
measurement artefacts**



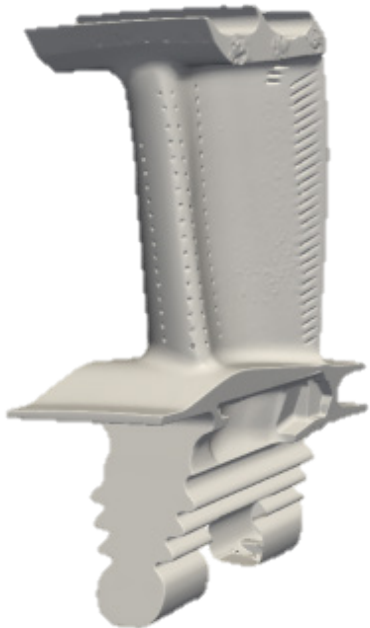
**Capture information for
maintenance & overhaul**



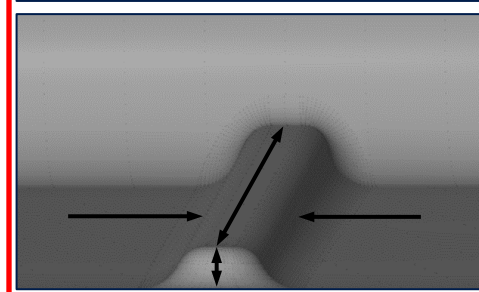
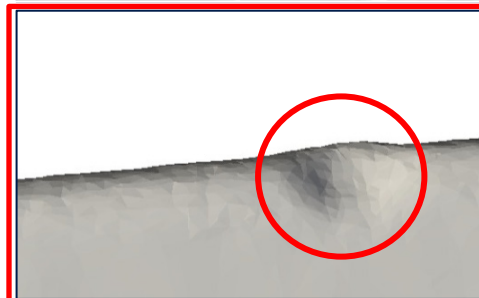
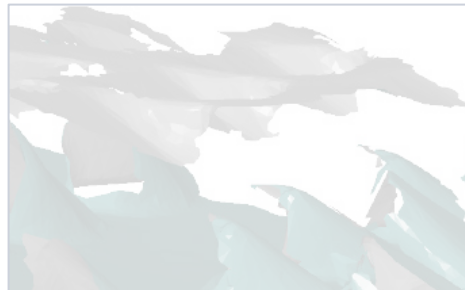
**Enable robust profile
parameterisation**



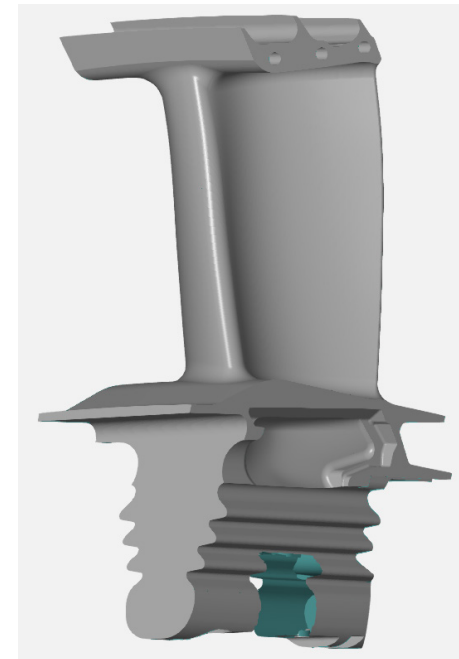
„Featured“ mesh

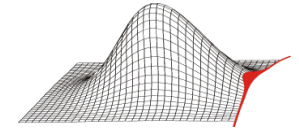


Information of
features

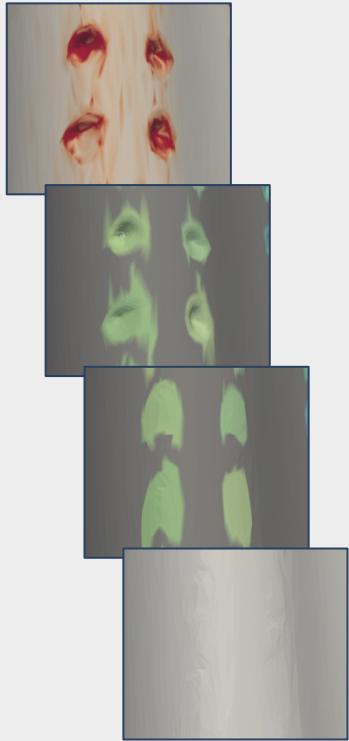


„Cleaned“ mesh





3) Mesh Pre-Processing



3.1 Motivation & Aim

3.2 Feature Detection

3.2.1 Underlying Method

3.2.2 Procedure of Detection

3.2.3 Capability of Algorithm

3.2.4 Idea of Application

3.4 Cleaning of the Mesh

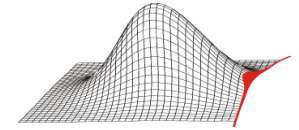
3.4.1 Removal of Detected Features

3.4.2 Closing of the Mesh Holes

3.5 Conclusion

3.2 Feature Detection

3.2.1 Underlying Method



Main principle: curvature investigation

Mean curvature

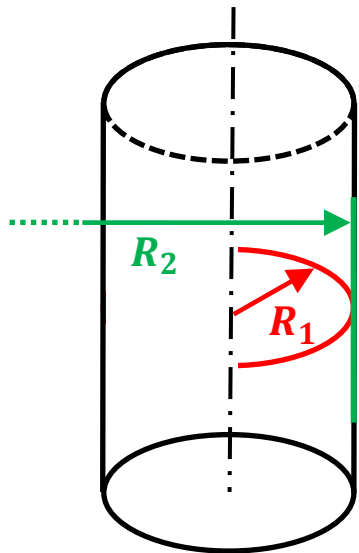
$$H = \frac{1}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

≠ 0 for parabolic points (cylinder)

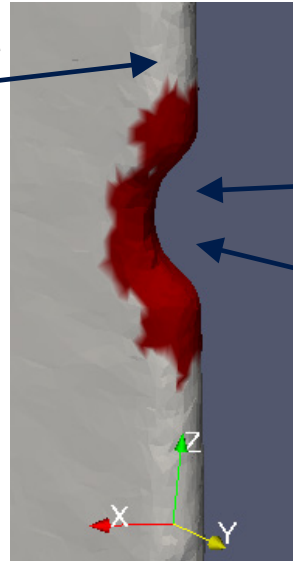
Gauß curvature

$$K = \frac{1}{R_1 R_2}$$

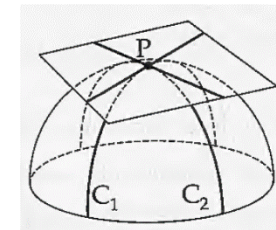
= 0 for cylinders etc.



≈ *Leading edge*

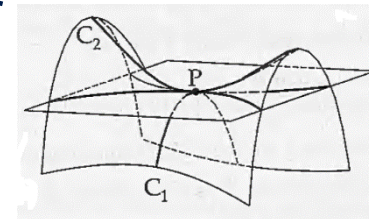


Dents etc.



elliptic:

$$K > 0$$

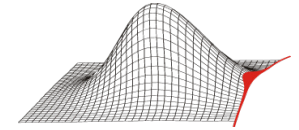


hyperbolic:

$$K < 0$$

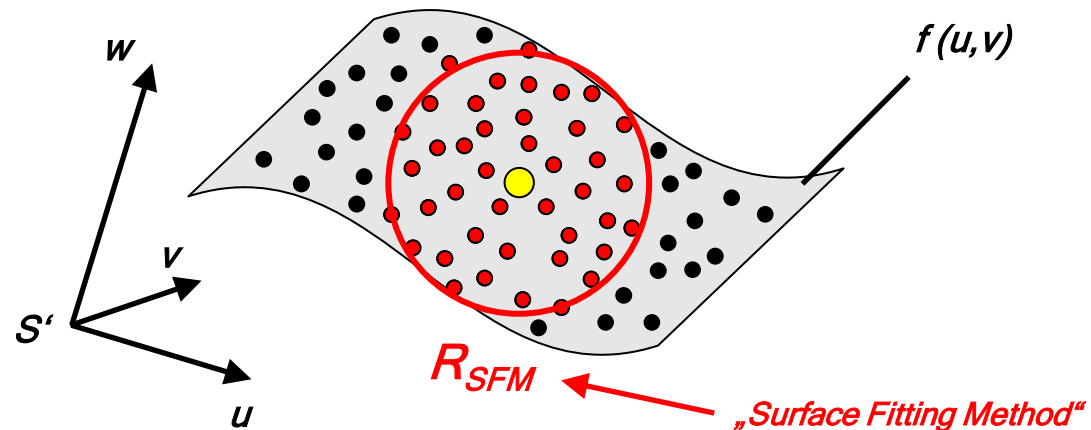
3.2 Feature Detection

3.2.2 Procedure of Detection



For each point ● of the mesh:

regression surface → analytical surface function $f(u, v)$ in system S'



Connection between
surface function & curvature:

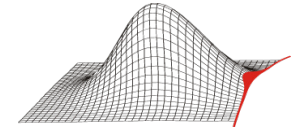
$$K[f(u, v)] = \frac{\frac{\partial^2 f}{\partial u^2} \frac{\partial^2 f}{\partial v^2} - \left[\frac{\partial^2 f}{\partial u \partial v} \right]^2}{\left(1 + \left[\frac{\partial f}{\partial u} \right]^2 + \left[\frac{\partial f}{\partial v} \right]^2 \right)^2}$$

$$H[f(u, v)] = \frac{\frac{\partial^2 f}{\partial u^2} \left(1 + \left[\frac{\partial f}{\partial v} \right]^2 \right) - 2 \frac{\partial f}{\partial u} \frac{\partial f}{\partial v} \frac{\partial^2 f}{\partial u \partial v} + \frac{\partial^2 f}{\partial v^2} \left(1 + \left[\frac{\partial f}{\partial u} \right]^2 \right)}{2 \left(1 + \left[\frac{\partial f}{\partial u} \right]^2 + \left[\frac{\partial f}{\partial v} \right]^2 \right)^{3/2}}$$

Entire mesh gets
curvature distribution

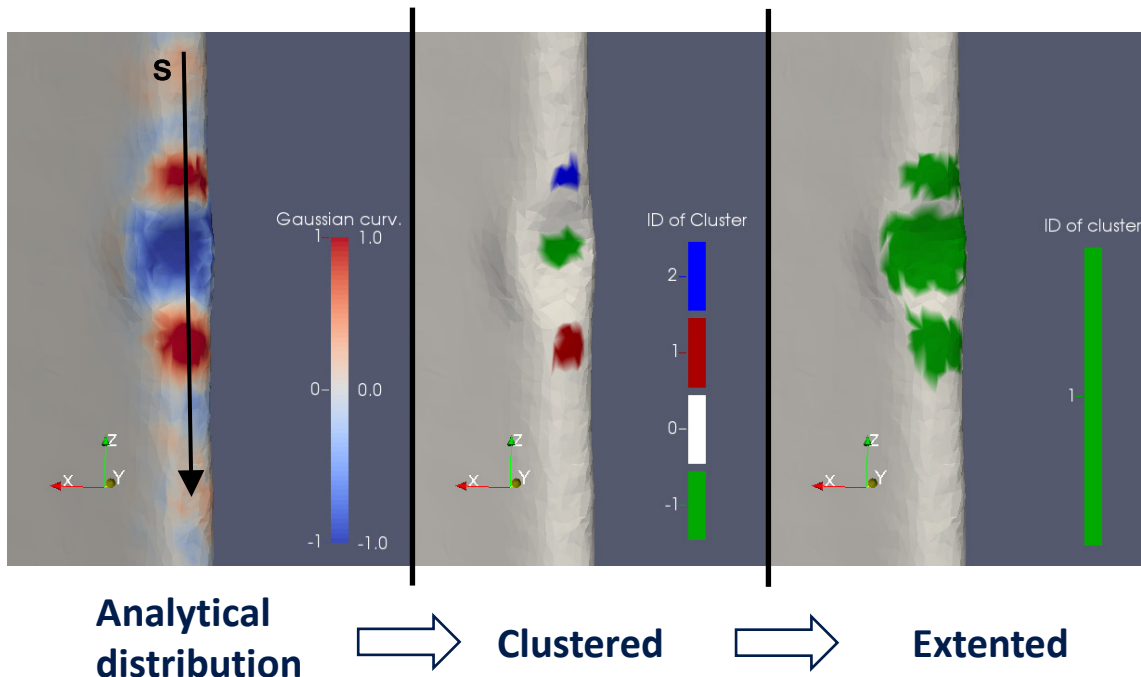
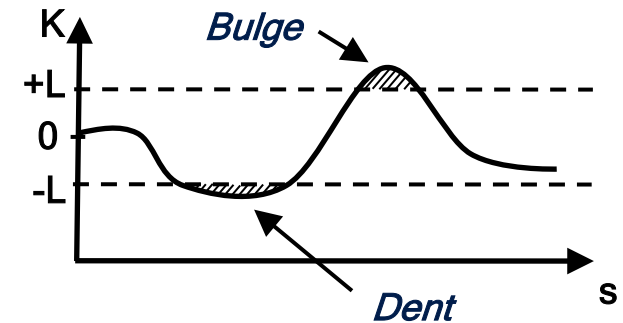
3.2 Feature Detection

3.2.2 Procedure of Detection



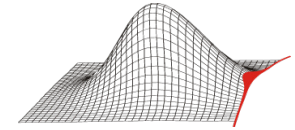
- Discretise analytical (smooth) distribution
- Separate actual feature from adjacent regions

→ ,clustering‘



3.2 Feature Detection

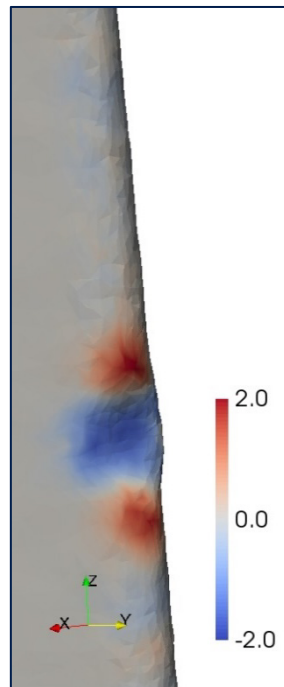
3.2.3 Capability of Algorithm



Different possible features detectable by curvature distribution: dents, cooling holes, etc.



Leading edge fan blade



Gaussian curvature distribution



Identification of impact

Sensitivity of algorithm adjustable:

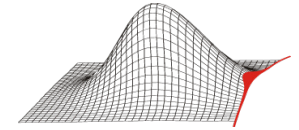
- limit value of curvature for clustering
- radius for surface regression

Application example:

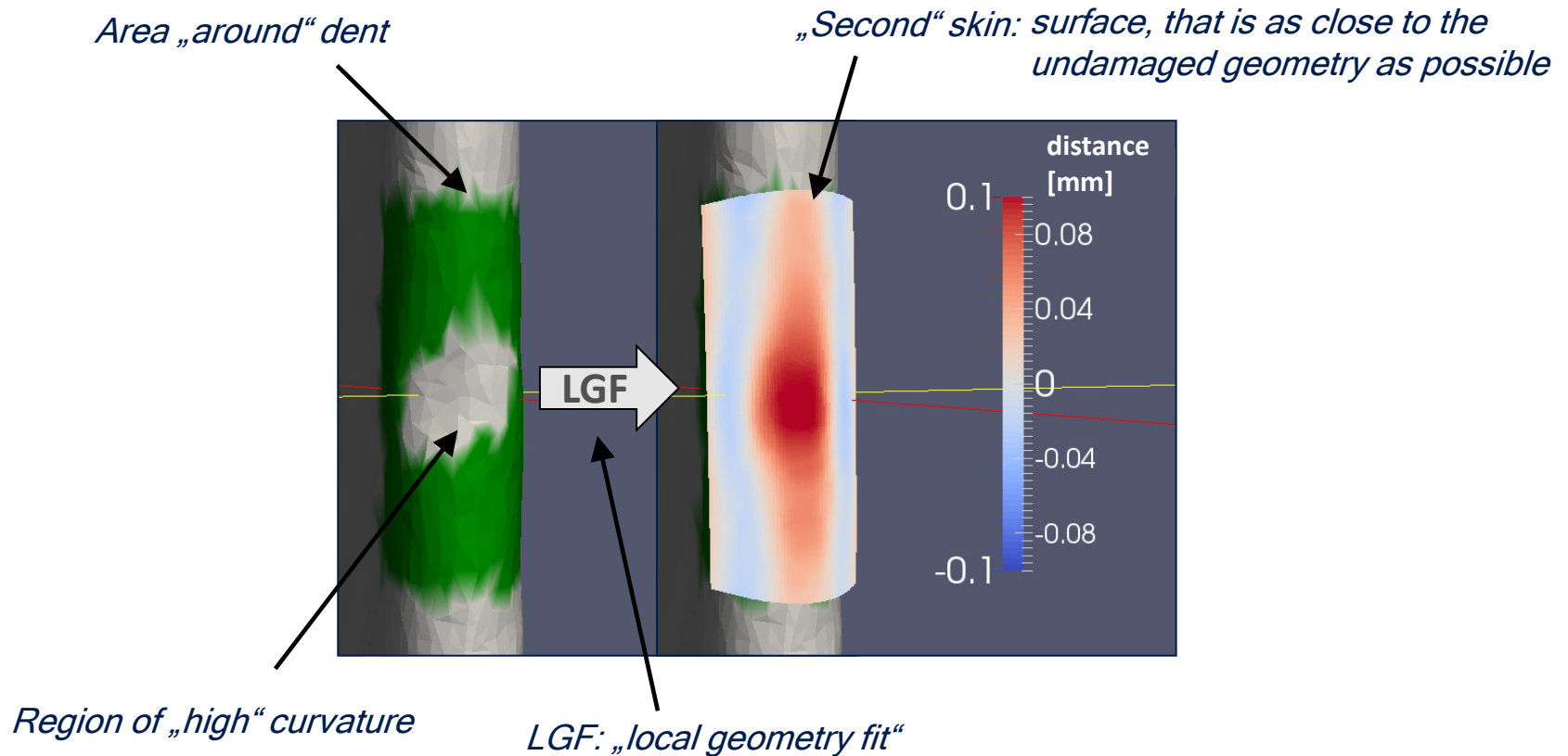
- in combination with a dent depth computation algorithm
- automatic assessment of blades regarding FODs

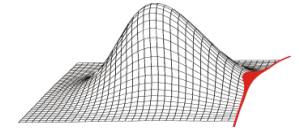
3.2 Feature Detection

3.2.4 Idea of Application

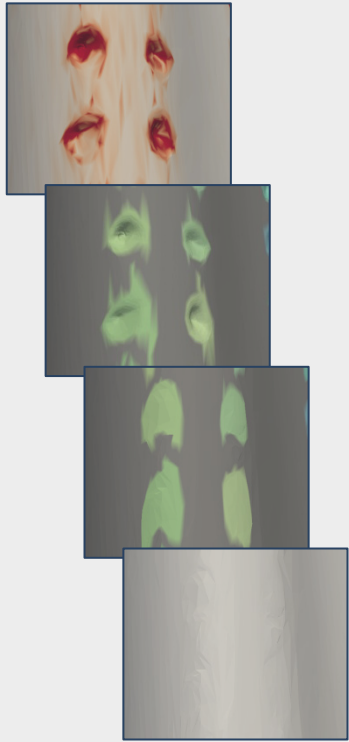


- Application for maintenance purposes
- Here: measuring the impact depth of a leading edge dent





3) Mesh Pre-Processing



3.1 Motivation & Aim

3.2 Feature Detection

3.2.1 Underlying Method

3.2.2 Procedure of Detection

3.2.3 Capability of Algorithm

3.2.4 Idea of Application

3.4 Cleaning of the Mesh

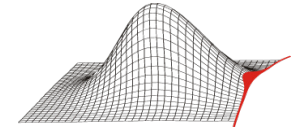
3.4.1 Removal of Detected Features

3.4.2 Closing of the Mesh Holes

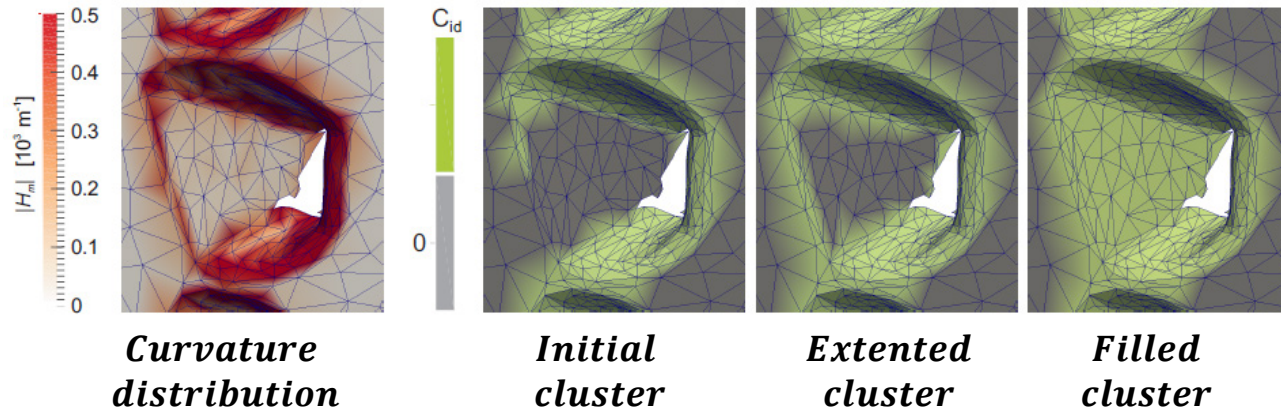
3.5 Conclusion

3.4 Cleaning of the Mesh

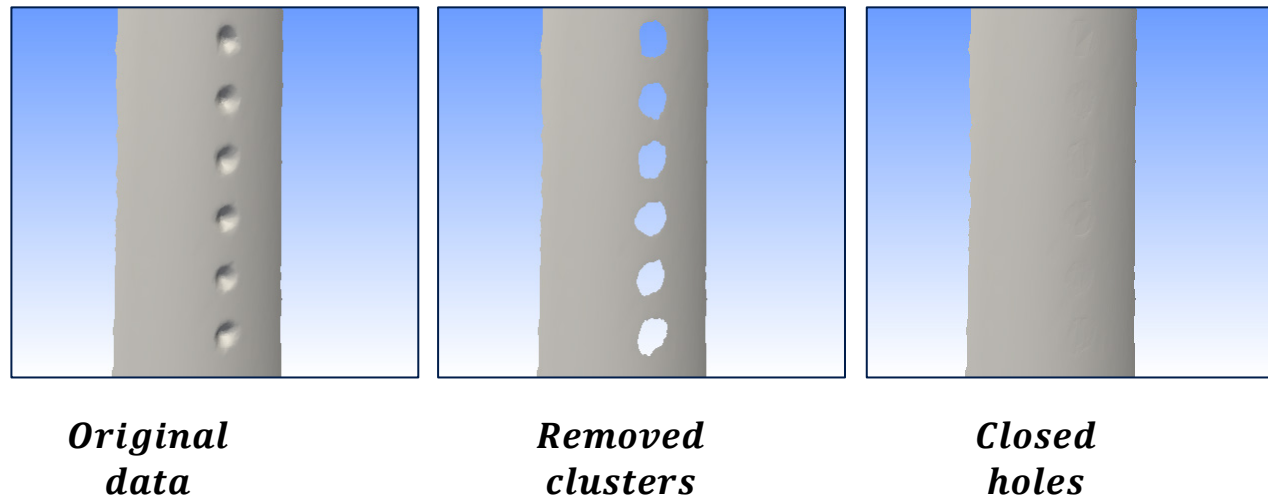
3.4.1 Removal of Detected Features



Preparation

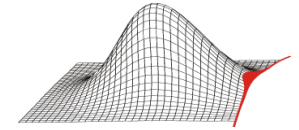


Removal & closing

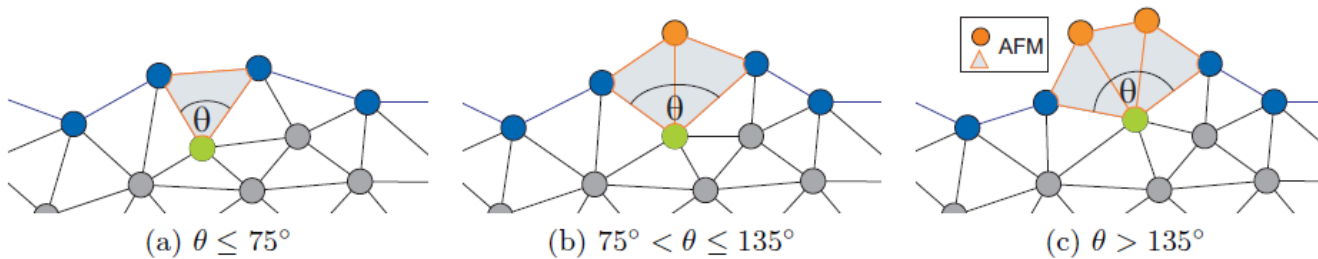


3.4 Removal of Detected Features

3.4.1 Closing of the Mesh Holes

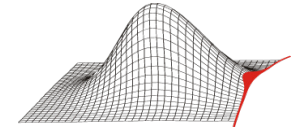


- When detected, deletion of features/ artefacts relatively simple
 - Closing of holes is significant step
 - 2 algorithms implemented:
 - Advancing Front Meshing (AFM):
 - Pros: simple, quick, robust
 - Cons: does not consider curved surfaces
- further smoothing step necessary



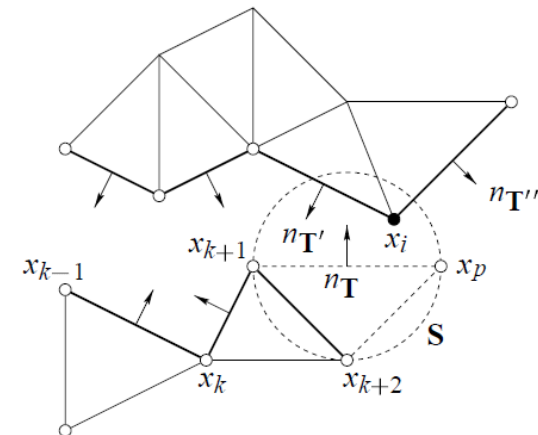
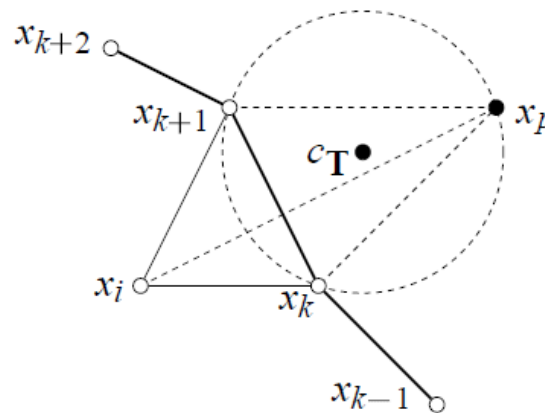
3.4 Removal of Detected Features

3.4.1 Closing of the Mesh Holes



- **Marching Triangles (MT):**
 - high sophisticated algorithm for many advanced geometry applications
 - Pros: follows the curved nominal geometry; resulting triangle sizes can be adapted automatically; equally distributed nodes
 - Cons: quite complex implementations; slower than simple algorithms like AFM

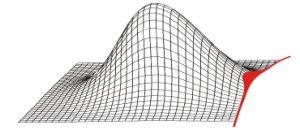
Base principle:
check of Delaunay
constraint for new triangle



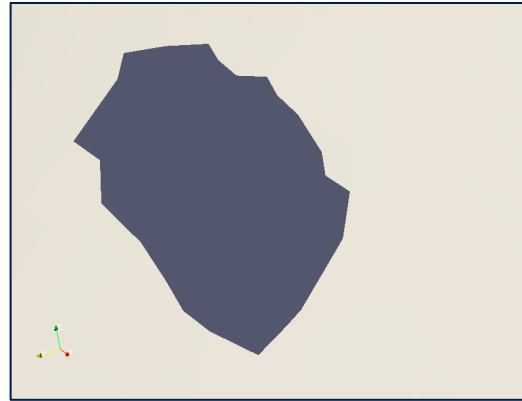


3.4 Removal of Detected Features

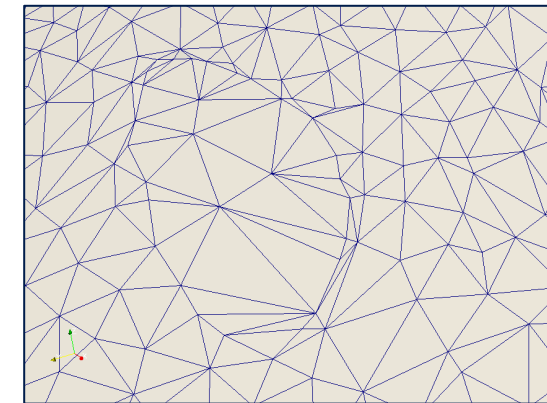
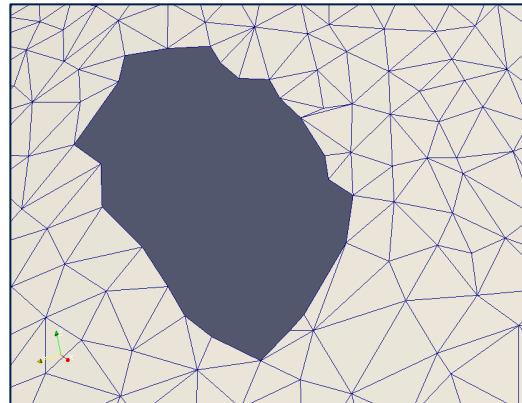
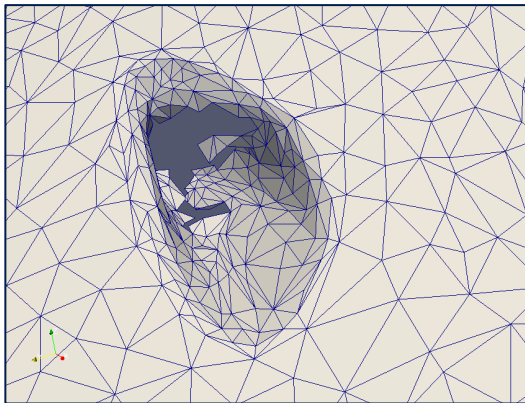
3.4.1 Closing of the Mesh Holes



Mesh
surface



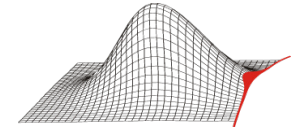
Mesh
grid



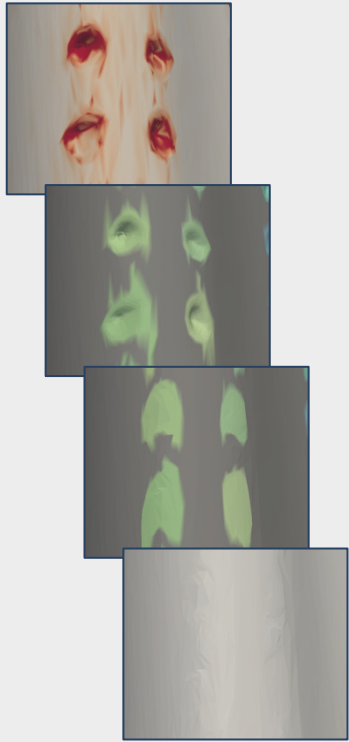
Feature (cooling hole)

Deleted feature

Closed mesh hole



3) Mesh Pre-Processing



3.1 Motivation & Aim

3.2 Feature Detection

3.2.1 Underlying Method

3.2.2 Procedure of Detection

3.2.3 Capability of Algorithm

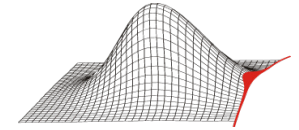
3.2.4 Idea of Application

3.4 Cleaning of the Mesh

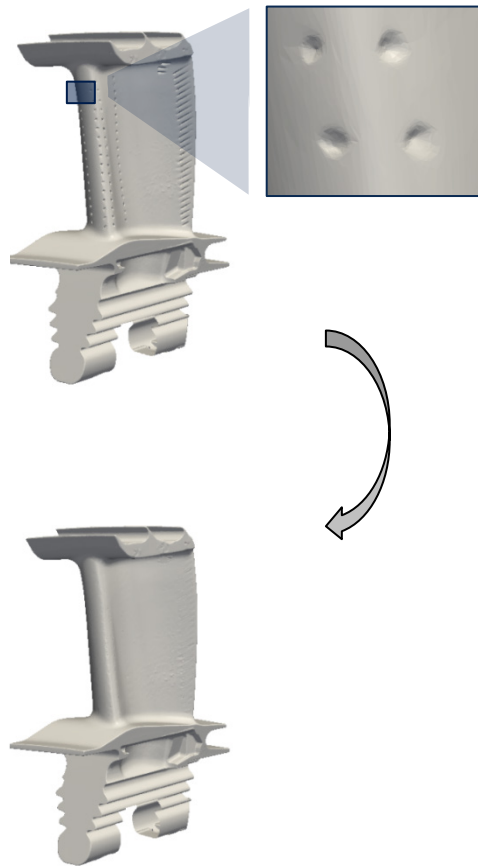
3.4.1 Removal of Detected Features

3.4.2 Closing of the Mesh Holes

3.5 Conclusion

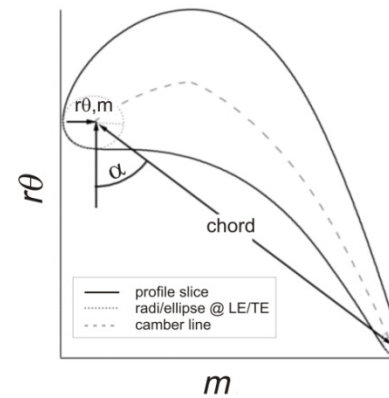
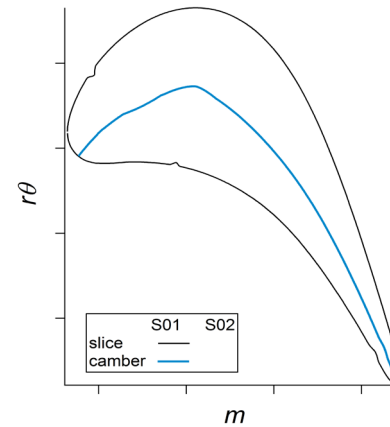


Mesh with artefacts



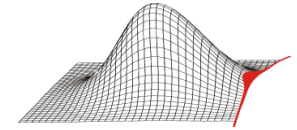
Cleaned mesh

Parameterisation erroneous



[1] Lange et al. 2009
[2] Scharfenstein et al. 2013

Correct parameterisation possible



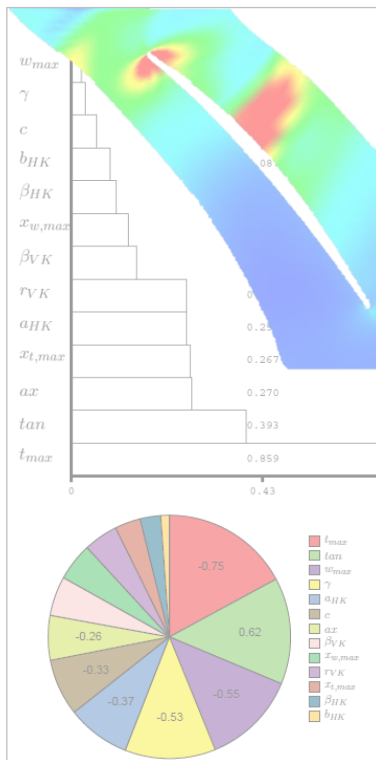
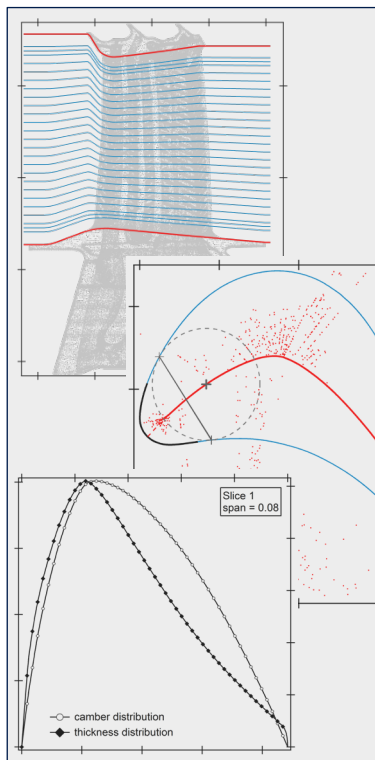
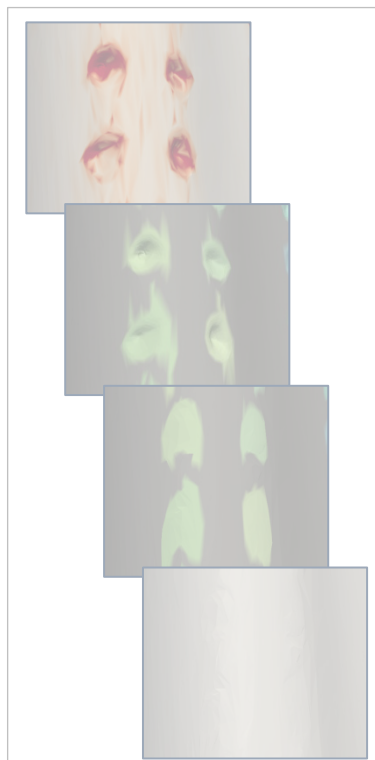
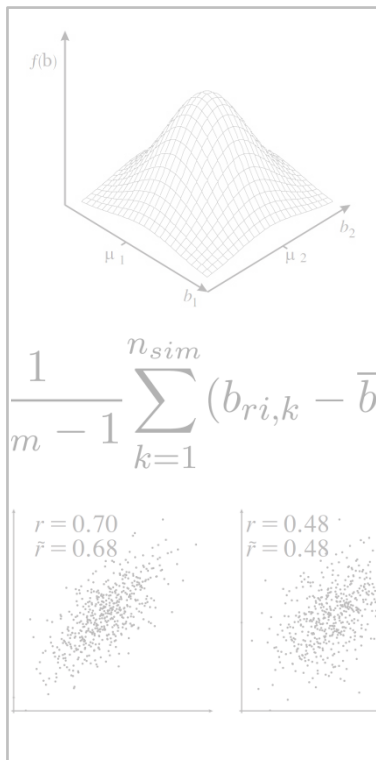
1) Introduction

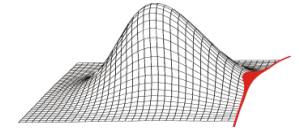
2) Optical Measurement

3) Mesh Pre-Processing

4) Parameterisation and Rebuild

5) Applications



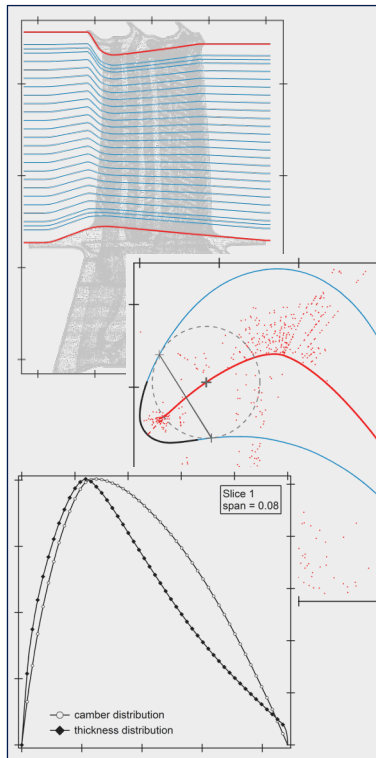


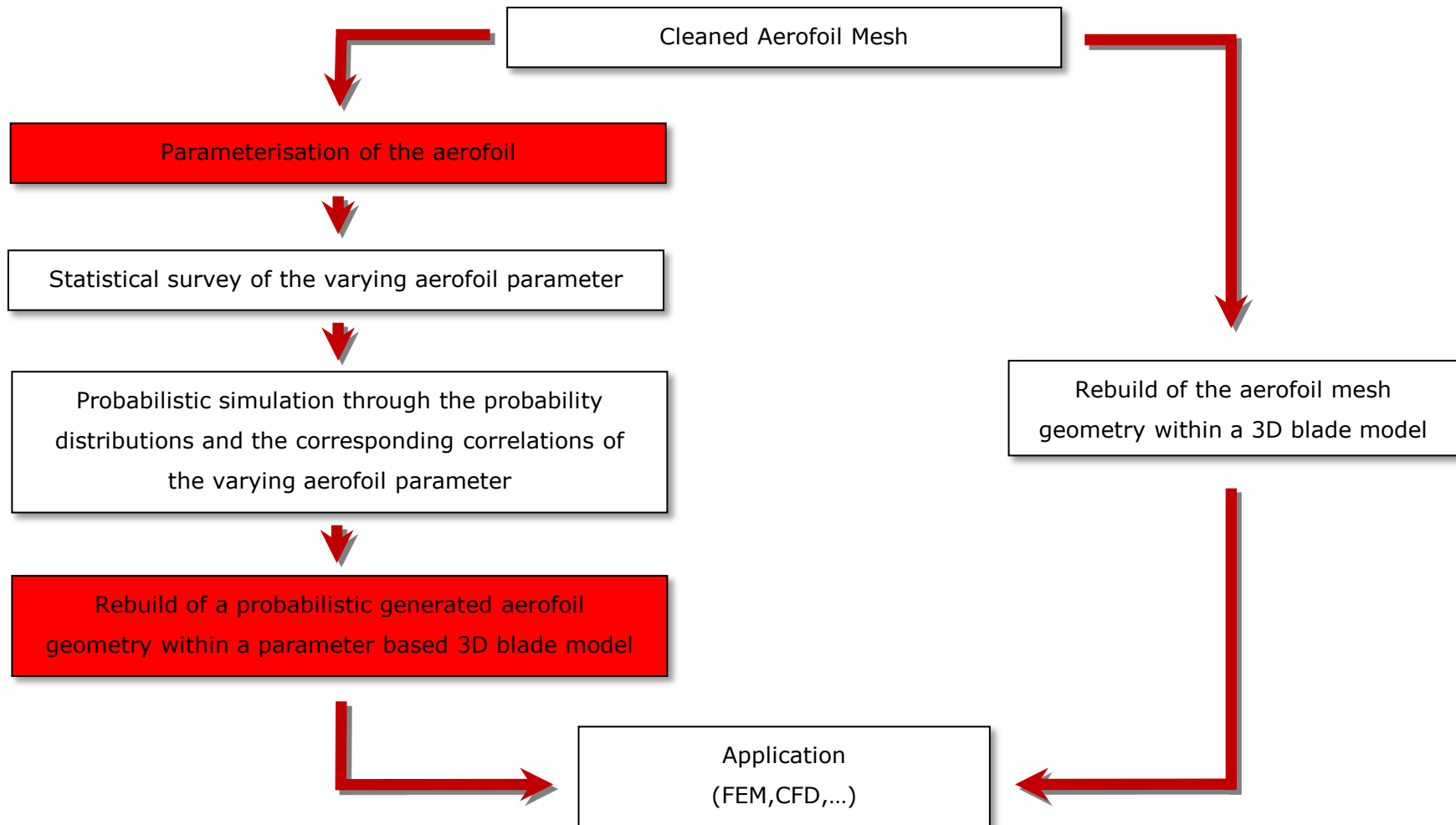
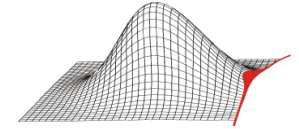
4) Parameterisation and Rebuild

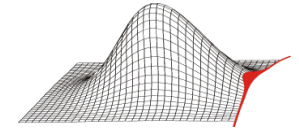
4.1 Parameterisation

4.2 Parameter based Rebuild

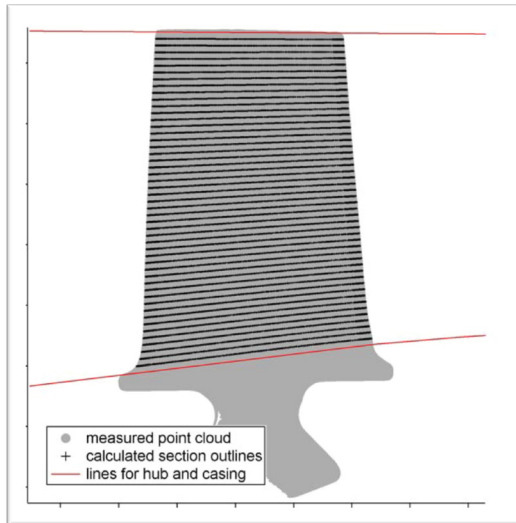
4.3 One to One Rebuild



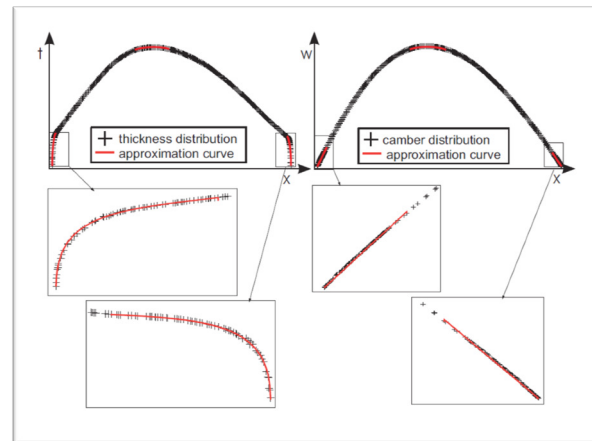
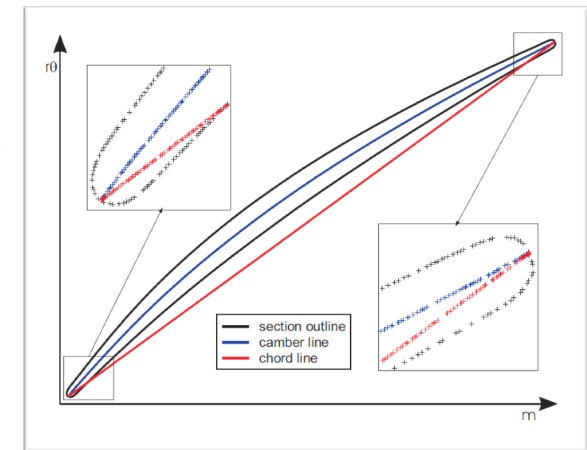




A) cut aerofoil into sections

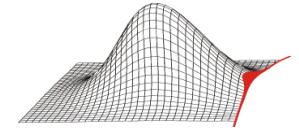


B) extraction of the profile geometry and skeleton line calculation

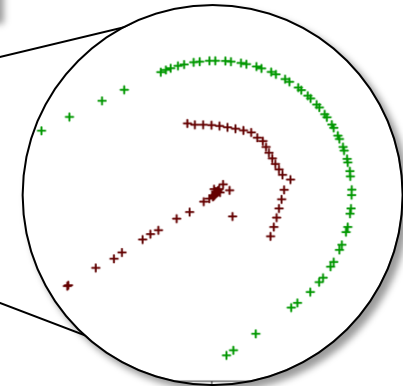
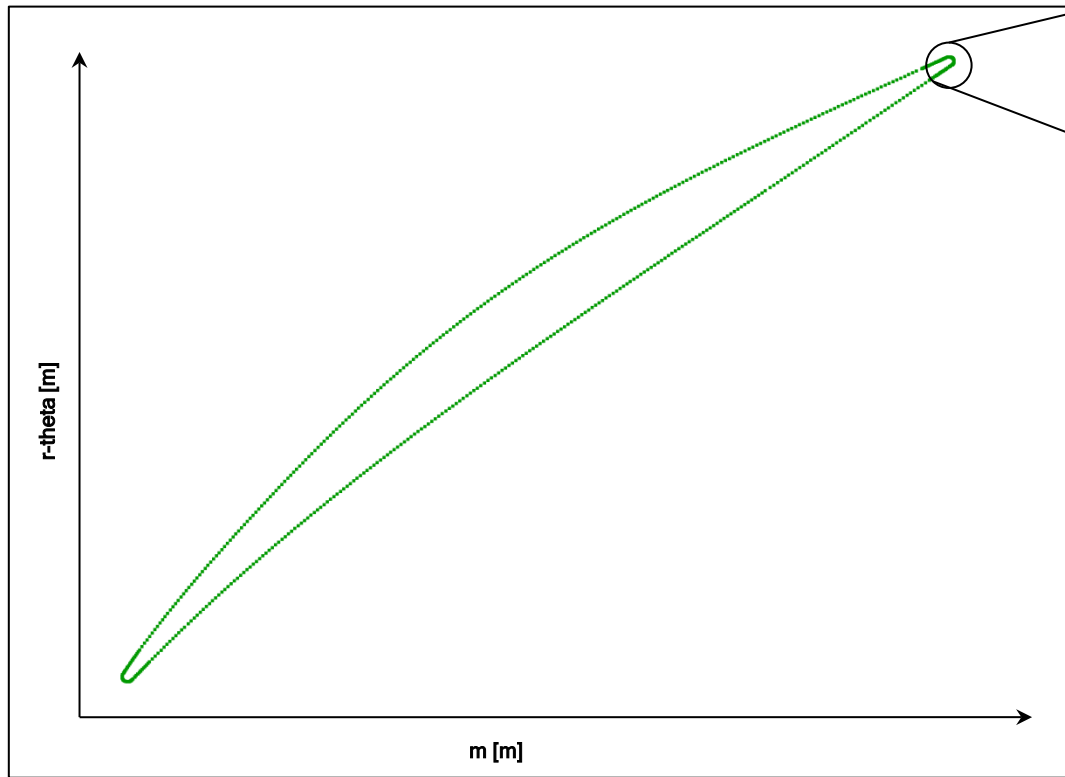


C) evaluation of all aerofoil parameter

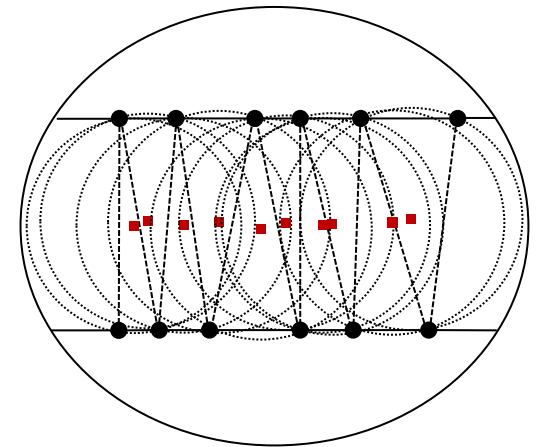
[1] A. Lange: Introduction of a parameter based compressor blade model for considering measured geometry uncertainties in numerical simulation, ASME Turbo Expo, GT2009-59937, Orlando, 2009

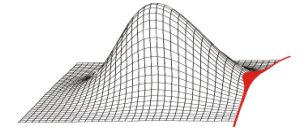


B.1) generation of the skeleton line with Delaunay-Voronoi-triangulation

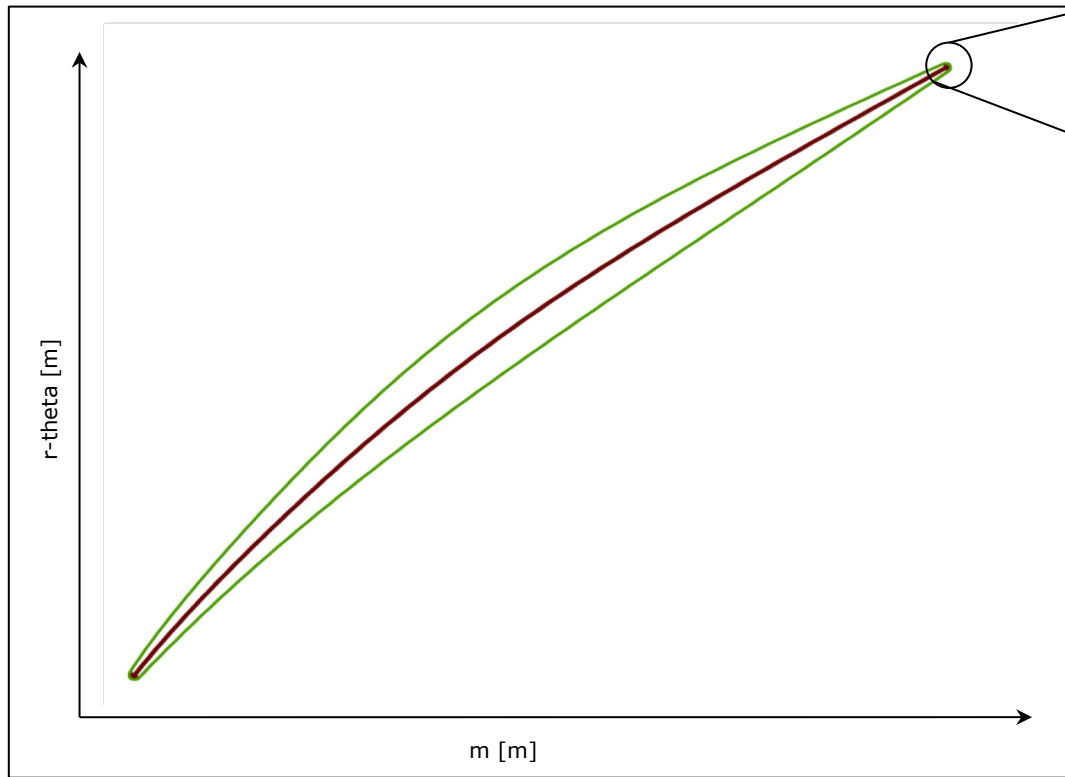


generation of the Delaunay triangles
with the corresponding Voronoi points





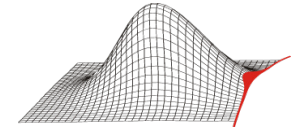
B.2) higher details on skeleton line with more points on profile outline



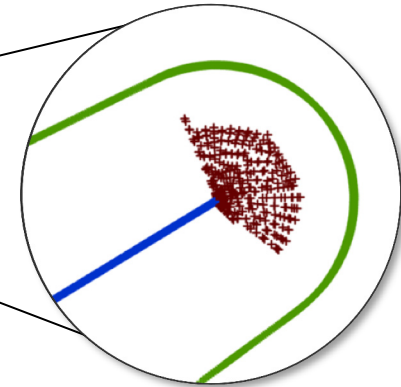
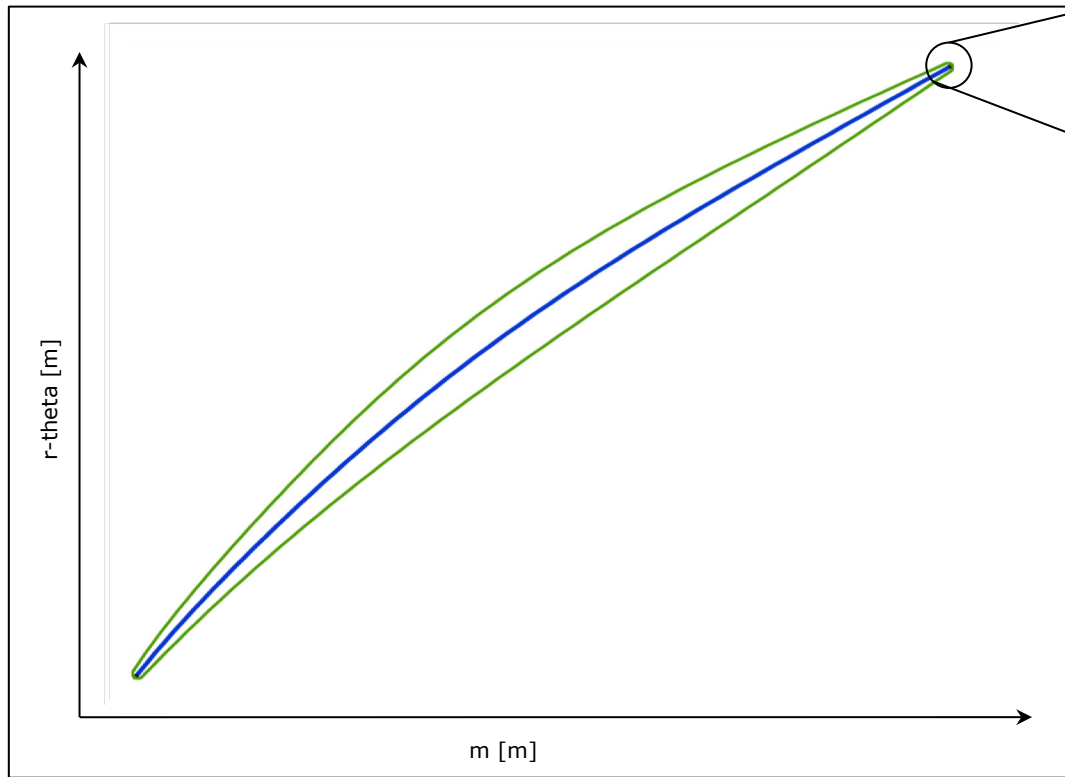
Status 1

Voronoi skeleton line splits up at leading and trailing edge

Slide 46

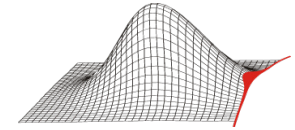


B.3) extrapolation of the skeleton line

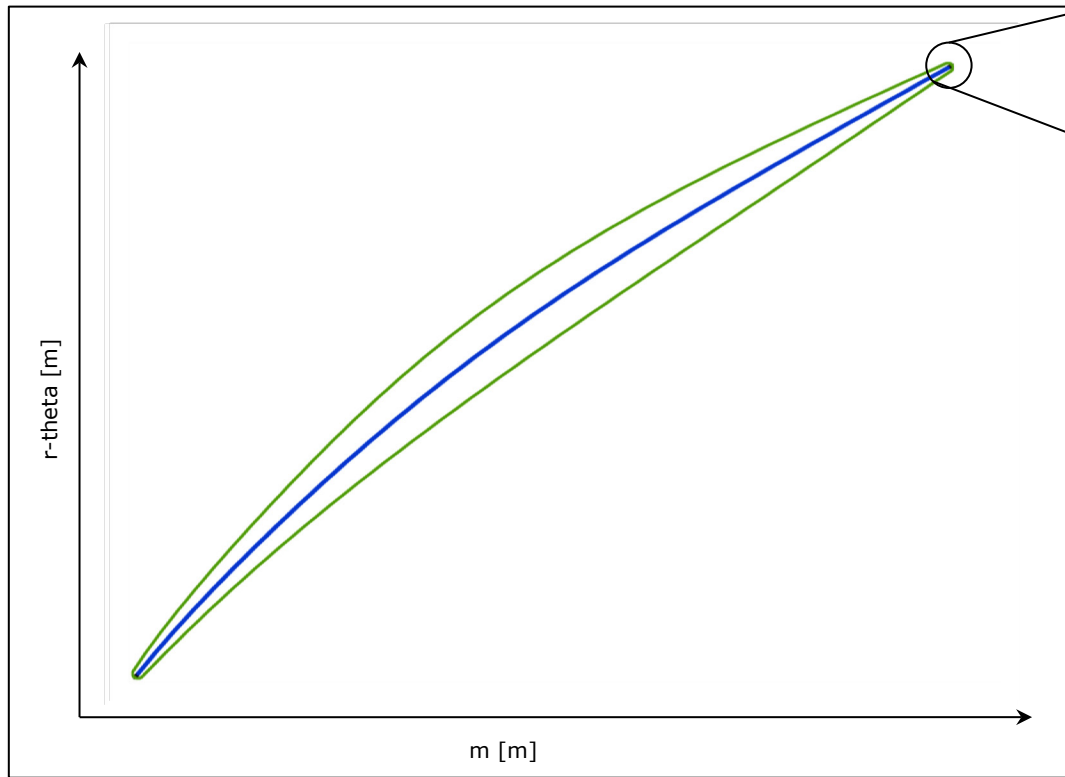


Status 2

the skeleton line will be cut off
after the splitting points at
leading and trailing edge

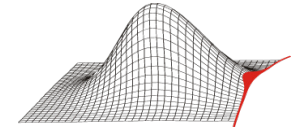


B.3) extrapolation of the skeleton line

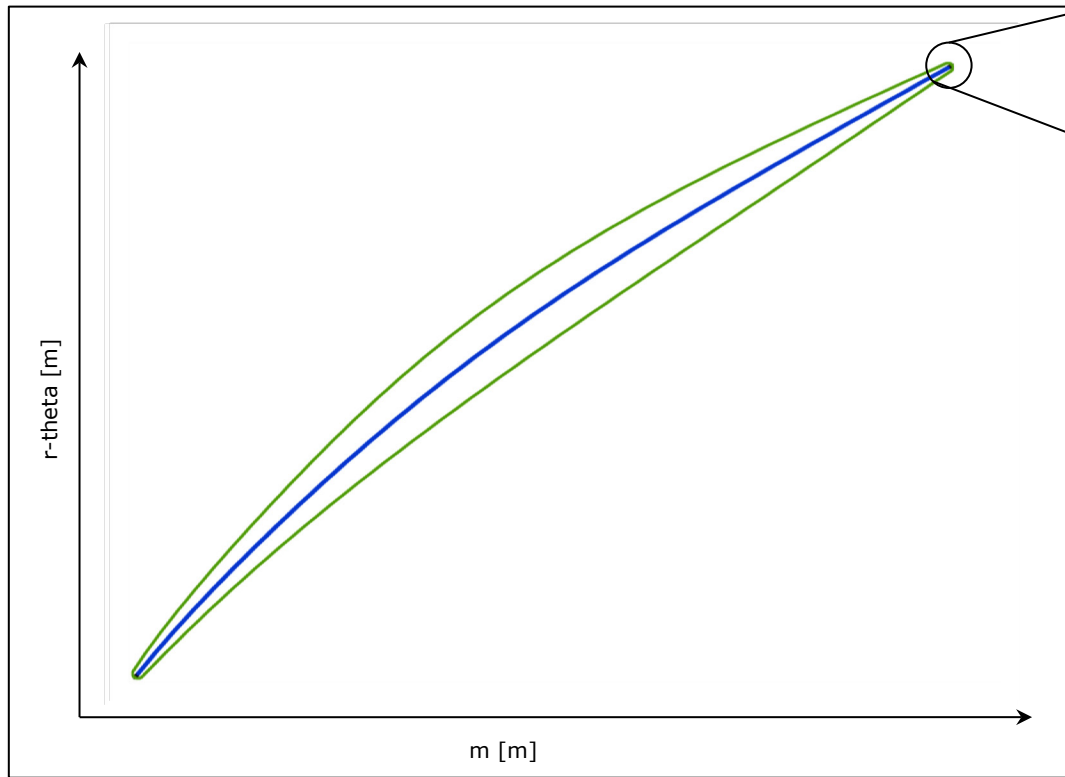


Status 3

the skeleton line will be extrapolated at leading and trailing edge and the cross points towards the profile outline will be calculated

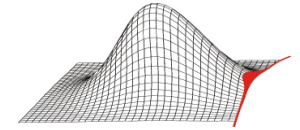


B.3) extrapolation of the skeleton line

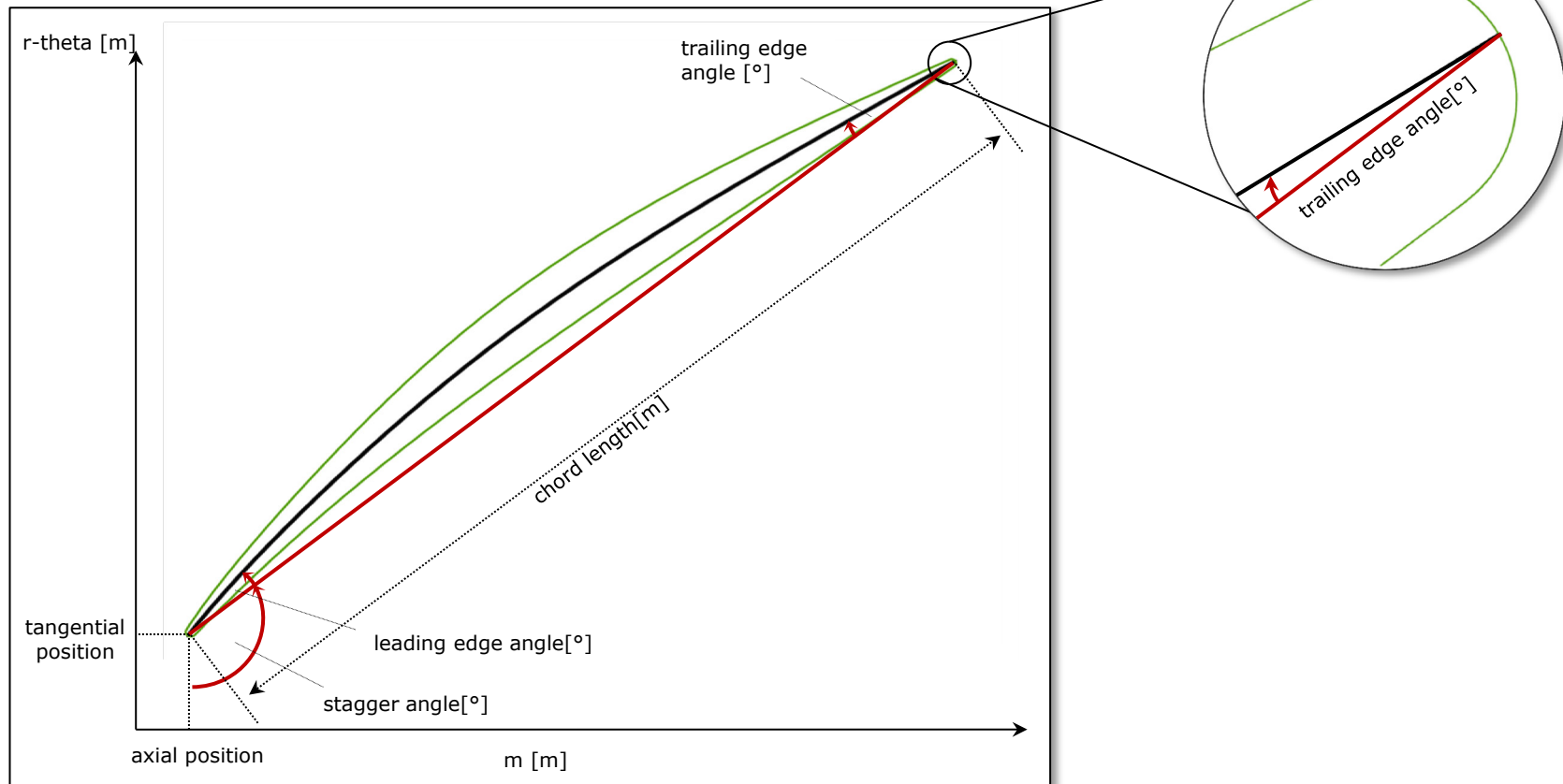


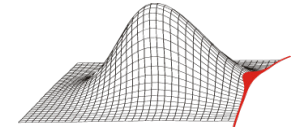
Status 4

the final skeleton line will be generated and discretised

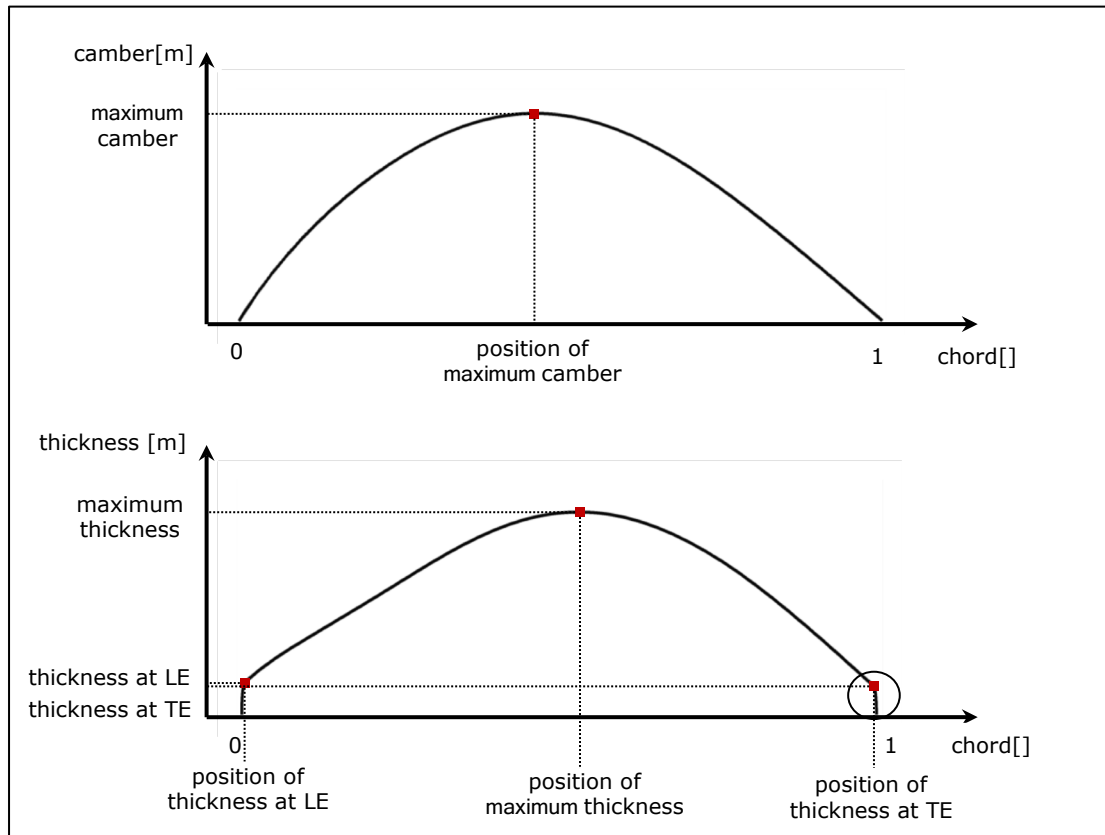


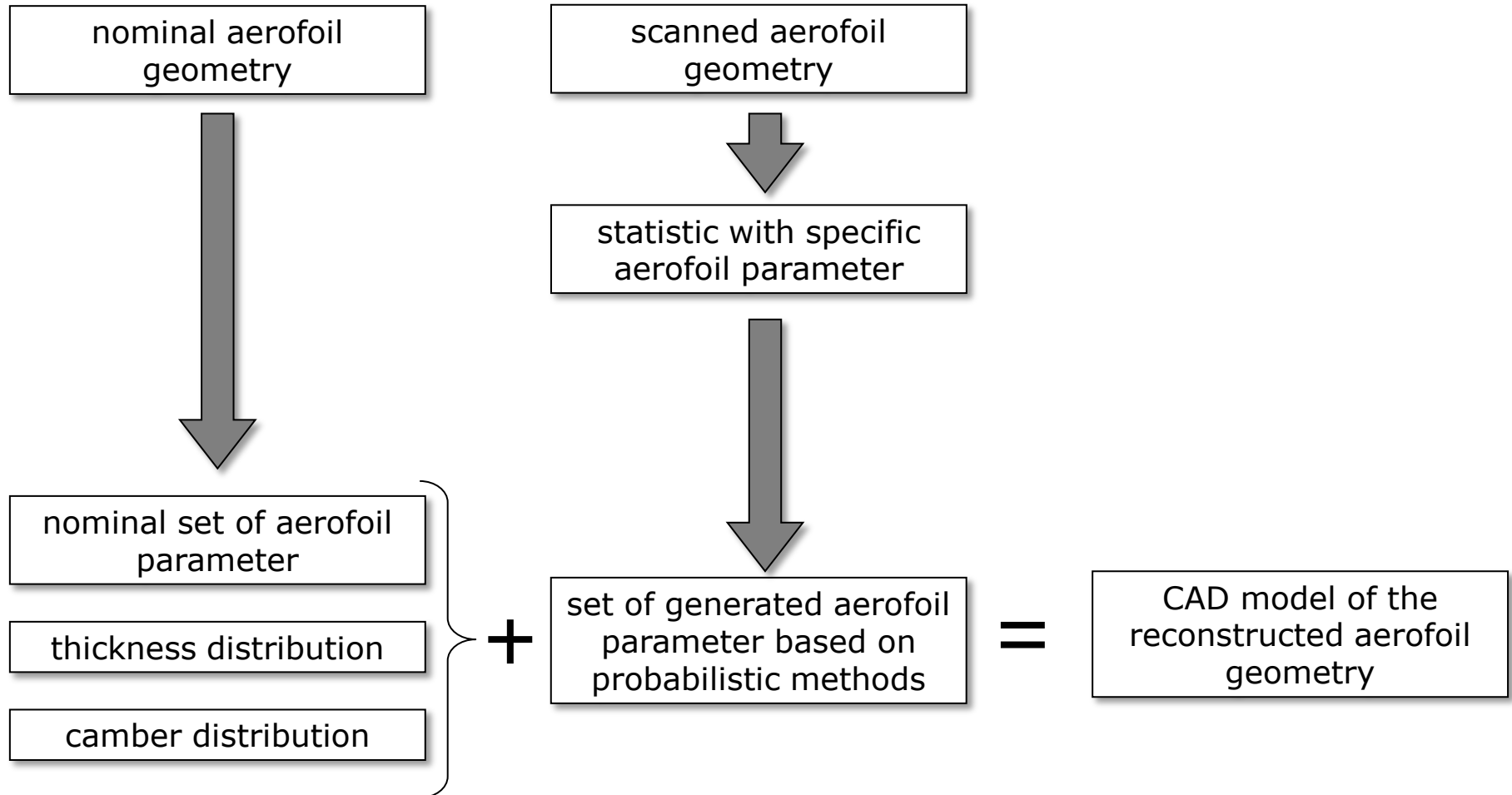
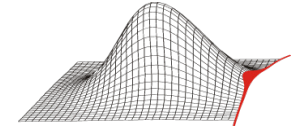
C.1) evaluation of the profile parameter

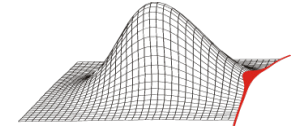




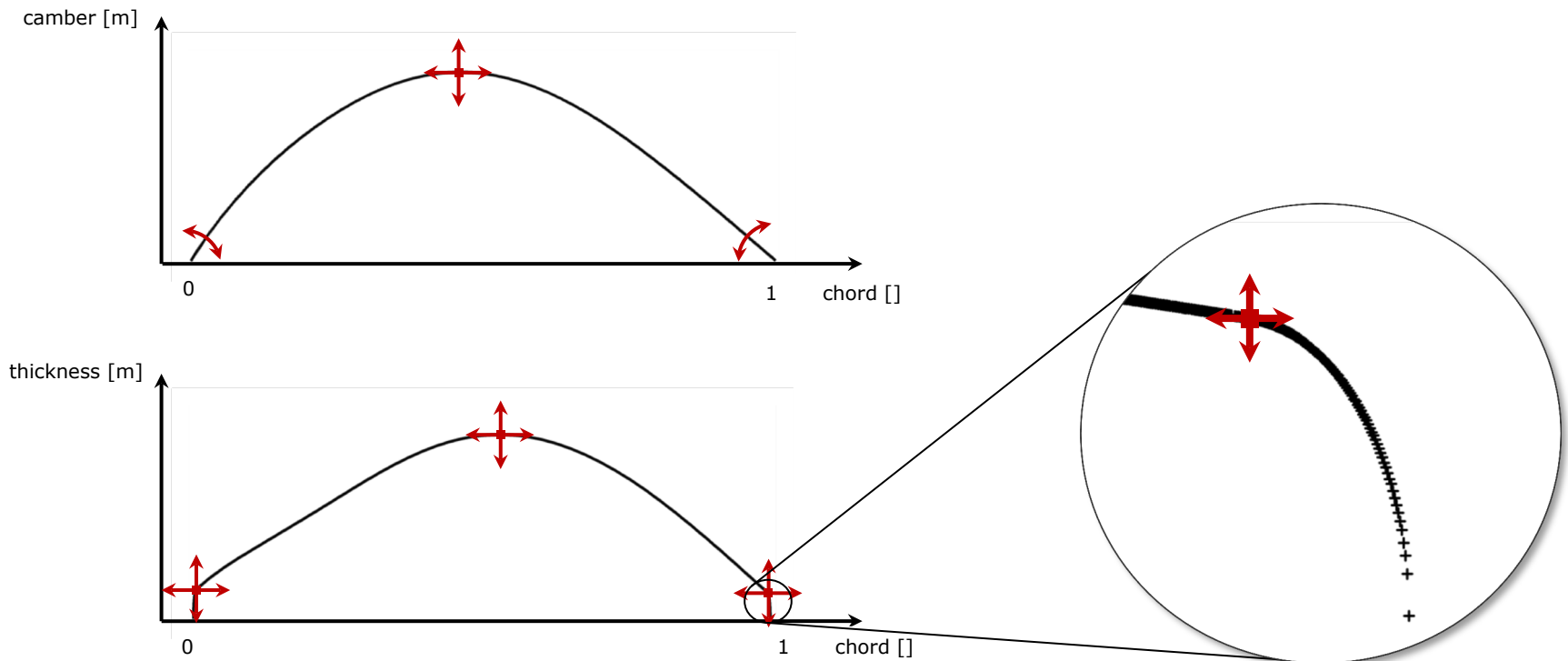
C.2) evaluation of the profile parameter

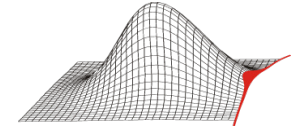




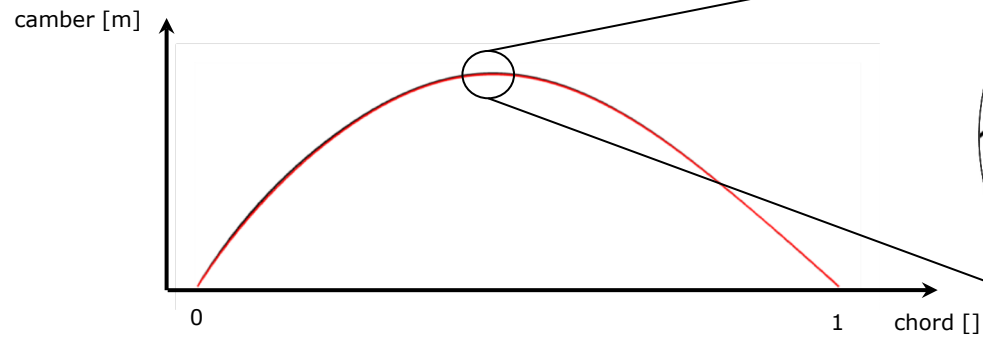


morphing of thickness and camber

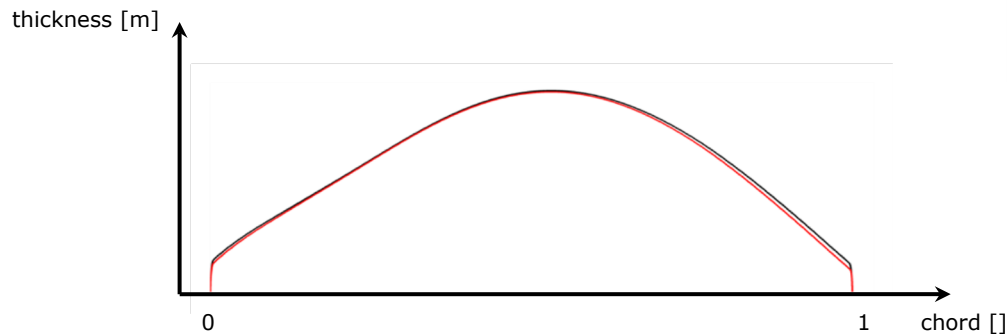


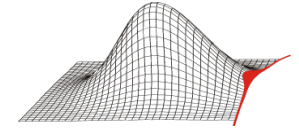


morphing of thickness and camber

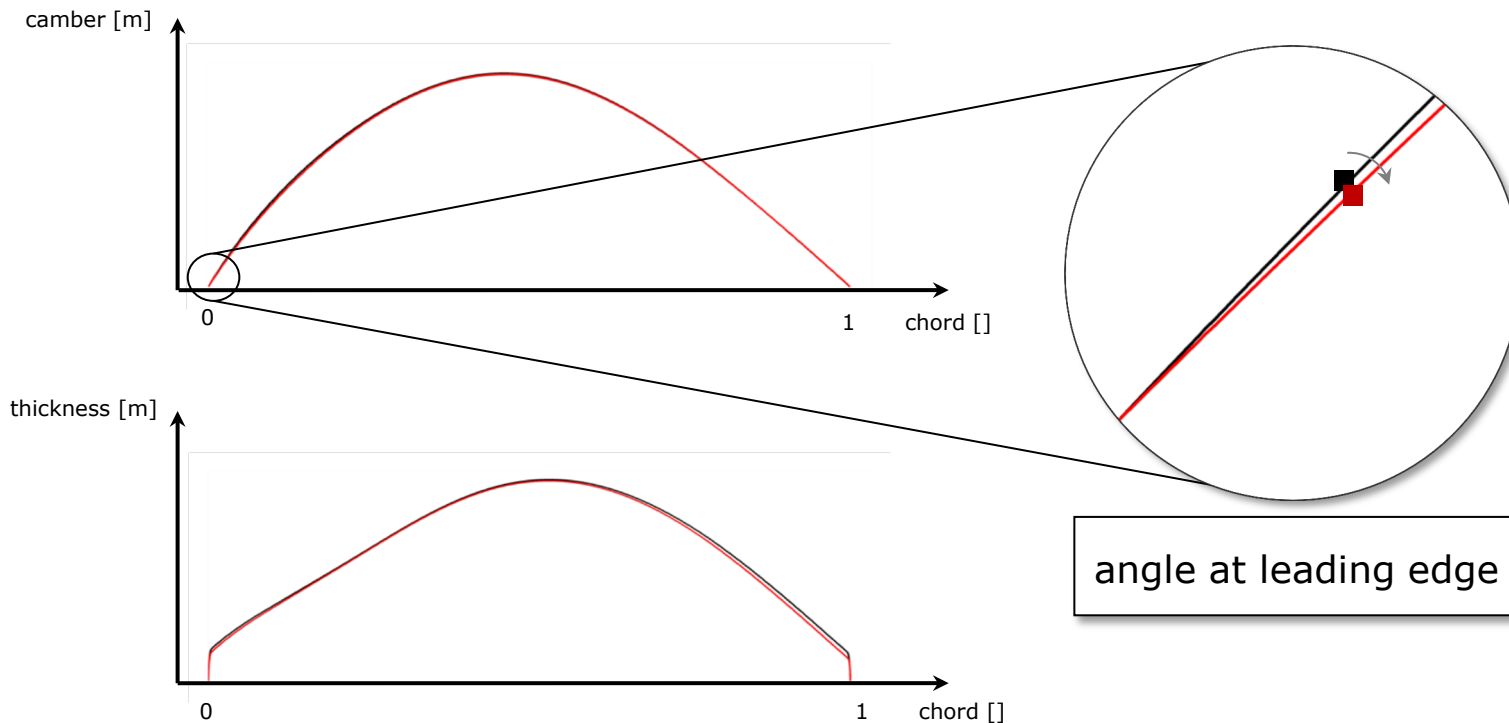


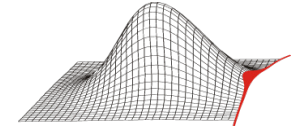
maximum camber



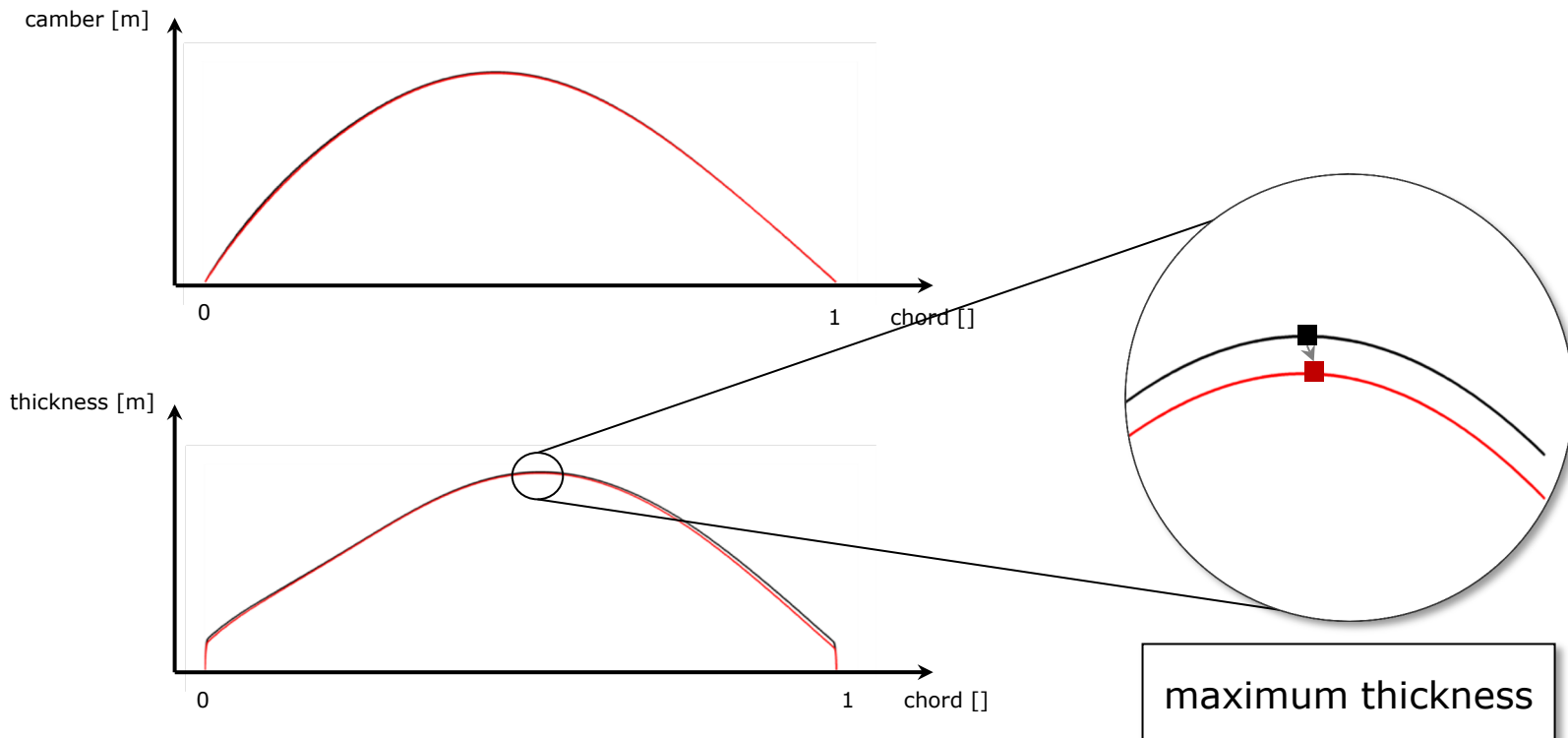


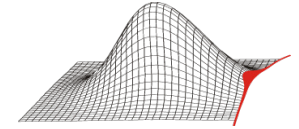
morphing of thickness and camber



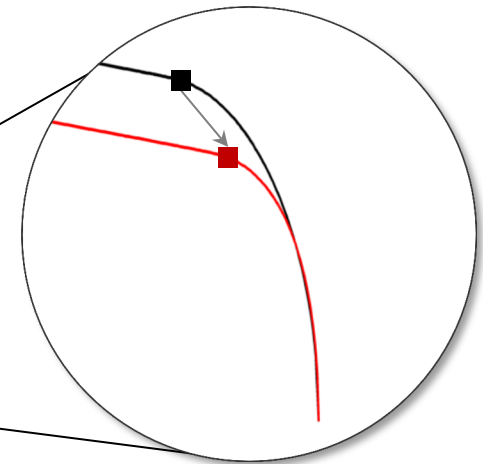
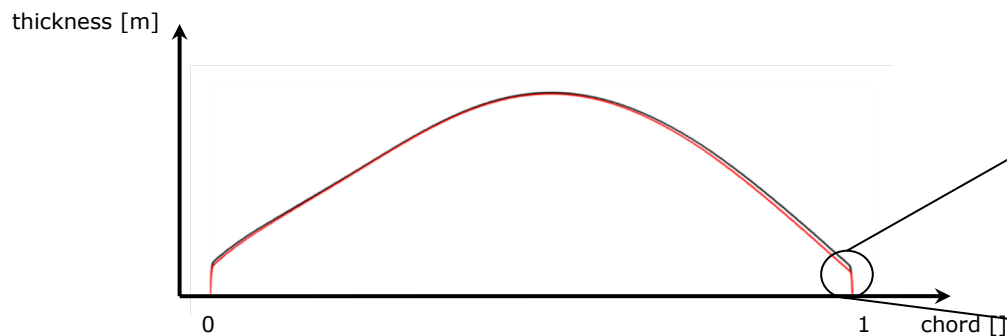
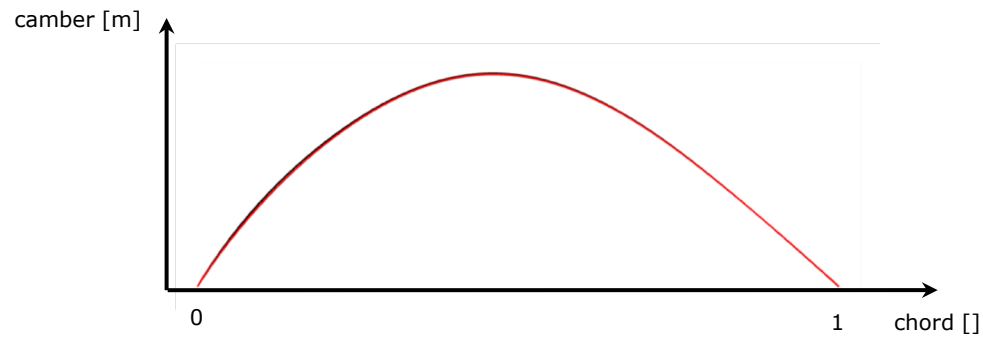


morphing of thickness and camber

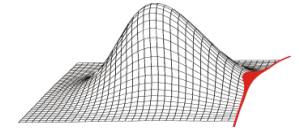




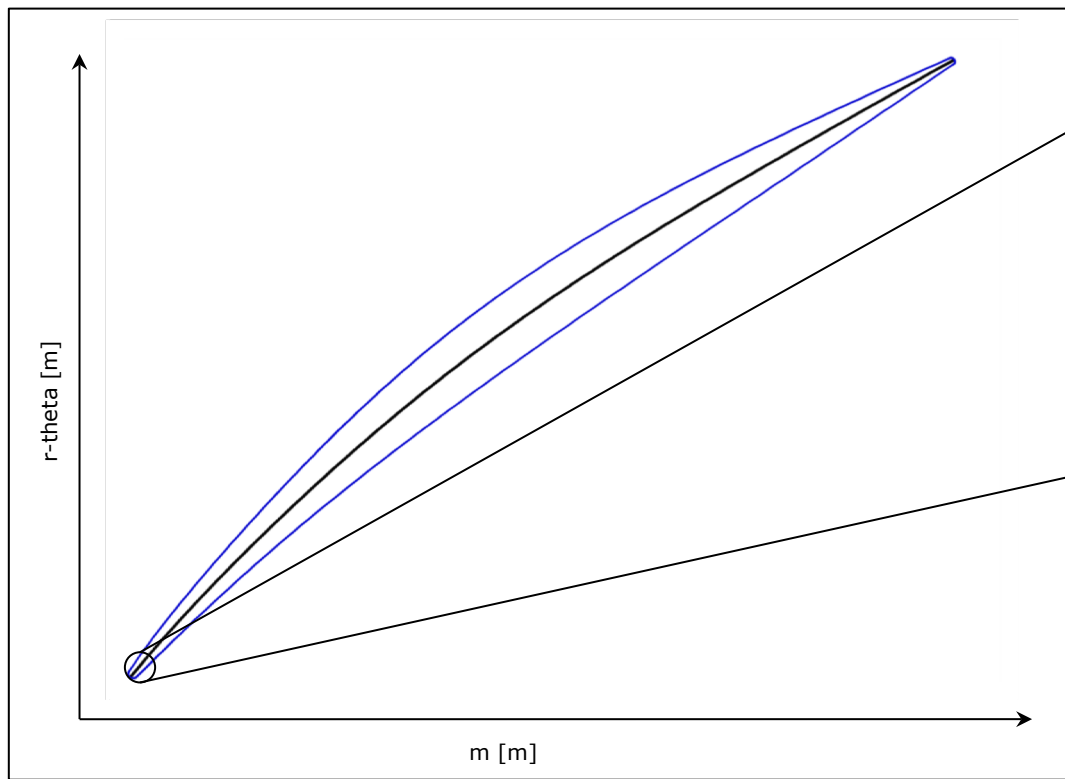
morphing of thickness and camber



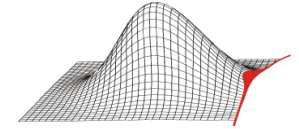
thickness at trailing edge



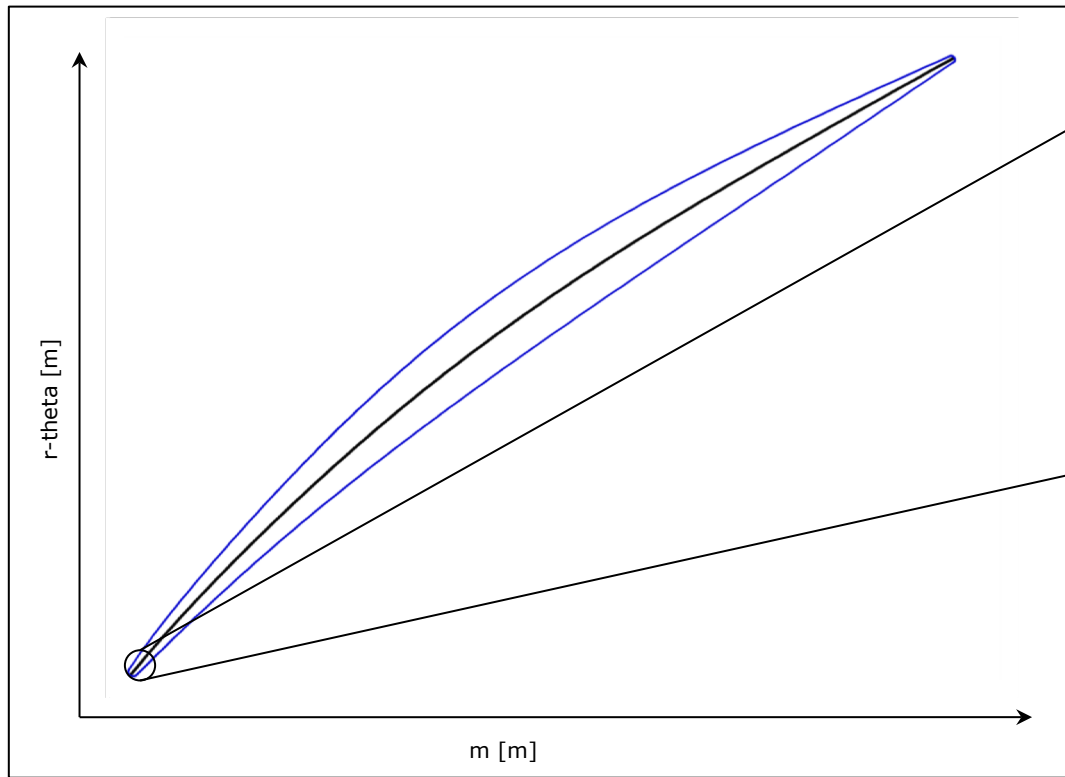
reconstruction of the blade aerofoil



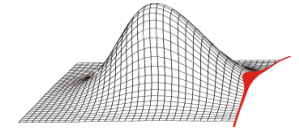
rebuild of the profile outline
with 4 patches (LE,TE,PS,SS)



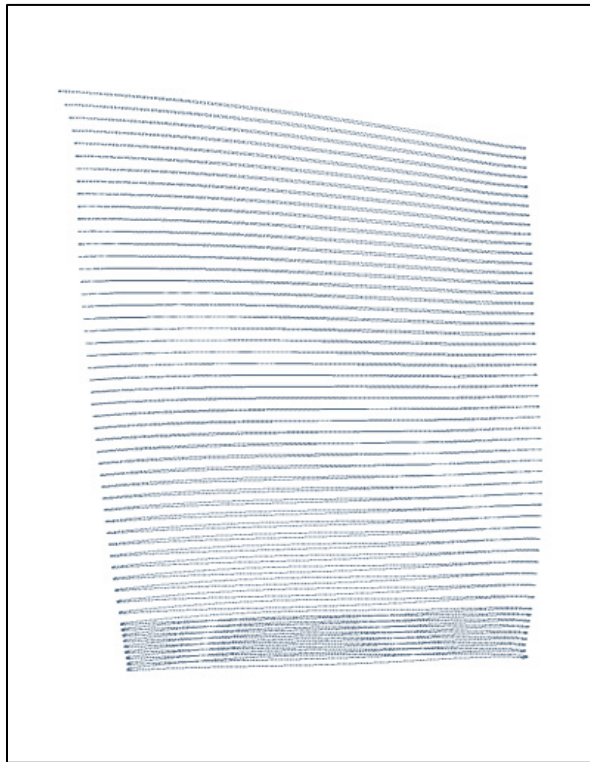
reconstruction of the blade aerofoil



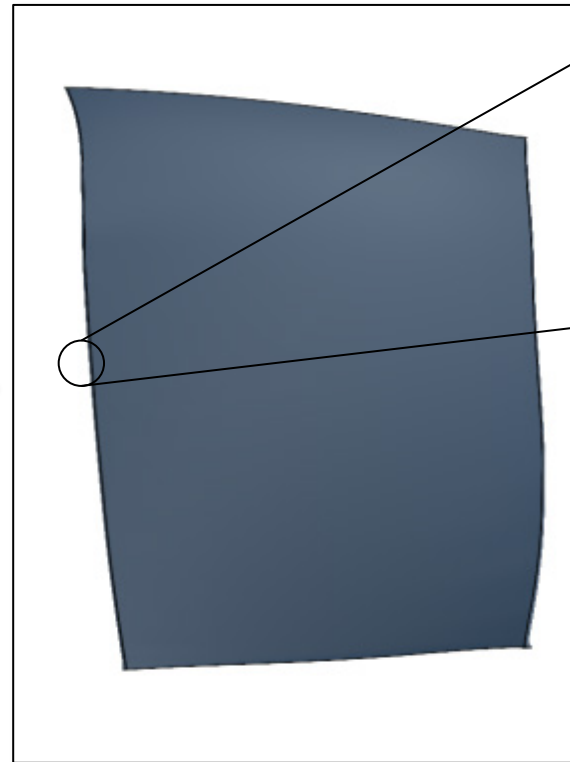
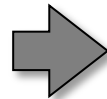
generate a smooth discretization
of the profile outline



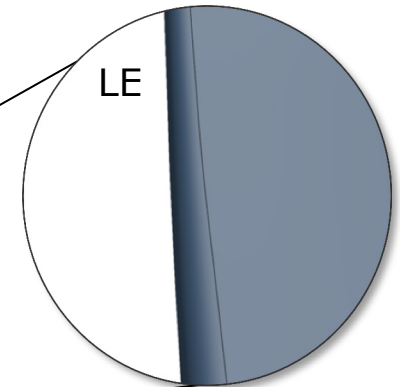
reconstruction of the blade aerofoil

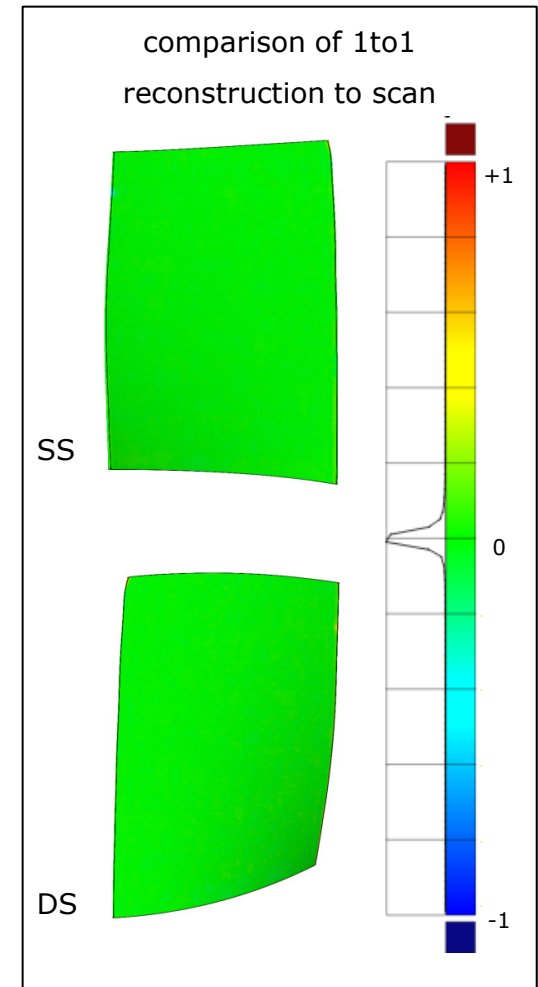
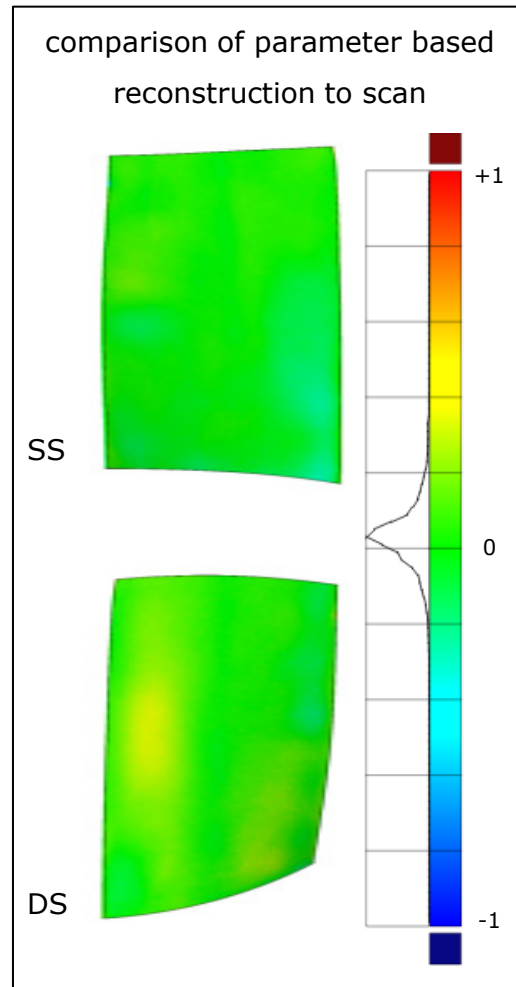
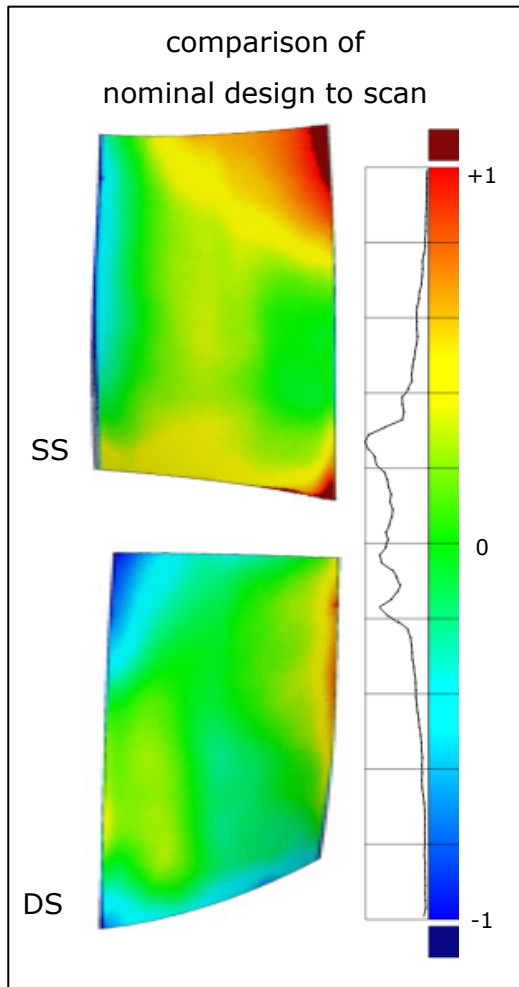
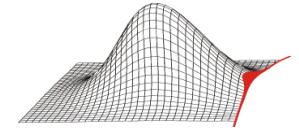


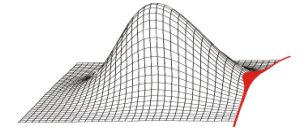
reconstructed profile sections



reconstructed 3D model





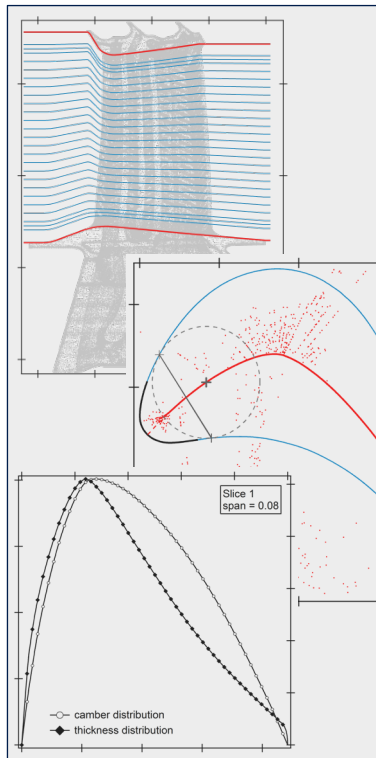


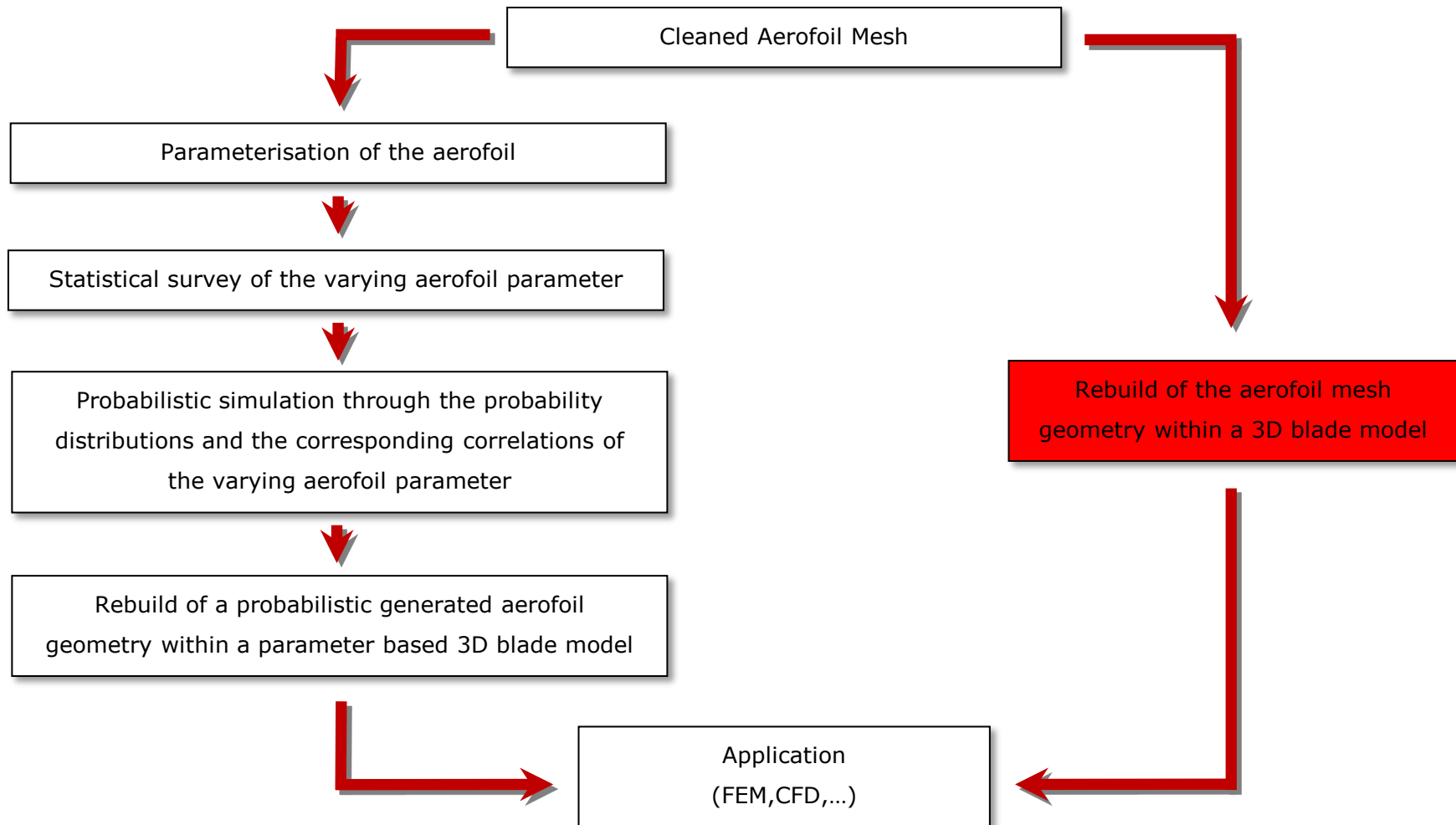
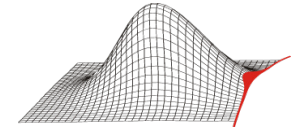
4) Parameterisation and Rebuild

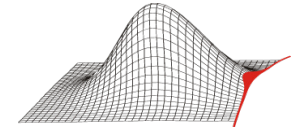
4.1 Parameterisation

4.2 Parameter based Rebuild

4.3 One to One Rebuild

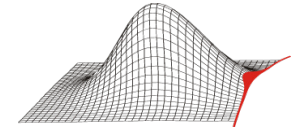




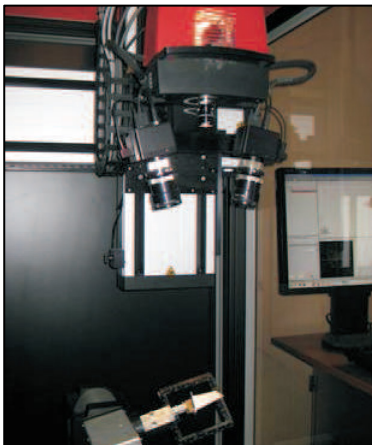


- reverse-engineering: One to One implementation of the manufactured component back into the development process for further design-processing and validation (CFD, FEM)
- advantages over common surface reconstruction software:
 - customised especially for turbo machinery components
 - robust
 - fast and automatic process
 - output of selected parameters for a subsequent analysis of component-characteristics

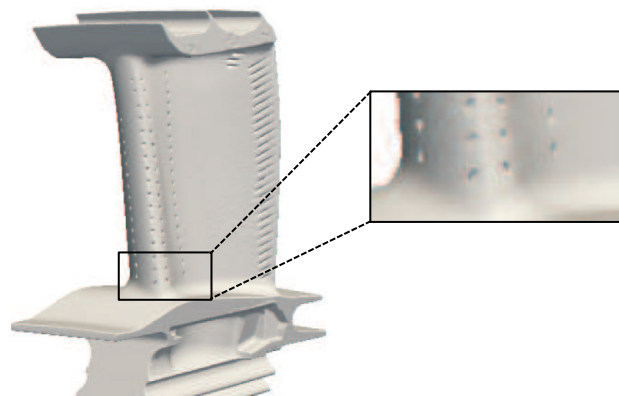
4.3.2 Process Operation



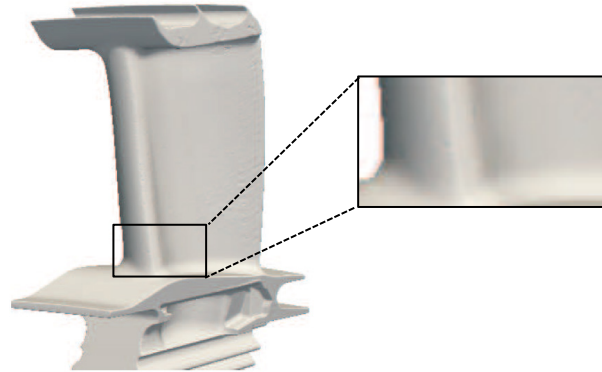
optical 3D measurement



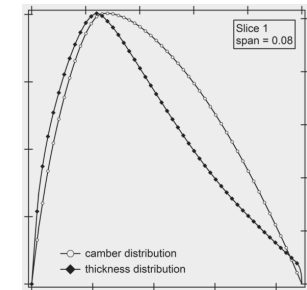
STL-mesh



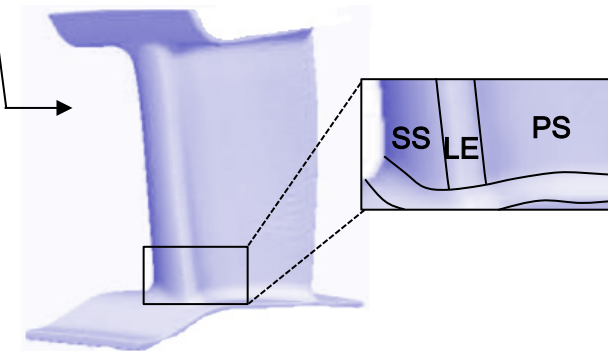
pre-processed STL-mesh
(closed cooling holes)

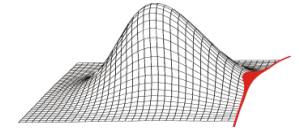


parameter extraction &
parameter based rebuild

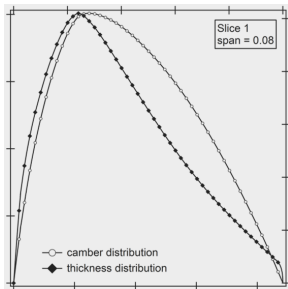


one to one rebuild



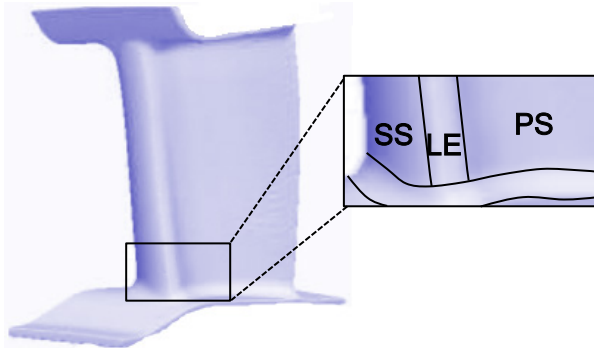


parameter extraction & parameter based rebuild

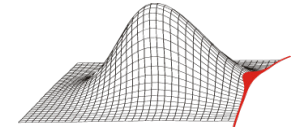


- rebuild the component as accurate as possible with the least possible amount of parameters (with acceptable deviations):
 - probabilistic simulations
 - optimisation
 - estimate manufacturing scatter

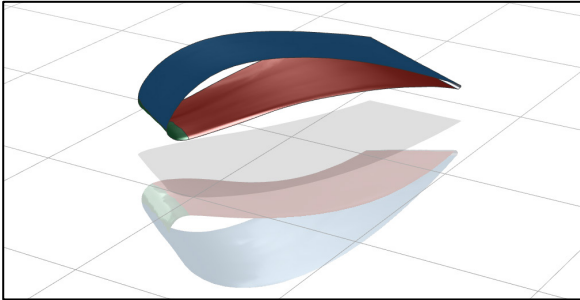
one to one rebuild



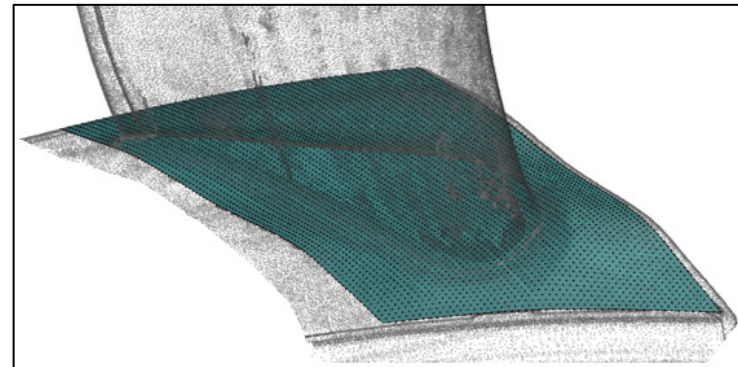
- rebuild the component as close to reality as possible:
 - validation purposes (comparison to measurements)
 - analysis of component characteristics



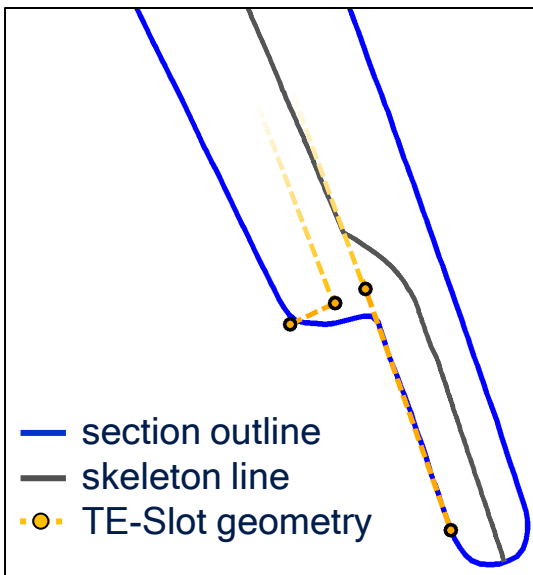
Aerofoil



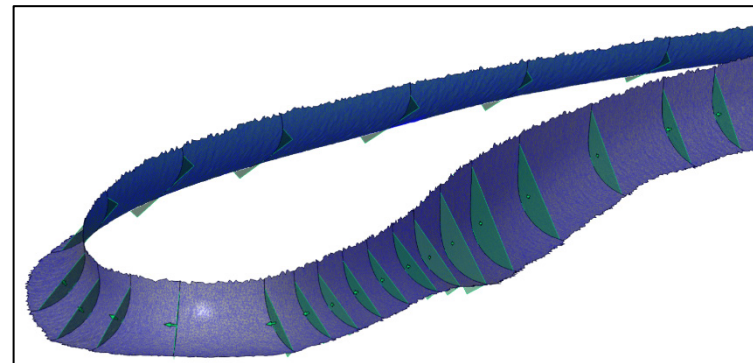
Profiled Endwall (PEW)

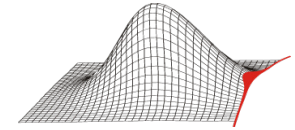


Trailing Edge Slot

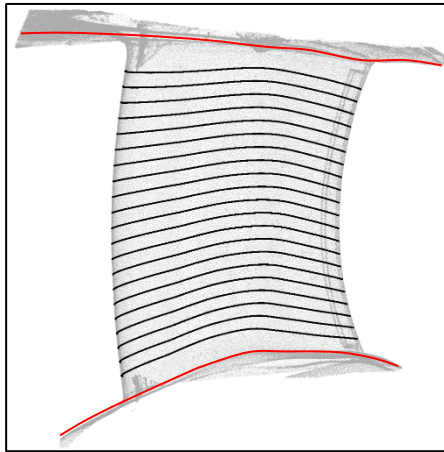


Fillet





1) profile partition into n slices



- measured point cloud
- generated section outlines
- lines for hub and casing

2) 3D to 2D coordinate transformation (X,Y,Z to rθ,m)

$$dm^2 = dr^2 + dz^2$$

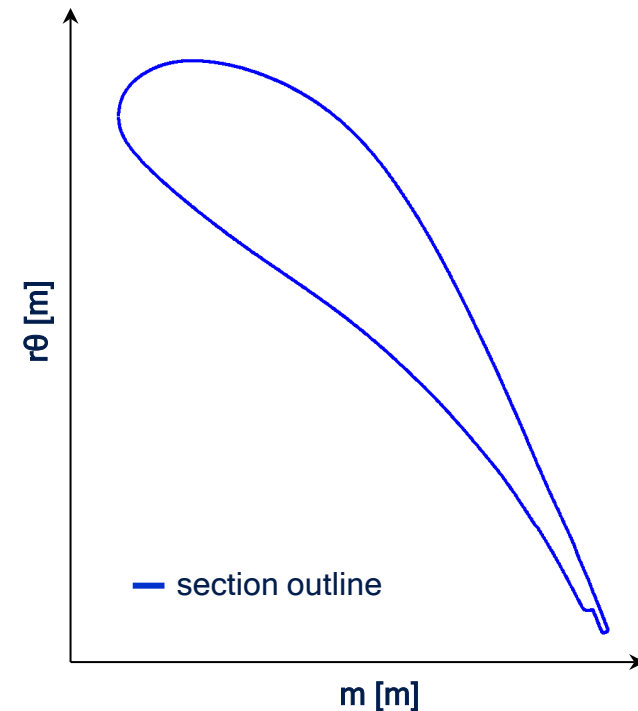
$$m_k = \begin{cases} m_{k-1} + \sqrt{(r_k - r_{k-1})^2 + (z_k - z_{k-1})^2} & \text{for } \Delta z_k \geq 0 \\ m_{k-1} - \sqrt{(r_k - r_{k-1})^2 + (z_k - z_{k-1})^2} & \text{for } \Delta z_k < 0 \end{cases}$$

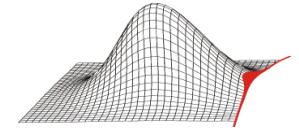
$$r\theta_k = r_k\theta_k$$

$$\theta_k = \arctan \frac{y_k}{x_k}$$

$$r_k = \sqrt{y_k^2 + x_k^2}$$

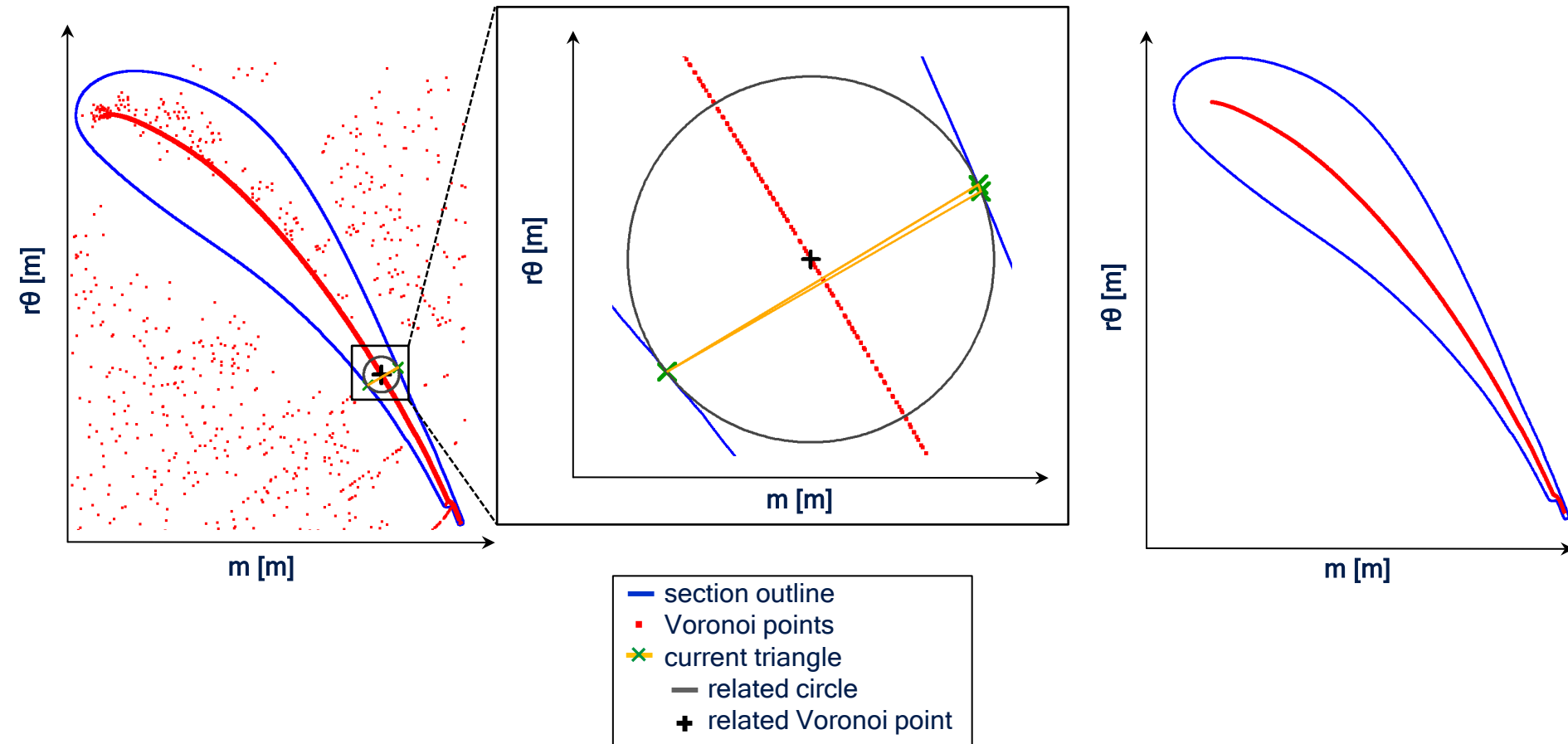
3) extracted profile contour

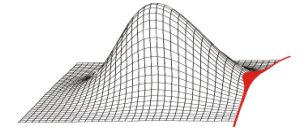




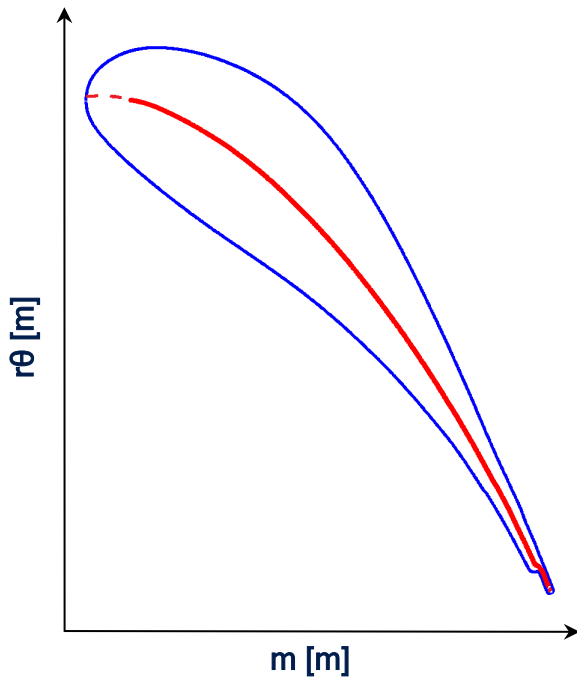
4) Delaunay - Voronoi - Triangulation

5) filter created points

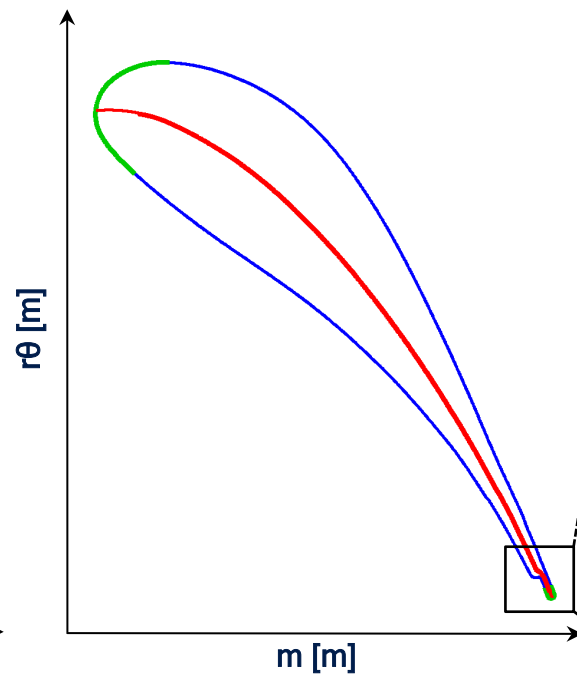




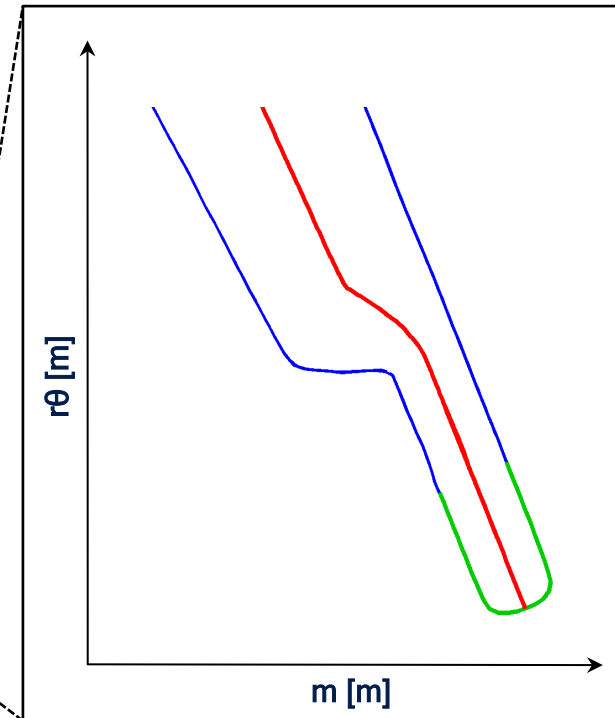
6) skeleton line extrapolation



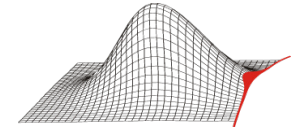
7) LE & TE patch detection



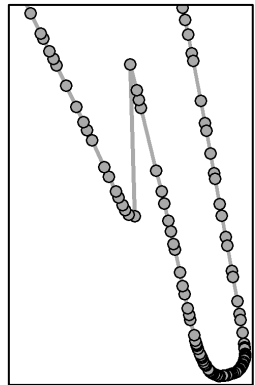
Trailing Edge Slot



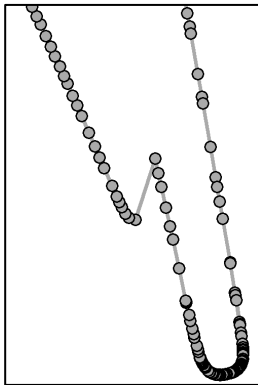
- section outline
- skeleton line
(voronoi points)
- LE / TE patch



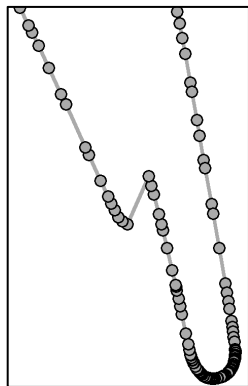
intersection points between
streamwise slices and mesh



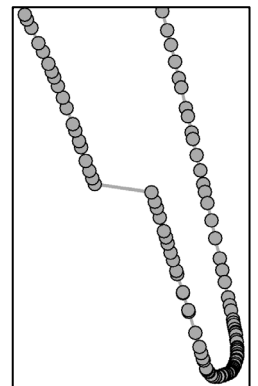
20 % span



40 % span

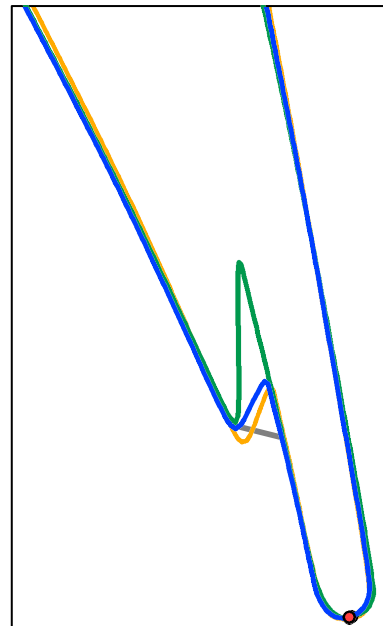


60 % span



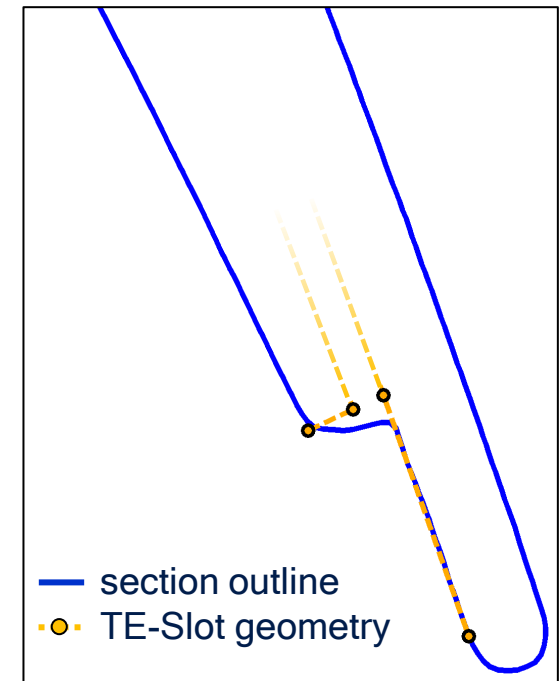
80 % span

superimposed

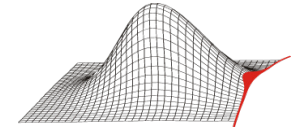


- TE Position
- different section outlines

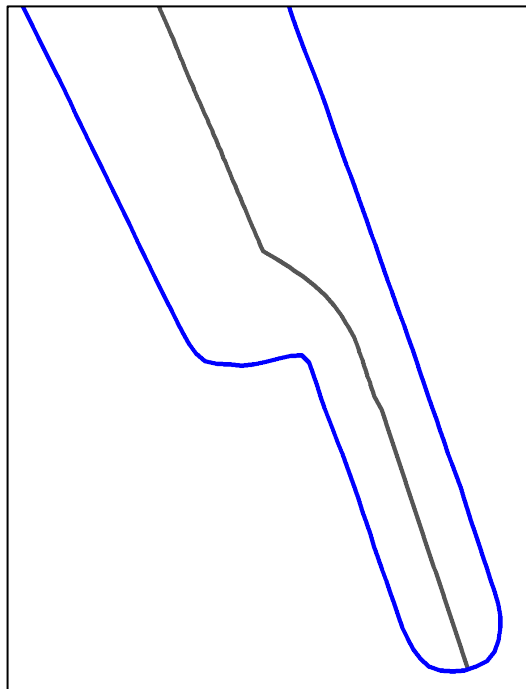
TE-Slot geometry



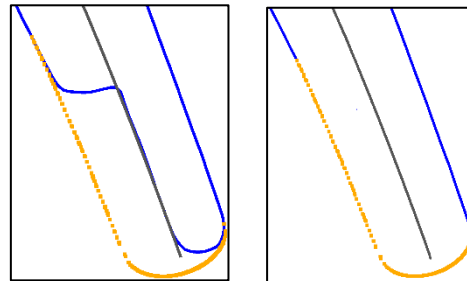
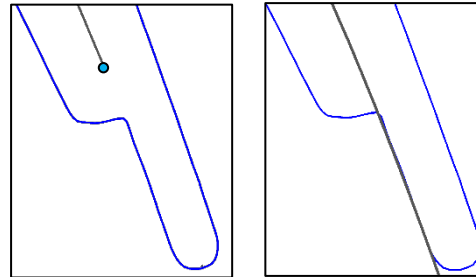
4.3.3 Extraction Methods - Aerofoil



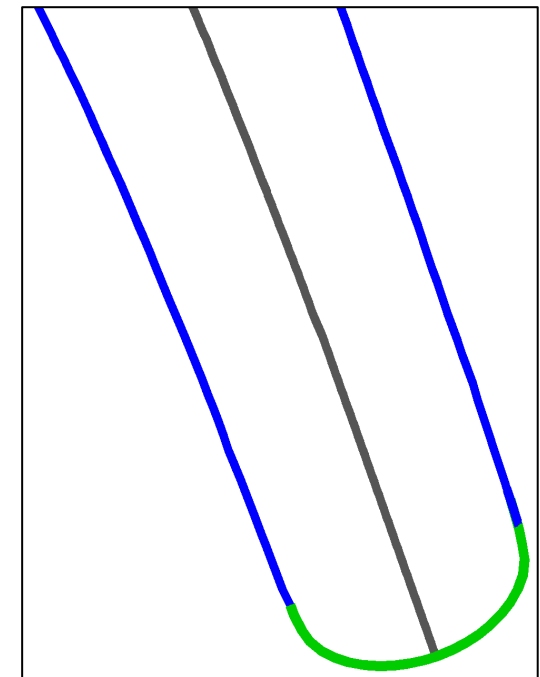
measured component



— section outline
— skeleton line

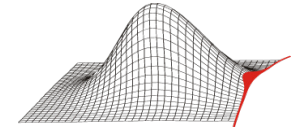


closed component



— section outline
— skeleton line
— TE patch

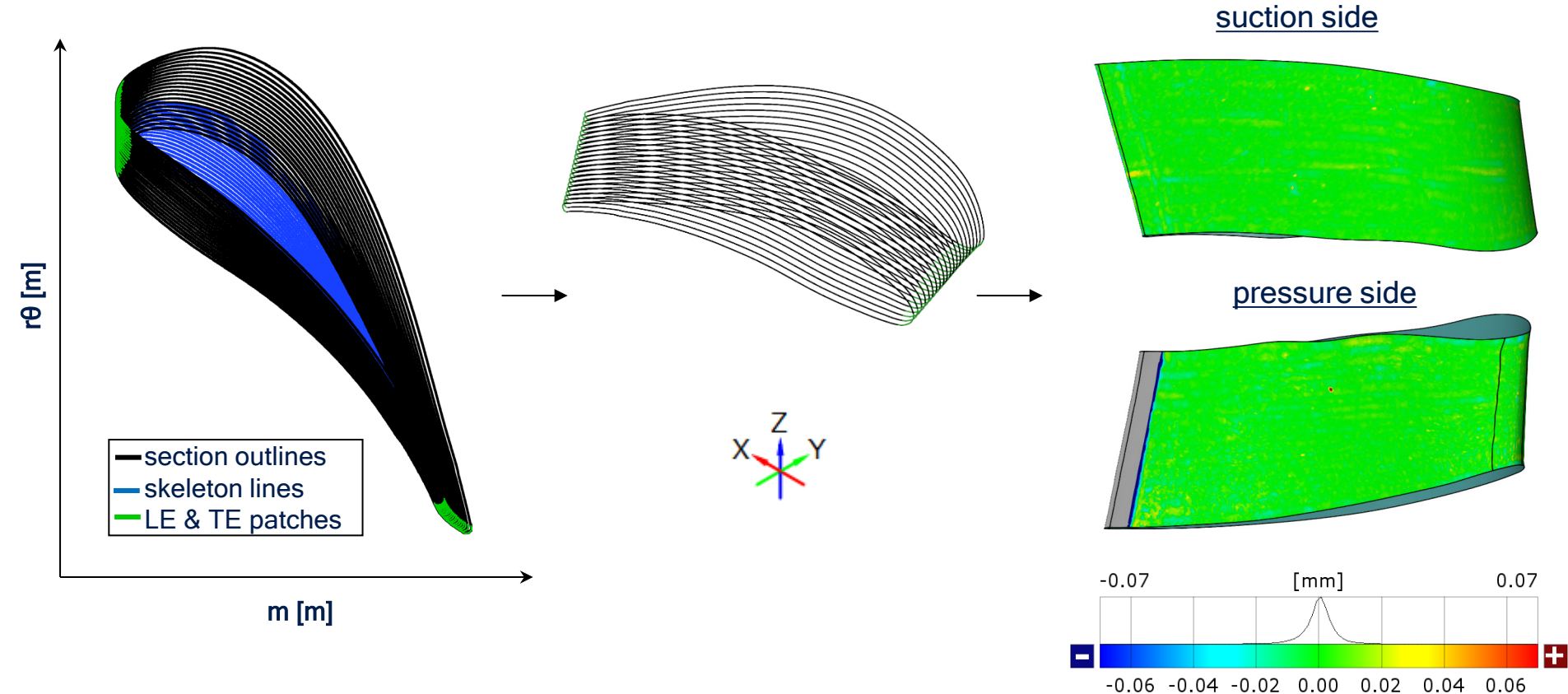
4.3.3 Extraction Methods - Aerofoil

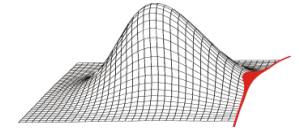


1) extracted profile contours and calculated skeleton lines

2) 2D to 3D coordinate transformation ($r\theta, m$ to X, Y, Z)

3) recreation of the aerofoil with common CAD-software

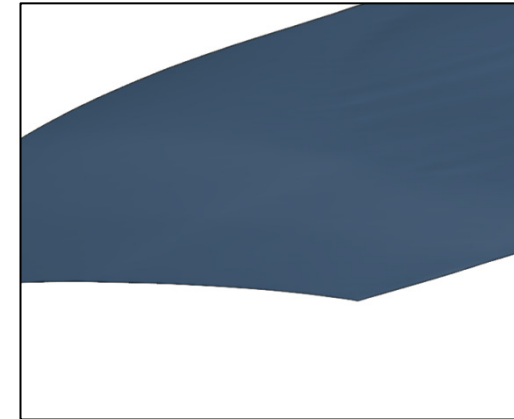
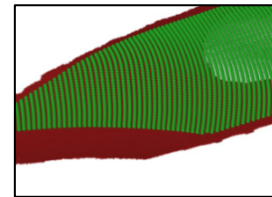
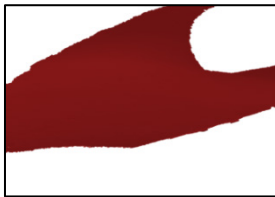
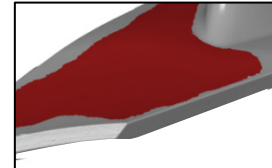
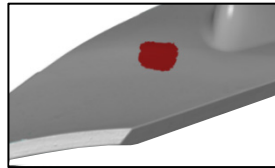
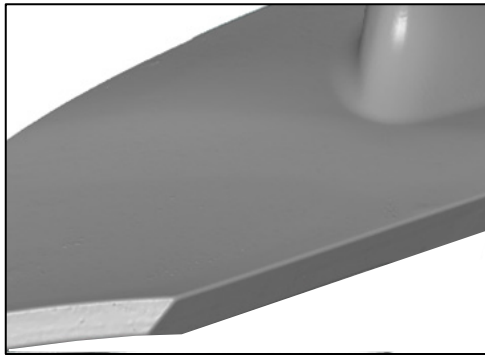




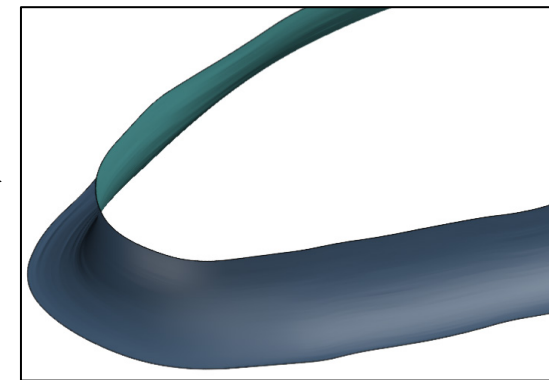
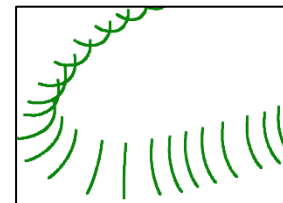
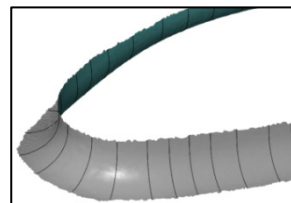
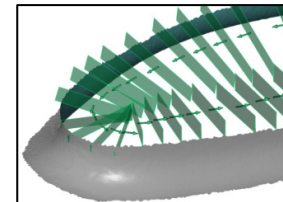
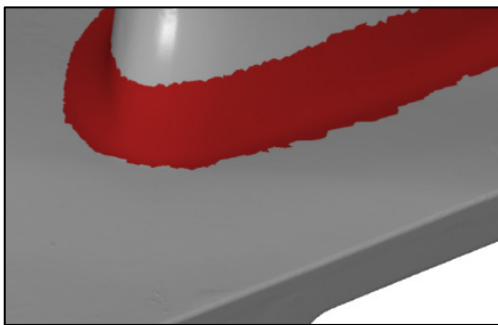
STL - mesh

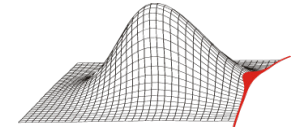
→ CAD - geometry

Profiled Endwalls

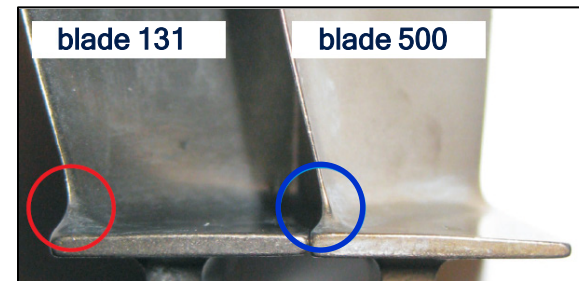
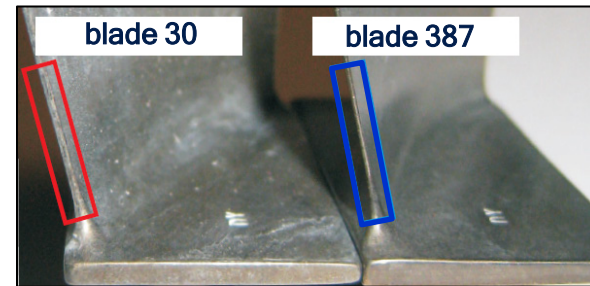
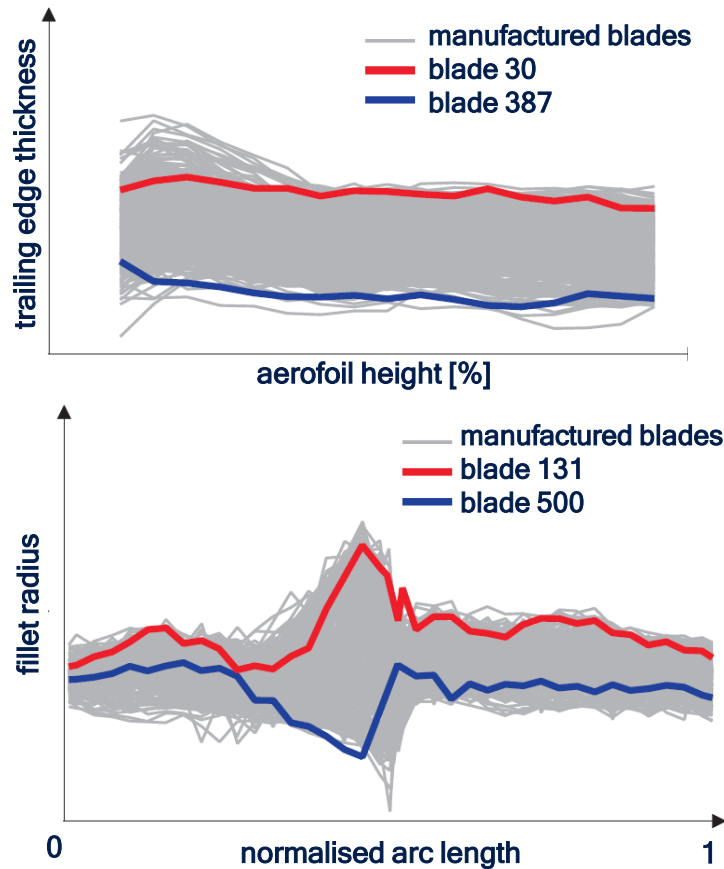


Fillet

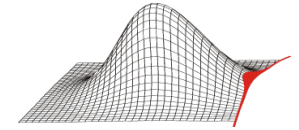




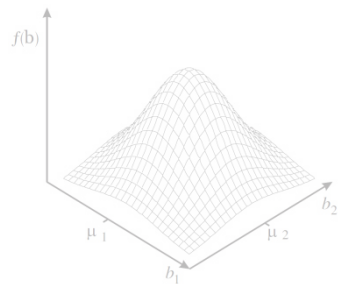
examples for obtained blade-characteristics[2]:



[2] Heinze, K.: *Eine Methode für probabilistische Untersuchungen zum Einfluss von Fertigungsstreuungen auf die hochzyklische Ermüdung von Verdichterschaufeln*, Dissertation



1) Introduction



$$\frac{1}{m-1} \sum_{k=1}^{n_{sim}} (b_{ri,k} - \bar{b})$$

$r = 0.70$
 $\bar{r} = 0.68$

$r = 0.48$
 $\bar{r} = 0.48$



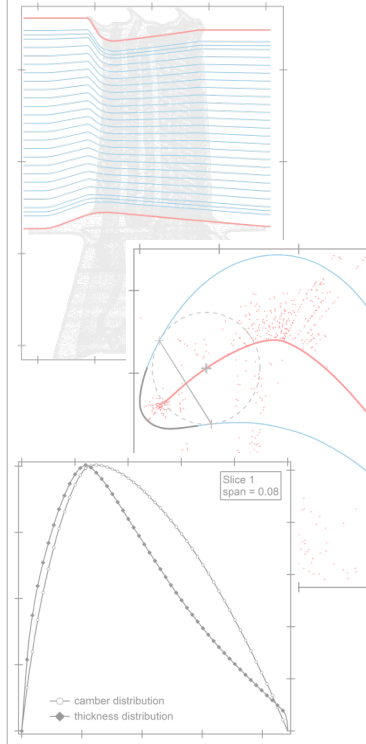
2) Optical Measurement



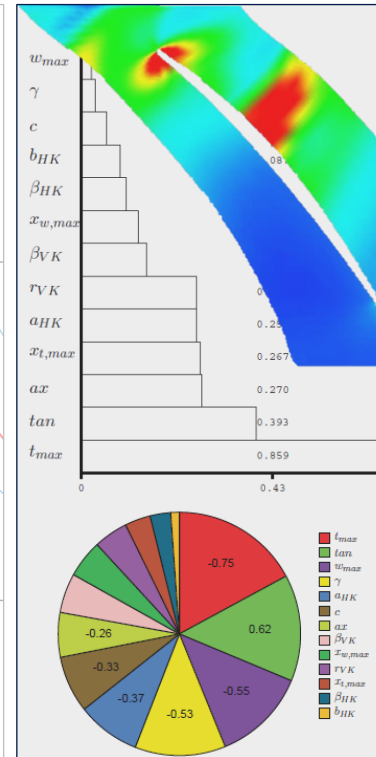
3) Mesh Pre-Processing

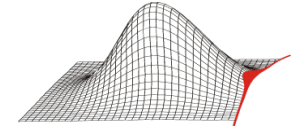


4) Parameterisation and Rebuild



5) Applications



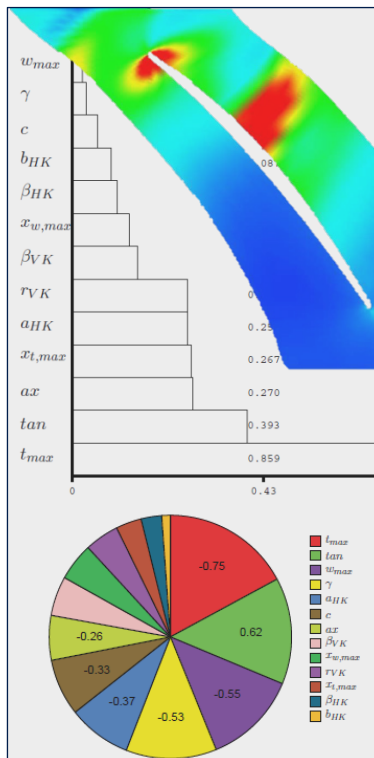


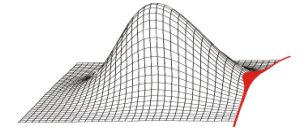
5) Applications

5.1 Non-deterministic CFD simulation

- 5.1.1 UQ method - Monte Carlo Simulation
- 5.1.2 Deterministic model
- 5.1.3 Incorporate the geometry into CFD mesh
- 5.1.4 Influence of simulation parameters
- 5.1.5 Sensitivity analysis
- 5.1.6 Statistics on passage

5.2 UQ applications





5) Applications

5.1 Non-deterministic CFD simulation

5.1.1 UQ method - Monte Carlo Simulation

5.1.2 Deterministic model

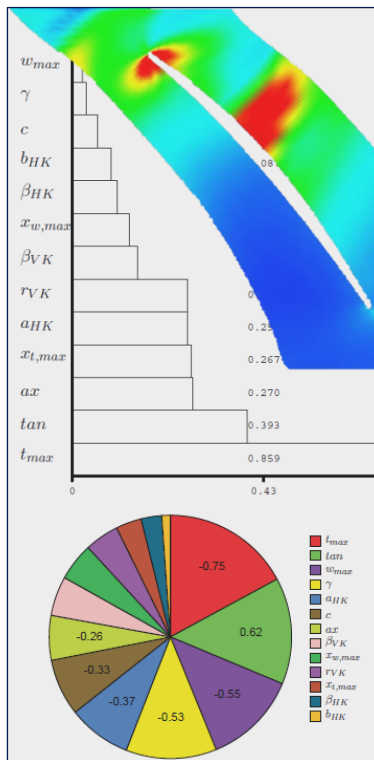
5.1.3 Incorporate the geometry into CFD mesh

5.1.4 Influence of simulation parameters

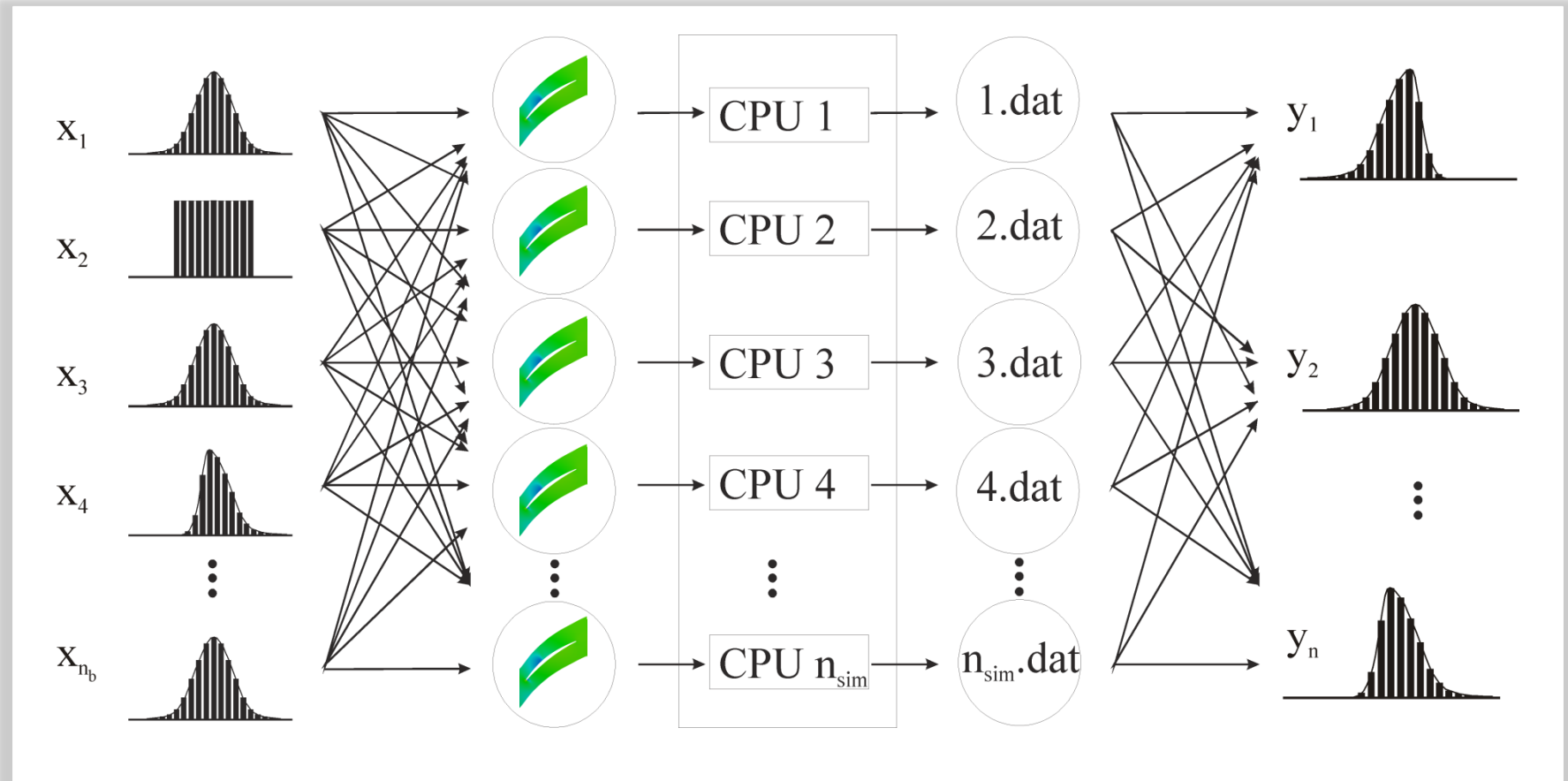
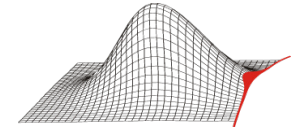
5.1.5 Sensitivity analysis

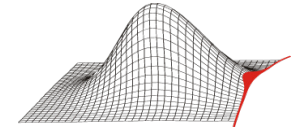
5.1.6 Statistics on passage

5.2 UQ applications



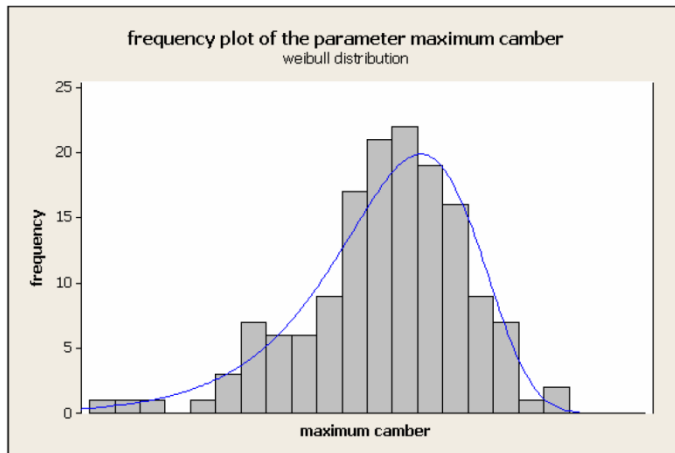
5.1.1 UQ method – Monte Carlo Simulation





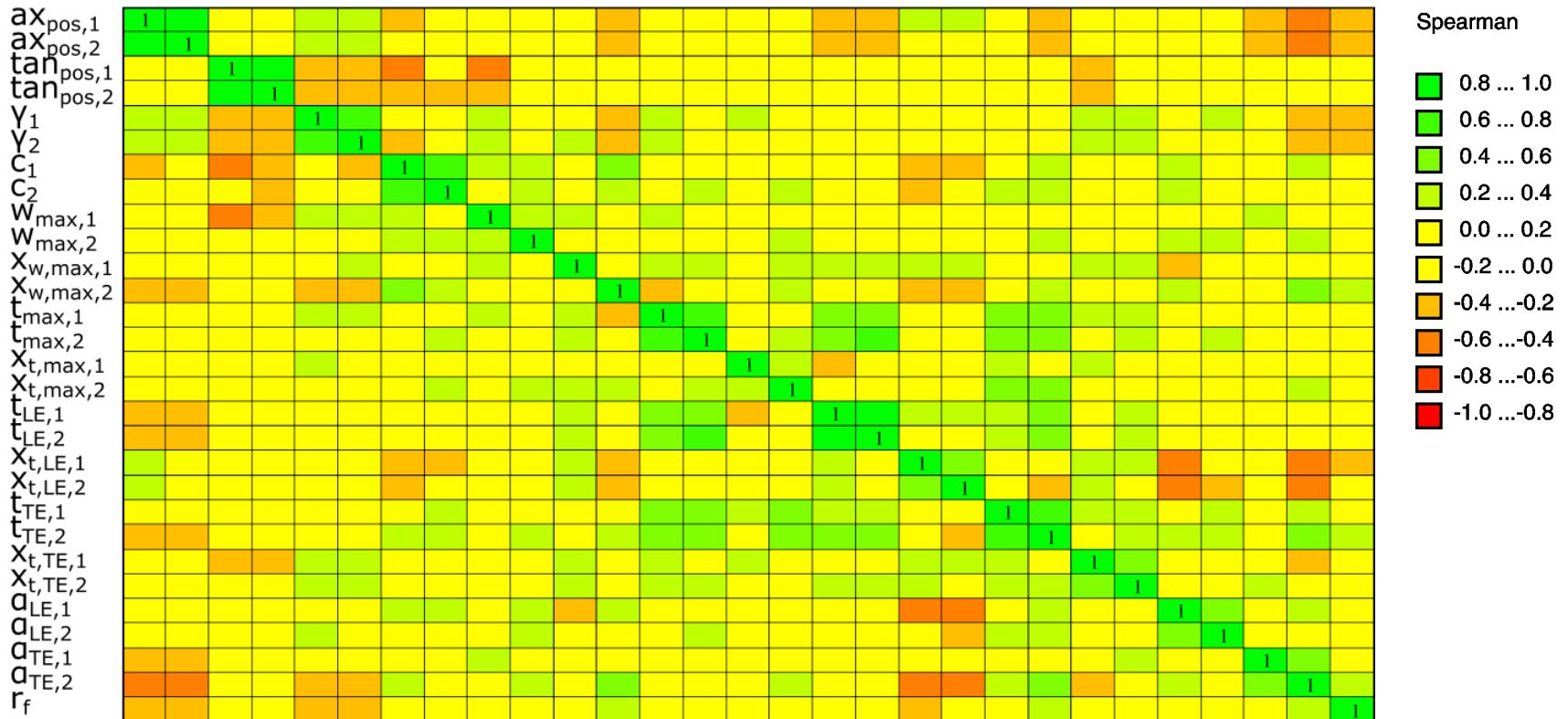
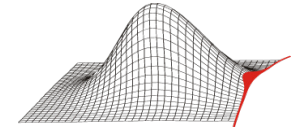
Identification of statistical parameters from measured blades

- PDF of marginal distribution with e.g. Anderson-Darling test



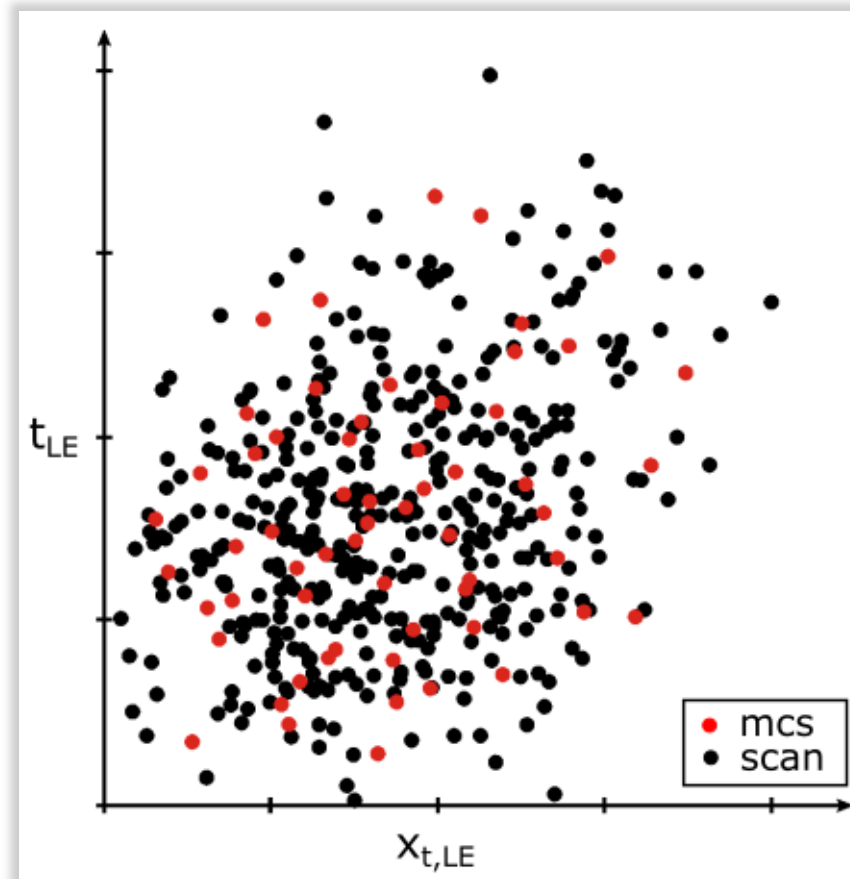
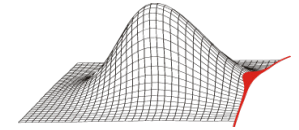
[Lange et al., 2010]

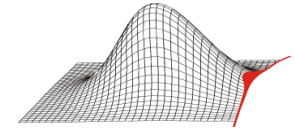
- Linking of the individual marginal distributions in terms of a multivariate distribution (rank correlation, copula)





5.1.1 UQ method – Monte Carlo Simulation





5) Applications

5.1 Non-deterministic CFD simulation

5.1.1 UQ method - Monte Carlo Simulation

5.1.2 Deterministic model

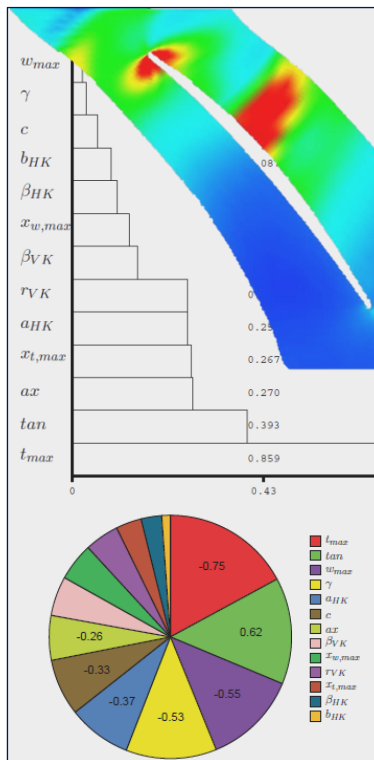
5.1.3 Incorporate the geometry into CFD mesh

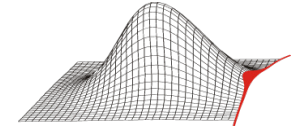
5.1.4 Influence of simulation parameters

5.1.5 Sensitivity analysis

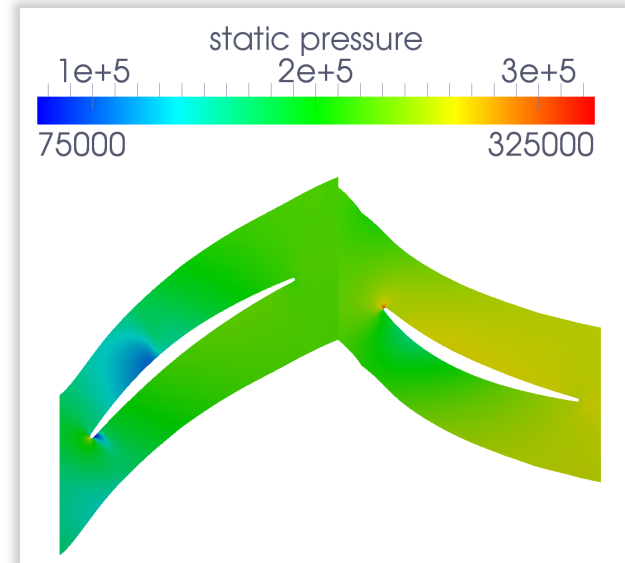
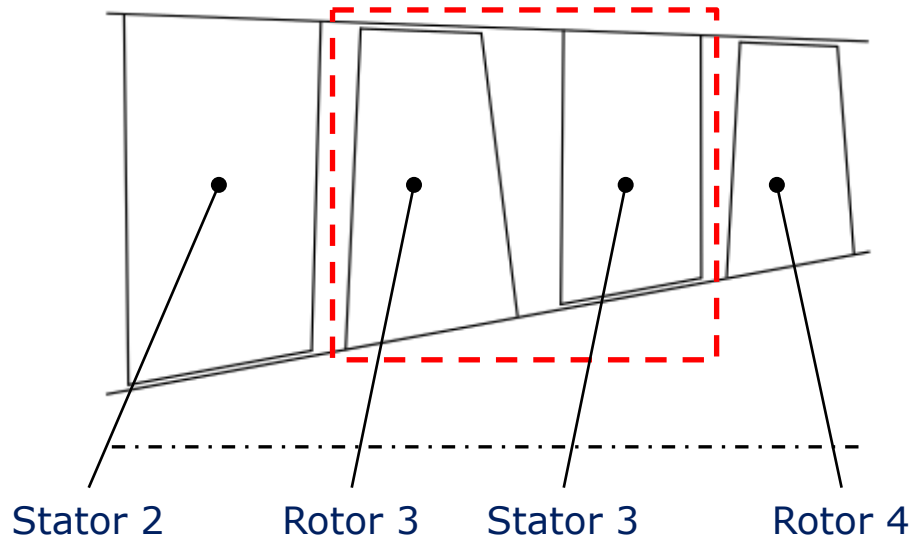
5.1.6 Statistics on passage

5.2 UQ applications

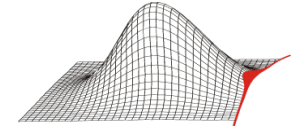




- Two-stage high pressure compressor (hpc)



- Deterministic CFD model
 - steady RANS calculation
 - boundary and initial conditions are given by radial profiles at inlet, fixed mass flow at the outlet
 - data transfer between the blocks is solved by mixing planes
 - constant grid setup

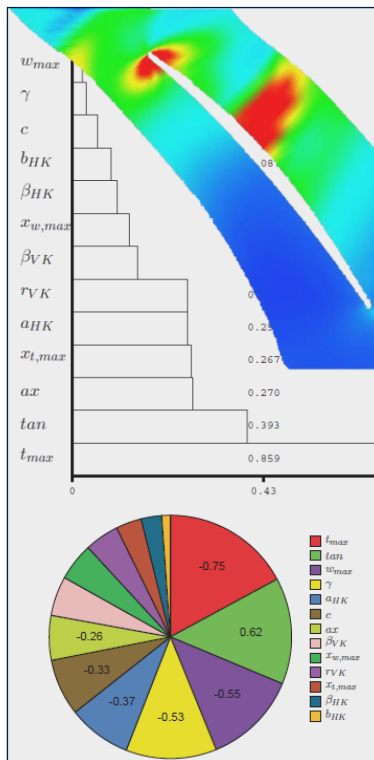


5) Applications

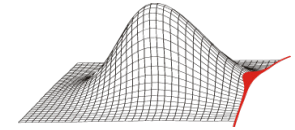
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5.2 UQ applications



5.1.3 Incorporate the geometry into CFD mesh

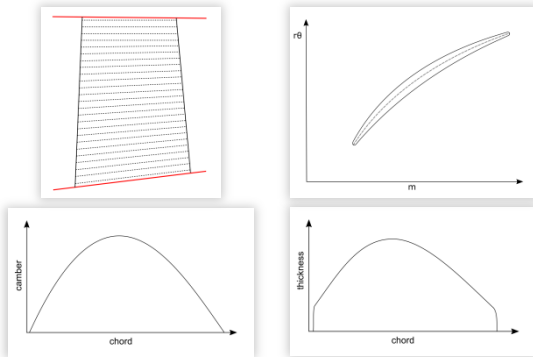


Blade2Parameter

CAD aerofoil generator

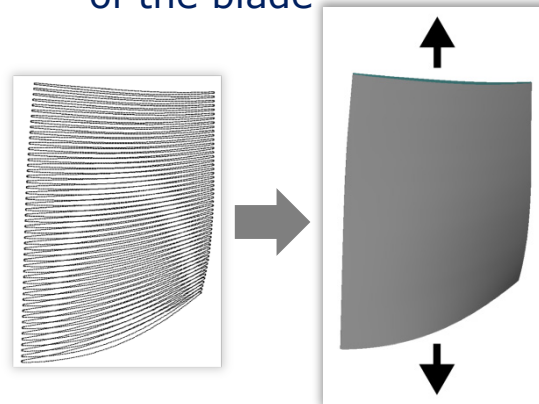
CFD input generator

- parametrisation of the mean model (basis)

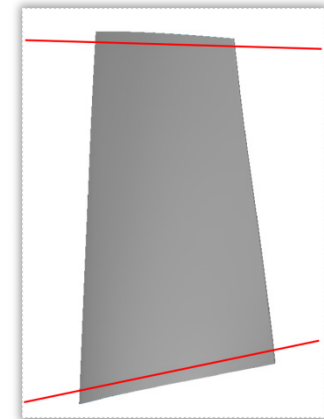


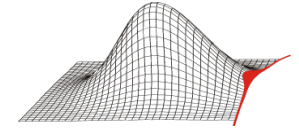
- Δ -parameter which resemble typical scatter of investigated compressor blades
- output of discrete profile sections

- generation of 3D surfaces and extension of the blade



- cut the blade according to the passage contour
- output of geometry file for mesh generation



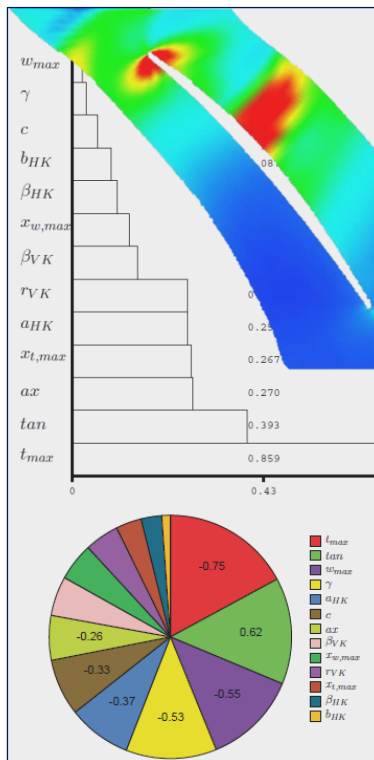


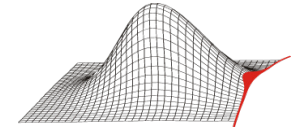
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5.2 UQ applications





Parameters:

– Grid Resolution

- Deterministic Grid Study

no. of grid nodes [$\cdot 10^6$]	0.45	0.9	1.8	3.6 (ref)
calculation time [h]*	0.5	1	2.5	6
η/η_{ref}	0.9847	0.9928	0.9977	1
M/M_{ref}	1.0074	1.0041	1.0040	1
$\Delta p_t/\Delta p_{t\text{ref}}$	0.9983	0.9998	1.0002	1

[Lange et al., 2010]

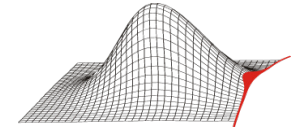
*parallel calculation on 4 Intel Itanium CPU

- Probabilistic Grid Study

no. of grid nodes [$\cdot 10^6$]	0.9	1.8 (ref)	$\tilde{r}(0.9, 1.8)$
$\sigma(\eta)/\sigma(\eta_{\text{ref}})$	1.0782	1	0.99
$\sigma(M)/\sigma(M_{\text{ref}})$	1.0493	1	1.00
$\sigma(\Delta p_t)/\sigma(\Delta p_{t\text{ref}})$	1.0534	1	1.00

[Lange et al., 2010]

5.1.4 Influence of simulation parameters



Parameters:

- Grid Resolution
- Number of Iterations

no. of iterations	150	300	600 (ref)
$\sigma(\eta)/\sigma(\eta_{ref})$	1.0565	1.0055	1
$\sigma(M)/\sigma(M_{ref})$	0.9983	1.0002	1
$\sigma(\Delta p_t)/\sigma(\Delta p_{t ref})$	0.9958	0.9988	1

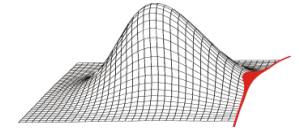
[Lange et al., 2010]

- Spanwise Averaging Sections

no. of averaging sections	1	2	4	7 (ref)
no. of parameters	18	34	66	114
$\sigma(\eta)/\sigma(\eta_{ref})$	0.9369	0.9722	0.9976	1
$\sigma(M)/\sigma(M_{ref})$	1.0064	1.0010	0.9981	1
$\sigma(\Delta p_t)/\sigma(\Delta p_{t ref})$	0.9801	0.9855	0.9966	1

[Lange et al., 2010]

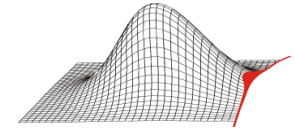
5.1.4 Influence of simulation parameters



Parameters:

- Grid Resolution
- Number of Iterations
- Spanwise Averaging Sections
- Number of MC realization

Usage of an adaptive sampling scheme – extended Latin Hypercube Sampling [Schmidt et al., 2014]

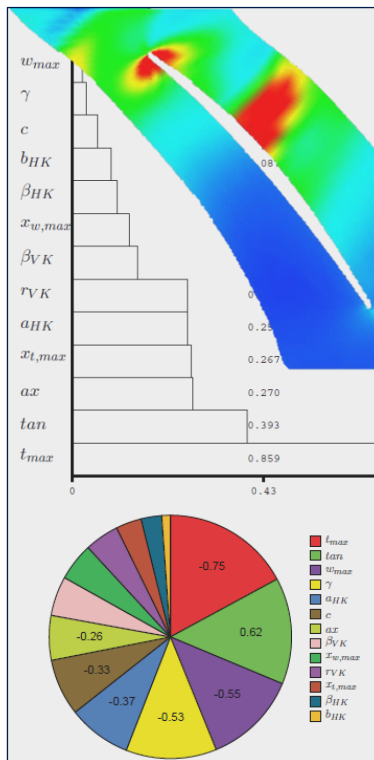


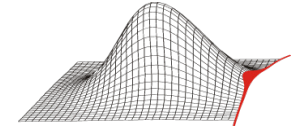
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5.2 UQ applications

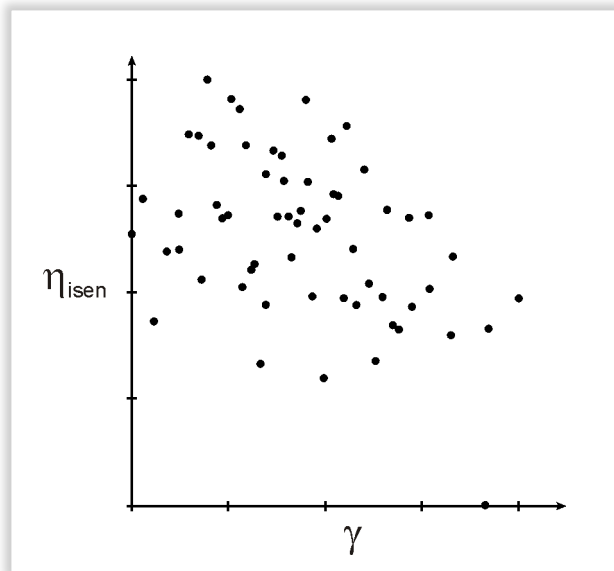




Result quantity: Isentropic efficiency of stage 3

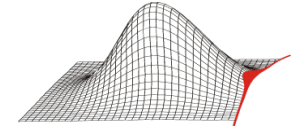
$$\eta_{isen} = \frac{\text{specific compressor work isentropic}}{\text{specific compressor work polytropic}}$$

Use of rank correlation coefficient:

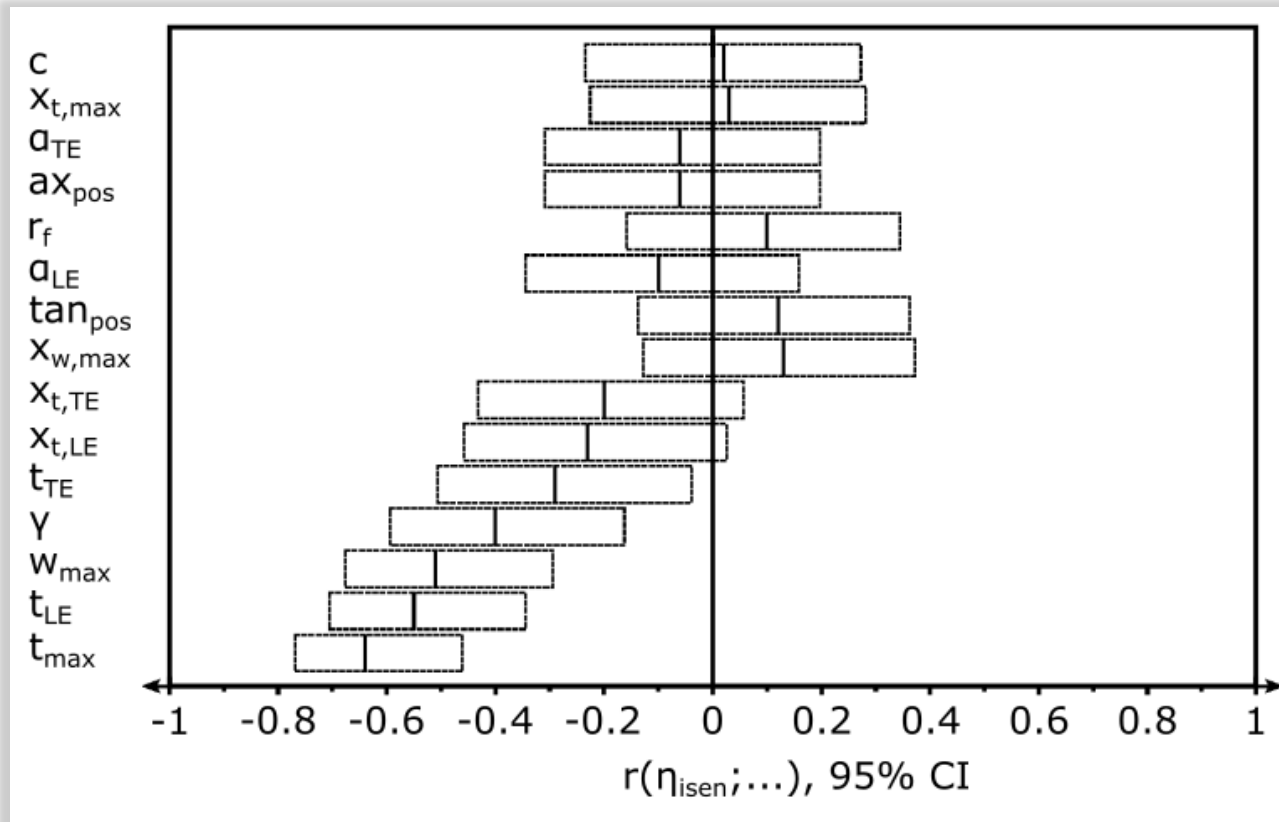


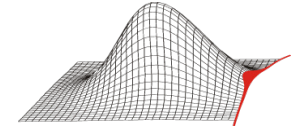
$$\tilde{r} = -0.40$$

5.1.5 Sensitivity analysis



Rank correlation coefficient for result quantity: Isentropic efficiency of stage 3

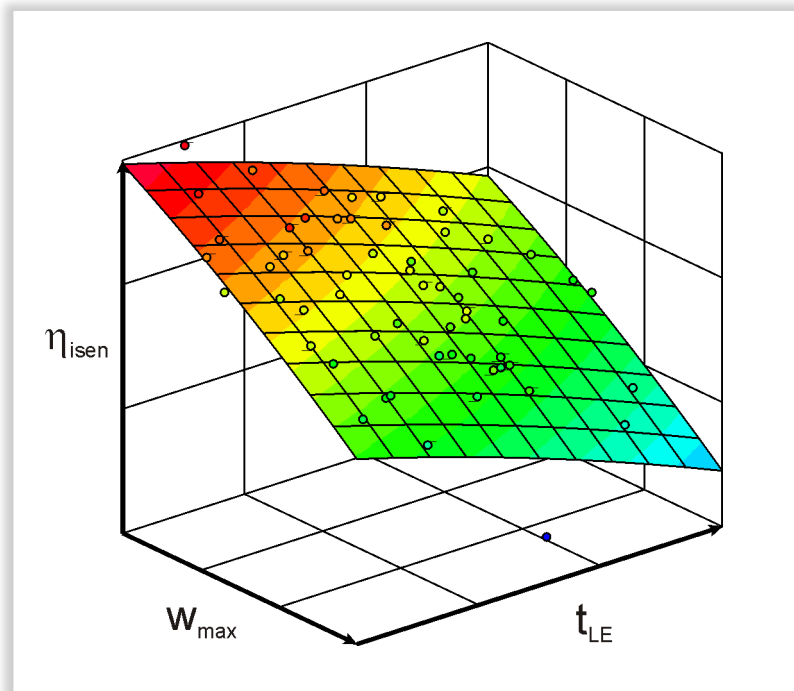




Result quantity: Isentropic efficiency of stage 3

Use of response surface methods:

$$\tilde{y}_i = c_0 + c_1 b_{1,i} + c_2 b_{2,i} + c_3 b_{1,i}^2 + c_4 b_{2,i}^2$$



Calculation of the
Coefficient of Importance:

$$CoI_{i,j} = R_i^2 - R_{i,j}^2$$

Evaluation of response surface with
Monte Carlo cross-validation.

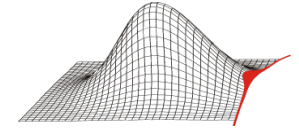
[Beschorner et al., 2014]

Polynomial of second order without
mixed terms delivers:

$$R_{mccv,\eta_{isen}} = 0.962$$

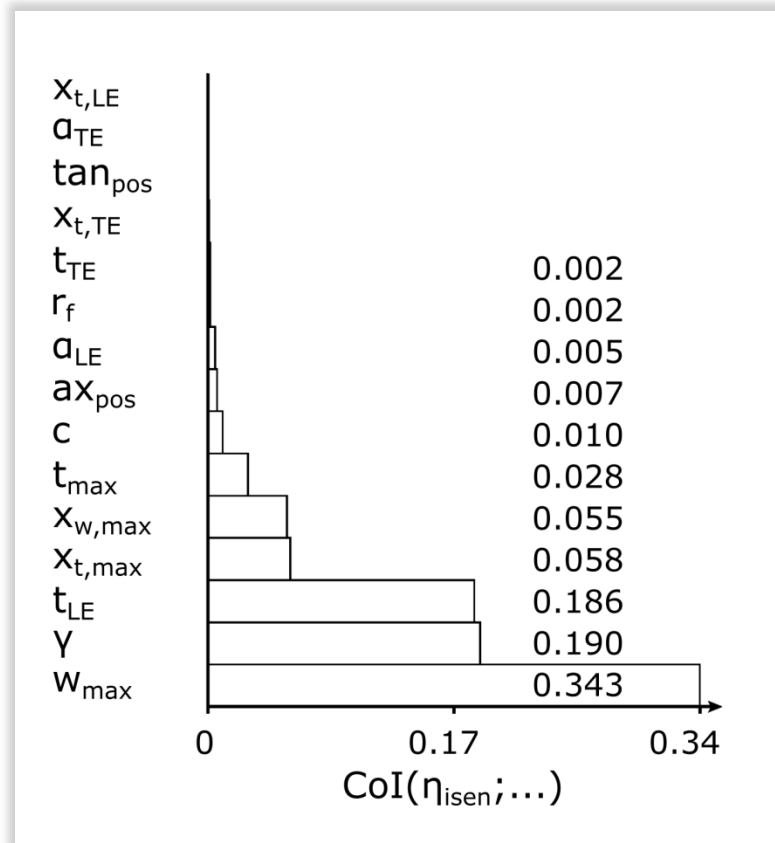
$$R_{\eta_{isen}} = 0.994$$

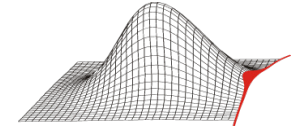
5.1.5 Sensitivity analysis



Result quantity: Isentropic efficiency of stage 3

Use of response surface methods:



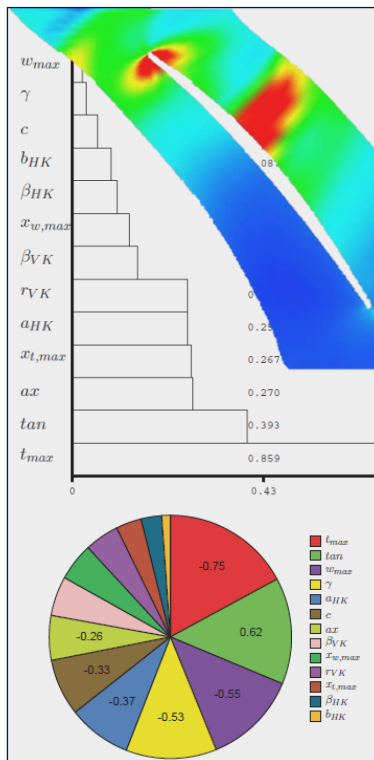


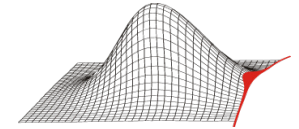
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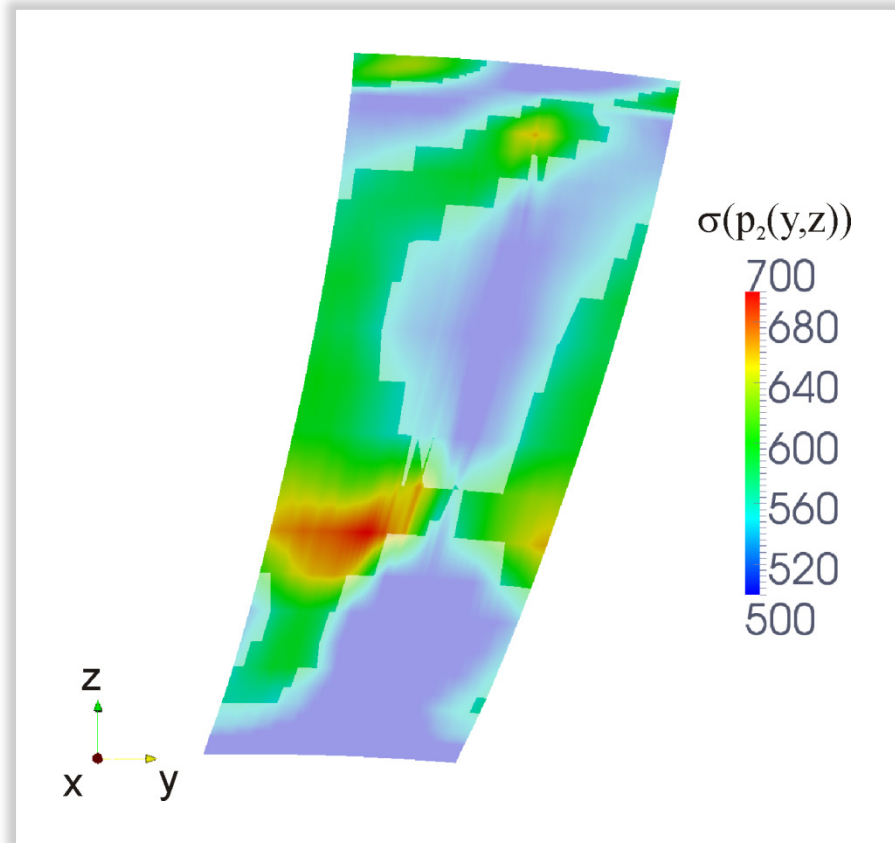
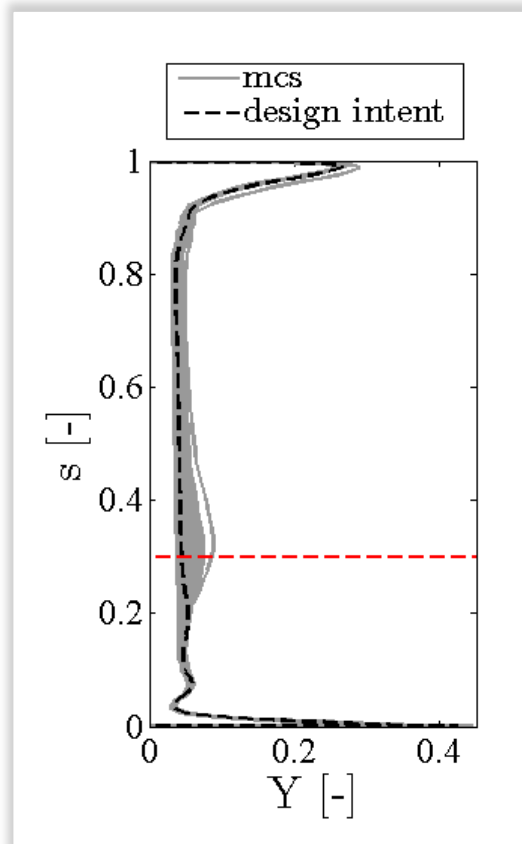
5.2 UQ applications

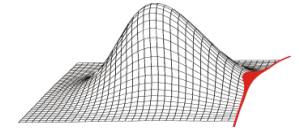




Result quantity: Loss of rotor 3

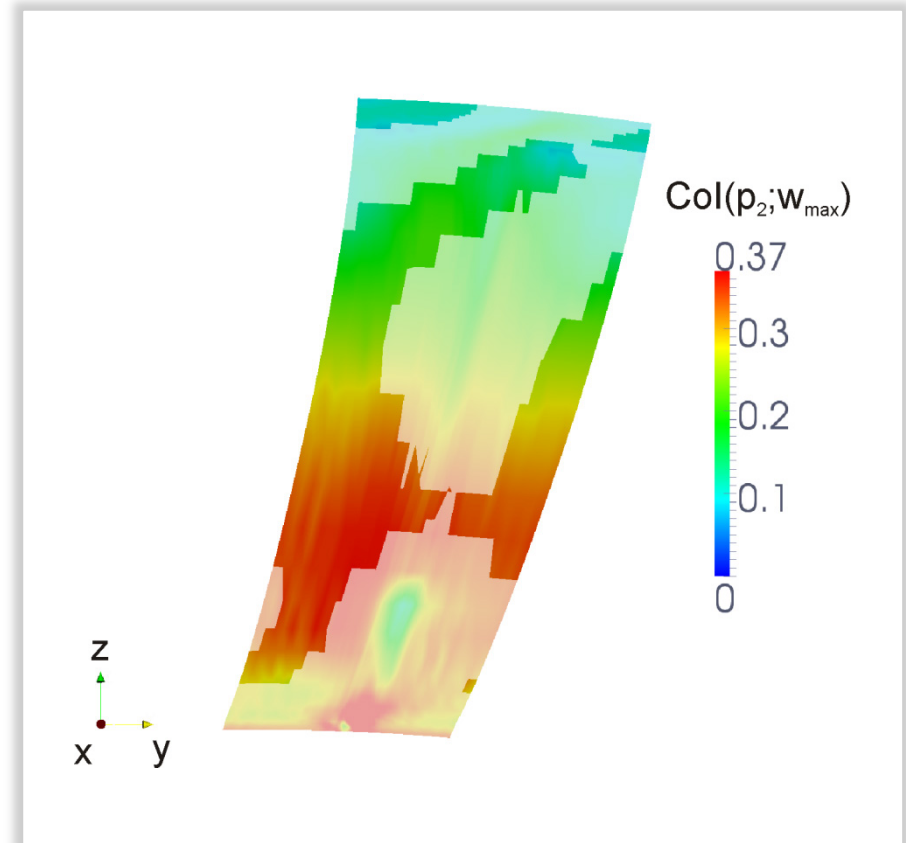
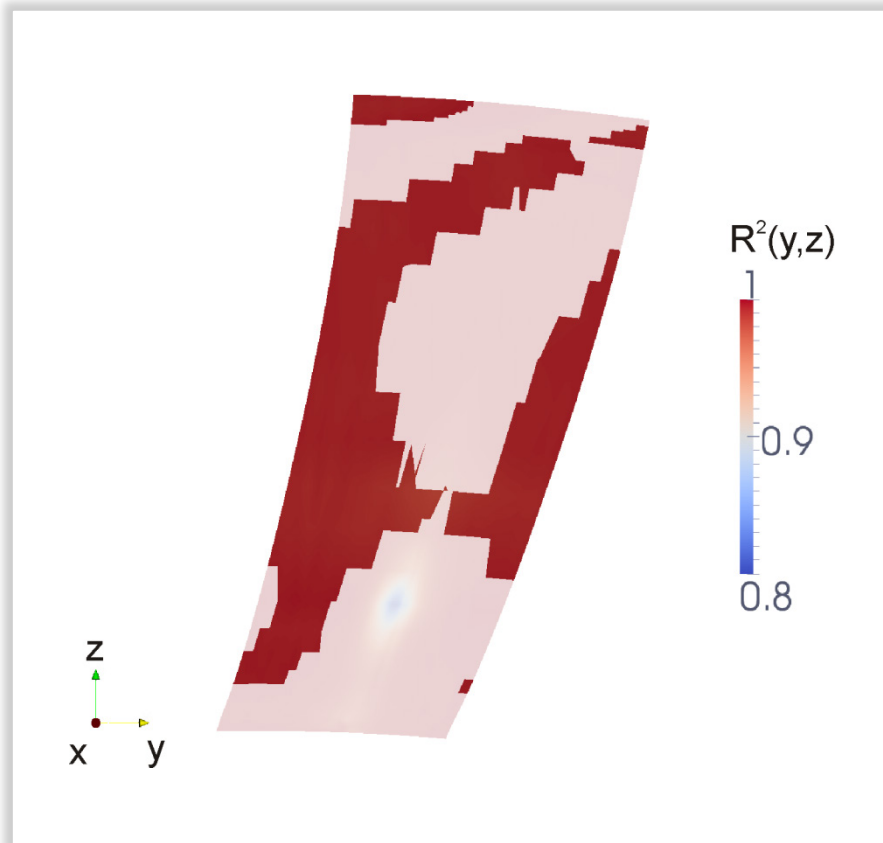
$$\gamma = \frac{p_{2,isen} - p_2}{p_{t,1} - p_1}$$

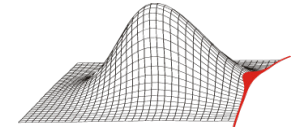




Result quantity: Loss of rotor 3

Use of response surface methods: polynomial of 2nd order without mixed terms



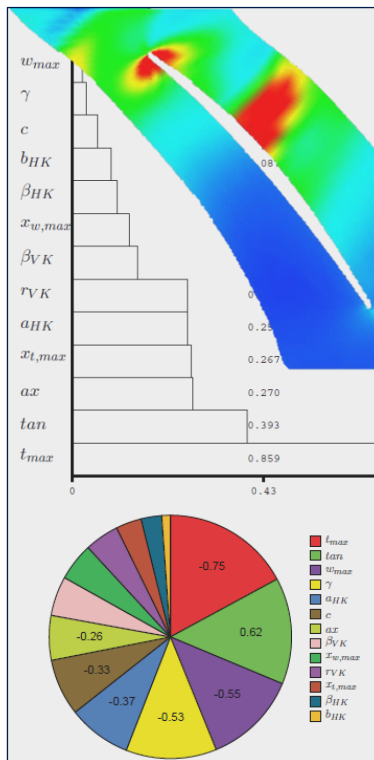


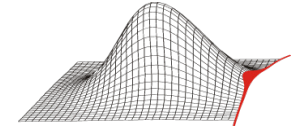
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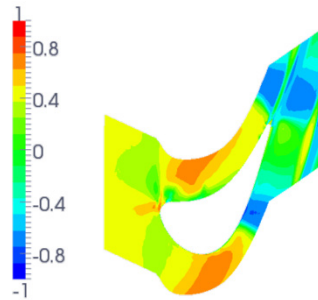
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5.2 UQ applications



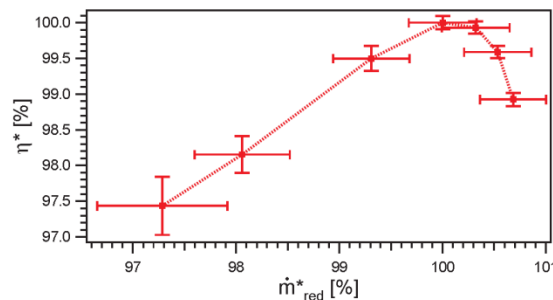


- Probabilistic CFD Analysis of High Pressure Turbine Blades considering real geometric Effects and non-axisymmetric assembling [Scharfenstein et al., 06/2013, Scharfenstein et al., 10/2013]



correlation of relative Mach number and stagger angle
[Scharfenstein et al., 10/2013]

- Probabilistic HCF-investigation of compressor blades considering manufacturing tolerances [Heinze et al., 2009, Heinze, 2015]
- Probabilistic CFD simulation of a high-pressure compressor considering manufacturing tolerances [Lange et al., 2010, Lange et al., 01/2012, Lange et al., 11/2012]



Normalized isentropic efficiency vs.
normalized reduced mass flow
[Lange et al., 11/2012]